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Comments on finite-time stability of time-delay systems[☆]

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ABSTRACT

Recently proposed conditions on finite-time stability in time-delay systems are revisited and it is shown that they are incorrect. General comments on possibility of finite-time convergence in time-delay systems and a necessary condition are given.

1. Introduction

The time-delay dynamical systems attract a lot of attention in different areas of practice (Chiasson & Loiseau, 2007; Erneux, 2009). Analysis of stability in these systems is complicated (Richard, 2003), the most of stability conditions deal with linear time-delay models, where the necessary and sufficient conditions have been obtained for some special cases (Gu, Kharitonov, & Chen, 2003; Hale, 1977; Kolmanovsky & Nosov, 1986). Prior to N.N. Krasovskii's papers on Lyapunov functionals and B.S. Razumikhin's papers on Lyapunov functions, L.E. El'sgol'ts (see El'sgol'ts & Norkin, 1973, and references therein) considered the stability problem of the solution $x(t) \equiv 0$ of time-delay systems by proving that the function $t \to V(x(t))$ is decreasing. Here V is some Lyapunov function. He showed that it is only possible in some rare special cases. Therefore, there are two generic methods for stability analysis in time-delay systems: the Lyapunov–Krasovskii approach and the Lyapunov–Razumikhin method. The former one is based on analysis of derivative for a functional, and it provides *qualitative* and *quantitative* estimates on the system convergence. The latter approach is based on derivative analysis of a function and, from the point of view of the convergence rate, it gives mainly a *qualitative* conclusion (stability/instability of a time-delay system can be detected without estimation of the convergence rate, see Theorem 1 below).

It is frequently important to quantify the rate of convergence in the system: exponential, asymptotic, finite-time or fixed-time (see the results obtained for ordinary differential equations in Dorato (2006), Moulay and Perruquetti (2006), Nersesov, Haddad, and Hui (2008), Polyakov (2012) and Roxin (1966)). Frequently, the homogeneity theory is used to evaluate finite-time or fixed-time stability in the delay-free case (Andrieu, Praly, & Astolfi, 2008; Bhat & Bernstein, 2005; Polyakov, 2012): for example, if a system is globally attractive and homogeneous of negative degree, then it is finite-time stable. There is a recent interest to analysis of finite-time stability behavior for time-delay systems (Karafyllis, 2006; Moulay, Dambrine, Yeganefar, & Perruquetti, 2008; Yang & Wang, 2012, 2013). The paper Karafyllis (2006) proposes design of a control, which implicitly contains some prediction mechanisms and time-varying gains in order to compensate the delay influence on the system dynamics and guarantee a kind of finite-time stability for the closed-loop system. The main result of Moulay et al. (2008) is given in Proposition 2: in order to establish finite-time

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stability for a functional differential equation it is necessary to find a Lyapunov–Krasovskii functional $V(\phi)$ whose derivative is upper bounded by a certain negative function of the functional $V(\phi)$ itself (it is a more restrictive condition than in the conventional Lyapunov–Krasovskii approach, where a function of $|\phi(0)|$ is required). Yang and Wang (2012, 2013) base their study on the Lyapunov–Razumikhin approach, they claim improvements over earlier results.

In this note, we argue that some key results in Yang and Wang (2012, 2013) are incorrect as stated, and we provide new insight on the features of finite-time stability for time-delay systems. The El'sgol'ts' arguments are recalled.

The outline of this work is as follows. The preliminary definitions and finite-time stability conditions for time-delay systems are given in Section 2. A counterexample to a key result in Yang and Wang (2012, 2013) is presented and discussed in Section 3. A necessary condition for finite-time stability for a class of time-delay systems and some supplementary comments are provided in Section 4. The result of Moulay et al. (2008) is quoted in concluding Section 5.

2. Preliminaries

Consider an autonomous functional differential equation of the retarded type (Kolmanovsky & Nosov, 1986):

$$dx(t)/dt = f(x_t), \quad t \geq 0, \tag{1}$$

where $x \in \mathbb{R}^n$ and $x_t \in C_{[-\tau,0]}$ is the state function, $x_t(s) = x(t+s), -\tau \leq s \leq 0$ (we denote by $C_{[-\tau,0]}, 0 < \tau < +\infty$ the Banach space of continuous functions $\phi : [-\tau, 0] \to \mathbb{R}^n$ with the uniform norm $\|\phi\| = \sup_{-\tau \leq \varsigma \leq 0} |\phi(\varsigma)|$, where $|\cdot|$ is the standard Euclidean norm); $f : C_{[-\tau,0]} \to \mathbb{R}^n$ is a continuous function, $f(0) = 0$. The representation (1) includes pointwise or distributed retarded systems. We assume that (1) has a solution $x(t, x_0)$ satisfying the initial condition $x_0 \in C_{[-\tau,0]}$, which is defined on some finite time interval $[-\tau, T)$ (we will use the notation $x(t)$ to reference $x(t, x_0)$ if the origin of x_0 is clear from the context).

The upper right-hand Dini derivative of a locally Lipschitz continuous functional $V : C_{[-\tau,0]} \to \mathbb{R}_+$ along the system (1) solutions is defined as follows for any $\phi \in C_{[-\tau,0]}$:

$$D^+V(\phi) = \lim_{h \to 0^+} \sup \frac{1}{h}[V(\phi_h) - V(\phi)],$$

where $\phi_h \in C_{[-\tau,0]}$ for $0 < h < \tau$ is given by

$$\phi_h = \begin{cases} \phi(\theta + h), & \theta \in [-\tau, -h) \\ \phi(0) + f(\phi)(\theta + h), & \theta \in [-h, 0]. \end{cases}$$

For a locally Lipschitz continuous function $V : \mathbb{R}^n \to \mathbb{R}_+$ the upper directional Dini derivative is defined as follows:

$$D^+V[x_t(0)]f(x_t) = \lim_{h \to 0^+} \sup \frac{V[x_t(0) + hf(x_t)] - V[x_t(0)]}{h}.$$

A continuous function $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to the class $\mathcal{K}$ if it is strictly increasing and $\sigma(0) = 0$; it belongs to the class $\mathcal{K}_\infty$ if it is also radially unbounded.

2.1. Stability definitions

Let Ω be an open subset of $C_{[-\tau,0]}$ containing 0.

Definition 1 (*Moulay et al., 2008*)**.** The equilibrium $x = 0$ of (1) is said to be

(a) stable if there is $\sigma \in \mathcal{K}$ such that for any $x_0 \in \Omega$, $|x(t, x_0)| \leq \sigma(\|x_0\|)$ for all $t \geq 0$;

(b) asymptotically stable if it is stable and $\lim_{t \to +\infty} |x(t, x_0)| = 0$ for any $x_0 \in \Omega$;

(c) finite-time stable if it is stable and for any $x_0 \in \Omega$ there exists $0 \leq T^{x_0} < +\infty$ such that $x(t, x_0) = 0$ for all $t \geq T^{x_0}$. The functional $T_0(x_0) = \inf\{T^{x_0} \geq 0 : x(t, x_0) = 0 \ \forall t \geq T^{x_0}\}$ is called the settling time of the system (1).

If $\Omega = C_{[-\tau,0]}$, then the origin is called *globally* stable/asymptotically stable/finite-time stable.

For the forthcoming analysis we will need Lyapunov–Razumikhin theorem, which is given below (we have adapted to our case the formulation of Gu et al. (2003), where time-dependent functional differential equations are considered).

Theorem 1 (*Gu et al., 2003*)**.** *Let $\alpha_1, \alpha_2 \in \mathcal{K}$ and $\eta : \mathbb{R}_+ \to \mathbb{R}_+$ be a continuous nondecreasing function. If there exists a Lipschitz continuous function $V : \mathbb{R}^n \to \mathbb{R}$ such that*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \quad \forall x \in \mathbb{R}^n$$

and the derivative of V along a solution $x(t)$ of (1) satisfies

$$D^+V[x(t)]f(x_t) \leq -\eta(|x(t)|) \quad \text{if } V[x(t+\theta)] \leq V[x(t)] \tag{2}$$

for $\theta \in [-\tau, 0]$, then (1) is stable at the origin.

If, in addition, $\eta \in \mathcal{K}$ and there exists a continuous nondecreasing function $p(s) > s$ for $s > 0$ such that the condition (2) is strengthened to

$$D^+V[x(t)]f(x_t) \leq -\eta(|x(t)|) \quad \text{if } V[x(t+\theta)] \leq p\{V[x(t)]\}$$

for $\theta \in [-\tau, 0]$, then (1) is asymptotically stable at the origin.

If in addition $\alpha_1 \in \mathcal{K}_\infty$, then (1) is globally asymptotically stable.

2.2. Sufficient conditions of finite-time stability in time-delay systems

The following is stated in Yang and Wang (2013) as Lemma 1 (an extension of the Lyapunov–Razumikhin method for analysis of finite-time stability in (1)):

"*Consider the system (1) with $f(\phi) = F(\phi(0), \phi(-\tau)), \phi \in \Omega, F(0, 0) = 0$ and uniqueness of the solution in forward time. If there exist real numbers $\beta > 1, k > 0$, a Class-$\mathcal{K}$ function σ and a differentiable Lyapunov function, $V(x)$, of the system (1) such that*

$$\sigma(|x|) \leq V(x),$$

$$\dot{V} \leq -kV^{\beta^{-1}}(x), \quad x \in \Omega \tag{3}$$

hold along the trajectory of the system whenever $V[x(t+\theta)] \leq V[x(t)]$ for $\theta \in [-\tau, 0]$, then the system (1) is finite-time stable. If $\Omega = \mathbb{R}^n$ and σ is a Class-$\mathcal{K}_\infty$ function, then the origin is a globally finite-time stable equilibrium of the system (1). Furthermore, the settling time of the system (1) with respect to the initial condition $\phi \in C_\delta$ satisfies

$$T_0(\phi) \leq \frac{\beta}{k(\beta - 1)} V^{\frac{\beta-1}{\beta}}[\phi]$$

for all $t \geq 0$."

For completeness we are going to give the "proof" of this lemma from Yang and Wang (2013):

"Since $V(x)$ is a Lyapunov function for the system (1), applying Razumikhin Theorem (Gu et al., 2003), it is easy to know that the system (1) is asymptotically stable under the conditions of the lemma. Next, we need to prove that $T_0(\phi) < +\infty$. Based on Condition for $\dot{V}$, one can obtain $\int_{V(\phi)}^{0} \frac{dz}{z^{1/\beta}} \leq -k \int_0^t d\tau$, from which it follows that $T_0(\phi) \leq \frac{\beta}{k(\beta-1)} V^{\frac{\beta-1}{\beta}}[\phi]$ for all $t \geq 0$. Thus, the proof is completed".

In Yang and Wang (2012) the same conclusion is obtained for the time-varying system (1) using a similar argumentation.

3. Comments on Yang and Wang (2013)

We claim that Lemma 1 in Yang and Wang (2013) is incorrect as stated. Indeed, as we can conclude from the result of Theorem 1, Lemma 1 in Yang and Wang (2013) is based on the condition (2), which allows only stability (non asymptotic) to be concluded. However, even if we would ask the inequality (3) to be satisfied whenever $V[\phi(\theta)] \le p\{V[\phi(0)]\}$ for all $\theta \in [-\tau, 0]$, where $p : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuous nondecreasing function such that $p(s) > s$ for all $s > 0$, then only asymptotic stability can be proven following Theorem 1. The claim about a finite-time convergence stays wrong since for a solution $x(t, x_0)$ with initial condition $x_0 \in \Omega$ the Lyapunov–Razumikhin condition (3) defines the rate of convergence of V only for the set of time instants

$$\mathrm{T}^+_{x_0} = \left\{t \in \mathbb{R}_+ : \sup_{\theta\in[-\tau,0]} V[x(t+\theta, x_0)] \le p\{V[x(t, x_0)]\}\right\}$$

while for $t \in \mathrm{T}^-_{x_0}$ with

$$\mathrm{T}^-_{x_0} = \left\{t \in \mathbb{R}_+ : \sup_{\theta\in[-\tau,0]} V[x(t+\theta, x_0)] > p\{V[x(t, x_0)]\}\right\}$$

there is no restriction on convergence rate of V. It is exactly the arguments used in the "proof" of Lemma 1 from Yang and Wang (2013) given above: a finite-time rate of convergence is established for $t \in \mathrm{T}^+_{x_0}$ only, which is clearly not sufficient since, for an illustration, if $V[x(t)]$ is strictly decreasing, then $\mathrm{T}^-_{x_0} = [0, +\infty)$.

For another illustration consider the following counterexample:

$$\dot{x}(t) = -|2x(t) - x(t-\tau)|^{0.5}\mathrm{sign}[2x(t) - x(t-\tau)]. \qquad (4)$$

Using $V(x) = 0.5x^2$ we obtain

$$\dot{V} \le -\sqrt{2-\sqrt{2}}|x|^{1.5} = -kV^{\beta^{-1}} \quad \text{if } V[x(t-\tau)] \le 2V[x(t)]$$

for $\beta = 4/3$ and $k = \sqrt{2-\sqrt{2}}\,2^{3/4}$. Thus, by Theorem 1 the system is globally asymptotically stable, and from Yang and Wang (2012, 2013) since all conditions of Lemma 1 (Yang & Wang, 2013) are also satisfied, one would conclude that the system is finite-time stable. Consequently, for the initial conditions $x_0 \in C_{[-\tau,0]}$ such that $|x_0(0)| \le 1$ the settling time function would possess the estimate:

$$T_0(x_0) \le 3.$$

Take $\tau > 3$ and for any $\delta \in [-1, 1] \setminus \{0\}$ select initial conditions

$$x_0(s) = \begin{cases} 2\delta, & s \in [-\tau, -\tau+3] \\ \dfrac{\delta}{3-\tau}s + \delta, & s \in (-\tau+3, 0]. \end{cases}$$

Obviously, $\dot{x}(t, x_0) = 0$ and $x(t, x_0) = x_0(0) = \delta \ne 0$ for all $t \in [0, T_0(x_0)]$, therefore *the given settling time estimate is invalid*. In addition, the results of the system simulation for $x_0(s) = 1$ for $s \in [-\tau, 0]$ are shown in Fig. 1 for $\tau = 4$ and the simulation step $h = 10^{-5}$. Clearly the system is not finite-time stable and the settling time estimate is wrong.

A peculiarity of the Lyapunov–Razumikhin conditions of stability is that they are delay-independent (see Theorem 1), thus the estimate on the settling time $T_0(x_0)$ obtained in Lemma 1 of Yang and Wang (2013) is also delay independent. Consequently, it is possible to select delay value τ for a given initial condition $x_0 \in C_{[-\tau,0]}$ such that $T_0(x_0) < \tau$, as it has been performed in the counterexample above. Obviously, convergence to zero independently on the part of initial conditions $x_0(t)$ with $t \in [T_0(x_0) - \tau, 0]$ is possible only under special restrictions on f in (1), which we are going to analyze in the next section.

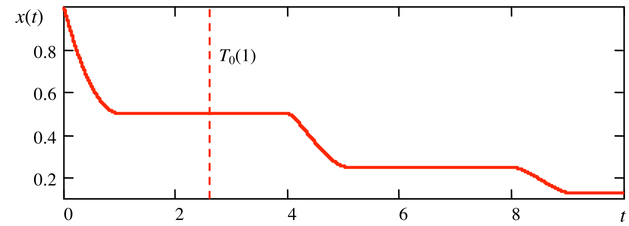

Fig. 1. A trajectory of the counterexample (4).

4. About necessary conditions for finite-time stability in time-delay systems

Assume that the system (1) is finite-time stable for some $\tau > 0$ and the settling-time functional is continuous and $T_0(0) = 0$. Then there is $x_0 \in C_{[-\tau,0]}$ such that $T_0(x_0) \le \tau$ and at the instant $T_0(x_0)$ the right-hand side of (1) is still dependent on the initial conditions x_0. Thus, without additional assumptions on the right-hand side $f(x_t)$ and its dependence on $x_t(0)$, or without skipping the continuity requirement of $T_0(x_0)$, an existence of finite-time stability phenomenon for time-delay systems is questionable.

In the remainder of this section we will consider

$$f(\phi) = F(\phi(0), \phi(-\tau))$$

for all $\phi \in \Omega$, where $F : \mathbb{R}^{2n} \to \mathbb{R}^n$ is a continuous function. Our conjecture is that the condition $F(0, z) = 0$ for any $z \in \mathbb{R}^n$ is "necessary" for a finite-time convergence of (1) from Ω to the origin.

Remark 1. We would like to stress that requirement on continuity of F is crucial for the consideration below. Indeed in discontinuous time-delay systems the finite-time stability may be easily observed without such a restriction on F. For example, it is a simple exercise to check that the system

$$\dot{x}(t) = -(1 + |x(t-\tau)|)\mathrm{sign}[x(t)] + x(t-\tau)$$

is globally finite-time stable (take $V(x) = 0.5x^2$, $\dot{V} \le -\sqrt{2V}$ and apply a variant of Proposition 2). In this discontinuous case $0 \in F(0, z)$ for any $z \in \mathbb{R}^n$.

Note that by the definition of finite-time convergence $x(t, x_0) = 0$ for all $t \ge T_0(x_0)$ and by the definition of $T_0(x_0)$ there is a non-empty set of time instants

$$\mathcal{T}_{x_0} = \{t \in [T_0(x_0) - \tau, T_0(x_0)] : x(t, x_0) \ne 0\}.$$

Proposition 1. *Let (1) be finite-time convergent in Ω, then*

$$\forall t \in \mathcal{T}_{x_0} : F[0, x(t, x_0)] = 0 \qquad (5)$$

for any $x_0 \in \Omega$.

Proof. Take $x_0 \in \Omega$ and the corresponding settling time $T_0(x_0)$. Assume that the necessary condition (5) is not satisfied, its negation implies that for some $x_0 \in \Omega$ there exists $t' \in \mathcal{T}_{x_0}$ such that $F(0, x(t', x_0)) \ne 0$ (the measure of $\mathcal{T}_{x_0}$ is not zero since $x(t)$ and F both are continuous), then $\dot{x}(t' + \tau) \ne 0$ and $x(t, x_0) \ne 0$ for some $t \ge T_0(x_0)$ that is a contradiction. □

A simple, but not equivalent, way to check this condition in practice is to verify that

$$F(0, z) = 0 \tag{6}$$

for any $z \in \mathbb{R}^n$. In Kolmanovskii and Myshkis (1999), a similar "necessary" condition has been indicated for the El'sgol'ts approach (El'sgol'ts & Norkin, 1973). Indeed, consider the case $n = 1$, take a Lyapunov function $V(x) = 0.5x^2$, which is a reasonable choice for the scalar case. We have $\dot{V}(t) = x(t)F[x(t), x(t-\tau)]$, if the inequality $\dot{V}(t) \le 0$ is satisfied around the origin, then we necessarily obtain (6).

In Yang and Wang (2012, 2013) the restriction $F(0, 0) = 0$ has been imposed that is not sufficient.

5. Discussion

Finite-time stability can be presented in time-delay systems, but only under rather strong restrictions on the right-hand side of the system. The most general sufficient conditions are given in Moulay et al. (2008):

Proposition 2 (*Moulay et al., 2008*)**.** *Let system (1) have unique solutions in forward time. If there exist a continuous functional $V : \Omega \to \mathbb{R}_+$, $\epsilon > 0$ and two functions α, r of class $\mathcal{K}$ such that $\dot{z} = -r(z)$ has a flow, $\int_0^\epsilon \frac{dz}{r(z)} < +\infty$ and for all $\phi \in \Omega$*

$$\alpha(|\phi(0)|) \le V(\phi), \qquad D^+V(\phi) \le -r[V(\phi)],$$

then the system (1) is finite-time stable with a settling time functional $T_0(\phi)$ satisfying the inequality:

$$T_0(\phi) \le \int_0^{V(\phi)} \frac{dz}{r(z)}.$$

The following example has been given in Moulay et al. (2008) for any $0 < \alpha < 1$:

$$\dot{x}(t) = -|x(t)|^\alpha \operatorname{sign}[x(t)]\{1 + x(t-\tau)^2\}. \tag{7}$$

Using the Lyapunov functional $V(\phi) = 0.5\phi(0)^2$ with $\dot{V}(\phi) \le -2^{\frac{1+\alpha}{2}} V^{\frac{1+\alpha}{2}}(\phi)$ the finite-time stability has been established. Note that for this example the "necessary" condition (6) is satisfied.

The system (7) is an example, where the El'sgol'ts' arguments can be applied ($V(\phi) = 0.5\phi(0)^2$ is in fact a Lyapunov function). Note that the result of Lemma 1 in Yang and Wang (2013) without the Razumikhin condition ("whenever...") is correct and in this case it is also a special case of Proposition 2. In such a reformulation the result extends (El'sgol'ts & Norkin, 1973) to finite-time stability, as well as the finite-time stability results of Roxin (1966) to time-delay systems.

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