Delayed Disturbance Attenuation via Measurement Noise Estimation

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***Abstract*—A novel control law is proposed to attenuate the influence of bounded disturbances and measurement noises for plants with vector output and sector-bounded nonlinearities. The control law is based on the estimation of measurement noises. Differently from the existing results, the ultimate bound of the closed-loop system depends only on one component of the noise vector (as well as on the disturbance). The proposed control law is extended to systems with uncertain input and output delays. The input-to-state stability conditions are given in terms of matrix inequalities. The efficiency and advantages of the results over the existing methods are demonstrated by numerical examples.**

***Index Terms*—Disturbance attenuation, Lyapunov–Krasovskii functional, time delay.**

I. Introduction

For practical implementation of control methods, it is important to take into account disturbances and measurement noises [1]–[3]. The following control methods are efficient in the presence of disturbances in the dynamics: methods based on high-gain observers [4], [5], sliding-mode observers [6], interval observers [7], [8], and control laws based on unknown input disturbances [9]. Under measurement noises, these methods may lose efficiency because the values of the ultimate bound and of the control law signal can be large. Some of these methods are studied under a special class of measurement noises; usually, it is high-frequency bounded noises (see, e.g., [4], [5], [10], and [11]). Differently from [4]–[11], the following methods are efficient in the presence of arbitrary bounded disturbances and measurement noises: H_∞-control [12], [13], invariant ellipsoid method [14], [15], and method of rejection of sinusoidal disturbances [16]–[18]. Although these methods reduce the influence of disturbances and noises, nevertheless, the state bounds depend on magnitudes of these uncertain signals.

The objective of this article is to design a method, which is efficient in the presence of both disturbances in the dynamics and noises in measurement.

Recently, a new control method has been suggested in [19] and [20]. This method decreases the resulting ultimate bound of the closed-loop system. Differently from [4]–[18], the above bound depends only on one component of measurement noise vector (as well as on the disturbance). Thus, the control law of [19] and [20] may reject noises with large magnitudes. However, the disturbance and noises considered in [19] and [20] are assumed to be differentiable and matched, whereas the influence of input and output delays is not taken into account.

Many papers have studied effects of input and output delays on control without compensation of disturbances and noises [21]–[33]. The control under input delay and compensation of disturbances are considered in [18] and [34]–[36], where the noises have not been compensated.

In this article, we extend [19] and [20] to the cases of:

1) nonsmooth and mismatched disturbances and nonsmooth noises;
2) unknown time-varying input and output time delays;
3) non-Hurwitz matrix and unknown parameters in the plant model.

The rest of this article is organized as follows. Problem formulation and main results for delay-free case are presented in Section II. An extension to the time-delay case is given in Section III. Section IV illustrates an efficiency of the proposed method and its advantages compared with the existing methods. Section V concludes this article.

Notations: Throughout this article, the superscript T stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space with vector norm $|\cdot|$; $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices with the norm $\|\cdot\|$; the notation $P>0$ for $P\in\mathbb{R}^{n\times n}$ means that P is symmetric and positive definite; $\lambda_{\min}(P)$ stands for the minimum eigenvalue of the matrix P; $E_i=[0,\ldots,0,1,0,\ldots,0]^{\mathrm{T}}$ is a vector, where the ith component is equal to 1 and other components are equal to 0; $\tilde{E}$ is the $(n-1)\times n$ matrix obtained from the identity matrix of order n by eliminating the ith row, i.e., $\tilde{E}^{\mathrm{T}}=[E_1,\ldots,E_{i-1},E_{i+1},\ldots,E_n]$; I is the identity matrix of corresponding order; the notation $\Theta(\chi)$ for $\chi\in\mathbb{R}$ means that $\lim_{\chi\to 0}\frac{\Theta(\chi)}{\chi}=C$, where C is a constant; $\mathrm{diag}\{\cdot\}$ denotes a block diagonal matrix; and "$*$" denotes a symmetrical block of a symmetric matrix.

II. Delay-Free Systems

A. Problem Formulation

Let a plant model be described by the following equations:

$$\dot{x}(t)=Ax(t)+Bu(t)+D\phi(x(t),t)+Gf(t) \quad (1)$$

$$y(t)=x(t)+\xi(t) \quad (2)$$

where $t\geq 0$, $x(t)\in\mathbb{R}^n$ is the unmeasured state vector, $n\geq 2$, $u(t)\in\mathbb{R}^m$ is the control signal, $y(t)\in\mathbb{R}^n$ is the measured signal, $f(t)\in\mathbb{R}^v$ is the disturbance, and $\lim_{t\to\infty}\sup_{t\geq 0}|f(t)|\leq\kappa_1,\kappa_1>0$, $\phi(x(t),t)\in\mathbb{R}^l$ is the unknown nonlinear function satisfying the condition

$$|\phi(x(t),t)|\leq\chi|x(t)|,\quad \chi>0 \quad (3)$$

$\xi(t)=[\xi_1(t),\ldots,\xi_n(t)]^{\mathrm{T}}$ is the measurement noise and $\kappa_2^j=\lim_{t\to\infty}\sup_{t\geq 0}|\xi_j(t)|$, $\kappa_2^j>0$, $j=1,..,n$. The matrices $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $D\in\mathbb{R}^{n\times l}$, and $G\in\mathbb{R}^{n\times v}$ and the constants κ_1, κ_2^j, $j=1,\ldots,n$, and χ are known.

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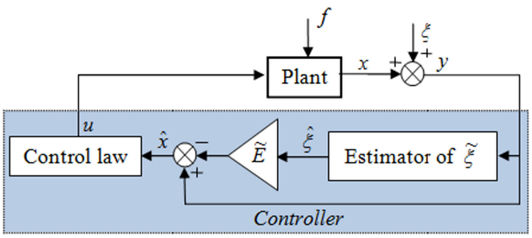

Fig. 1. Control scheme structure.

Assumption 1: The pair (A, B) is stabilizable.

Some additional assumptions will be given in Section II-C.

Our objective is to design the controller that guarantees the input-to-state stability (ISS) of (1) leading to ultimate bound

$$\lim_{t\to\infty} \sup_{t\geq 0} |x(t)| < \delta. \tag{4}$$

Similar to [19] and [20], in this article, $\delta = \Theta(\kappa_1^2 + (\kappa_2^i)^2)$, where the ith component is due to the designer choice. This is different from the existing results [12]–[18], [27], where $\delta = \Theta(\kappa_1^2 + (\kappa_2^i)^2 + \sum_{j=1, j\neq i}^n (\kappa_2^j)^2)$. Thus, if the values of $\lim_{t\to\infty} \sup_{t\geq 0} |\xi_j(t)|$, $j \in \{1, \ldots, i-1, i+1, \ldots, n\}$ are sufficiently large, then δ given in [12]–[18] and [27] is larger than δ given by the proposed control law and the results from [19] and [20]. However, differently from [19] and [20], in this article, the signals $f(t)$ and $\xi(t)$ may be not differentiable, as well as the nonlinear function $\phi(x(t), t)$ and the disturbance $f(t)$ are mismatched *w.r.t.* the control signal $u(t)$. Sufficient condition for our objective is given below in Theorem 1.

Let us briefly describe the design method. Since plant (1), (2) contains disturbances and noises, at least two measurement channels are required for getting information about these signals (thus, $n \geq 2$). Let us consider the ith equation in (1) and (2) for getting information about the disturbance $f(t)$. The other equations are used for getting information about noise. The output signal $y(t)$ contains noises; therefore, we design an algorithm that allows us to estimate the part of noise vector without the ith component, i.e., $\tilde{\xi}(t) = [\xi_1(t), \ldots, \xi_{i-1}(t), \xi_{i+1}(t), \ldots, \xi_n(t)]^{\mathrm{T}}$ (see "Estimator of $\tilde{\xi}$" in Fig. 2, where $\hat{\xi}(t)$ is the estimate of $\tilde{\xi}(t)$). Thus, having information about $\tilde{\xi}(t)$, the state vector estimate $\hat{x}$ is constructed and used for design the control law (see Fig. 1), reducing the influence of $f(t)$.

Remark 1: Assume that there exists the lth component of the vector ξ such that $\lim_{t\to\infty} \sup_{t\geq 0} |\xi_l(t)| < \lim_{t\to\infty} \sup_{t\geq 0} |\xi_k(t)|$ for $k \in \{1, \ldots, l-1, l+1, \ldots, n\}$. Therefore, in this case, we will use the matrix E_l (the lth component is equal to 1 and other components are equal to 0; see Notations) because with this choice, the ultimate bound δ in (4) will take the smallest value. Otherwise, E_i is chosen arbitrary. The matrix $\tilde{E}$ depends on choice of E_i (see Notations).

B. Control Law Design

Introduce the control law (see Fig. 1) in the form

$$u(t) = K\hat{x}(t) \tag{5}$$

where the matrix $K \in \mathbb{R}^{m\times n}$ is chosen such that the closed-loop system is ISS, the signal $\hat{x}(t)$ is the estimate of the state vector $x(t)$ obtained by

$$\hat{x}(t) = y(t) - \tilde{E}^{\mathrm{T}}\hat{\xi}(t). \tag{6}$$

Here, $\hat{\xi}(t)$ is the estimate of $\tilde{\xi}(t) = [\xi_1(t), \ldots, \xi_{i-1}(t), \xi_{i+1}(t), \ldots, \xi_n(t)]^{\mathrm{T}}$.

Furthermore, we design the algorithm for estimation of $\tilde{\xi}(t)$. Using relation

$$\xi(t) = \sum_{j=1}^n E_j \xi_j(t) = \tilde{E}^{\mathrm{T}}\tilde{\xi}(t) + E_i\xi_i(t)$$

rewrite $y(t)$ given by (2) as follows:

$$y(t) = x(t) + \tilde{E}^{\mathrm{T}}\tilde{\xi}(t) + E_i\xi_i(t). \tag{7}$$

Now, eliminate the ith equation in (7) and rewrite result w.r.t. $\tilde{\xi}(t)$. To this end, premultiplying (7) by $\tilde{E}$ and setting $\tilde{y}(t) = \tilde{E}^{\mathrm{T}}y(t)$, we have

$$\tilde{\xi}(t) = \tilde{y}(t) - \tilde{E}x(t). \tag{8}$$

Integrating (1) in t and employing (8), we obtain

$$\tilde{\xi}(t) = \tilde{y}(t) - \tilde{E}\int_0^t [Ax(s) + Bu(s) + D\phi(x(s), s) + Gf(s)]ds. \tag{9}$$

Denoting

$$\tilde{A} = \tilde{E}A\tilde{E}^{\mathrm{T}},\ \tilde{A}_1 = \tilde{E}A,\ \tilde{A}_2 = \tilde{E}AE_i$$
$$\tilde{B} = \tilde{E}B,\ \ \tilde{D} = \tilde{E}D,\ \tilde{G} = \tilde{E}G \tag{10}$$

and substituting $x(t)$ from (7) into (9), we have

$$\tilde{\xi}(t) = \int_0^t [\tilde{A}\tilde{\xi}(s) - \tilde{A}_1 y(s)]ds + \tilde{y}(t) - \int_0^t [\tilde{B}u(s) + \tilde{D}\phi(x(s), s) + \tilde{G}f(s) - \tilde{A}_2\xi_i(s)]ds. \tag{11}$$

The second row in (11) contains unknown functions $\phi(x(t), t)$, $f(t)$, and $\xi_i(t)$, while the first row in (11) can be used for designing the estimate of $\tilde{\xi}(t)$. Therefore, introduce the estimate of $\tilde{\xi}(t)$ (see "Estimator of $\tilde{\xi}$" in Fig. 1) in the form

$$\hat{\xi}(t) = \int_0^t \left[\tilde{A}\hat{\xi}(s) - \tilde{A}_1 y(s)\right] ds + \tilde{y}(t). \tag{12}$$

As a result, the proposed algorithm is presented by control law (5), (6) and noise estimator (12). In the next section, we derive the closed-loop system and formulate the sufficient condition for ISS.

C. Main Result

Consider the measurement noise estimation error

$$e(t) = \tilde{\xi}(t) - \hat{\xi}(t) \tag{13}$$

and, taking into account (6), rewrite control law (5) as follows:

$$u(t) = K[x(t) + \tilde{E}^{\mathrm{T}}e(t) + E_i\xi_i(t)]. \tag{14}$$

Substituting (14) into (1), we obtain

$$\dot{x}(t) = (A + BK)x(t) + BK\tilde{E}^{\mathrm{T}}e(t) + BKE_i\xi_i(t) + D\phi(x(t), t) + Gf(t). \tag{15}$$

Since (15) contains the variable $e(t)$, it is necessary to obtain dynamics of $e(t)$. Employing (11), (12), and (14), differentiate (13) in t and rewrite result in the form

$$\dot{e}(t) = (\tilde{A} - \tilde{B}K\tilde{E}^{\mathrm{T}})e(t) - \tilde{B}Kx(t) - \tilde{D}\phi(x(t), t) - \tilde{G}f(t) + (\tilde{A}_2 - \tilde{B}KE_i)\xi_i(t). \tag{16}$$

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Combine (15) and (16). To this end, introduce the following vectors and matrices:

$$x_a(t) = \mathrm{col}\{x(t), e(t)\}, \quad \psi(t) = \mathrm{col}\{\xi_i(t), f(t)\}$$

$$A_a = \begin{bmatrix} A+BK & BK\tilde{E}^{\mathrm{T}} \\ -\tilde{B}K & \tilde{A}-\tilde{B}K\tilde{E}^{\mathrm{T}} \end{bmatrix}, \quad G_a = \begin{bmatrix} D \\ -\tilde{D} \end{bmatrix}$$

$$F_a = \begin{bmatrix} BKE_i & G \\ \tilde{A}_2 - \tilde{B}KE_i & -\tilde{G} \end{bmatrix}. \tag{17}$$

Assumption 2: There exists K such that A_a is Hurwitz.

Remark 2: Assumption 2 holds if Assumption 1 is satisfied; the matrices $A+BK$ and $\tilde{A}$ are Hurwitz and $\|\tilde{B}\|$ is small enough.

Employing (17), rewrite (15) and (17) in the form

$$\dot{x}_a(t) = A_a x_a(t) + G_a \phi(x(t),t) + F_a \psi(t). \tag{18}$$

As a result, the closed-loop system (18) depends on $\xi_i(t)$ and $f(t)$ only, while the closed-loop systems in [12]–[18] depend on the whole vector $\xi(t)$ and the disturbance $f(t)$. The following result is thus in order.

Theorem 1: Let Assumptions 1 and 2 hold. Consider the closed-loop system consisting of plant (1), (2) under control law (5), where $\hat{x}(t)$ is defined by (6) and (12). Given a matrix K and a scalar α, if there exist constants $\beta > 0$ and $\tau > 0$ and matrix $P > 0$ that satisfy the following optimization problem:

maximize γ

subject to

$$\Psi_a = \begin{bmatrix} \Psi_{11} & PG_a & PF_a \\ * & -\tau I & 0 \\ * & * & -\beta I \end{bmatrix} < 0 \text{ and } P - \gamma I > 0 \tag{19}$$

where $\Psi_{11} = A_a^{\mathrm{T}} P + PA_a + 2\alpha P + \tau\chi^2\, C^{\mathrm{T}} C$ and χ is given by (3), then the solutions of the closed-loop system are ultimately bounded and (4) holds with $\delta = \sqrt{\frac{\beta[\kappa_1^2 + (\kappa_2^i)^2]}{2\alpha\gamma}}$.

Proof: For the ISS analysis of (18), introduce Lyapunov function in the form

$$V_1 = x_a^{\mathrm{T}} P x_a. \tag{20}$$

Employing (18) and (20), consider the following relation:

$$\dot{V}_1 + 2\alpha V_1 - \beta\psi^{\mathrm{T}}\psi = x_a^{\mathrm{T}}(A_a^{\mathrm{T}} P + PA_a + 2\alpha P)x_a$$
$$+ 2x_a^{\mathrm{T}} PG_a\phi(x(t),t) + 2x_a^{\mathrm{T}} PF_a\psi - \beta\psi^{\mathrm{T}}\psi. \tag{21}$$

Denoting

$$z(t) = col\{x_a(t), \phi(x(t),t), \psi(t)\}$$
$$\Psi = \Psi_a - \tau\mathrm{diag}\{\chi^2\, C^{\mathrm{T}} C, -I, 0\}$$

represent (21) as follows:

$$\dot{V}_1 + 2\alpha V_1 - \beta\psi^{\mathrm{T}}\psi = z^{\mathrm{T}}\Psi z. \tag{22}$$

Taking into account $x(t) = Cx_a(t)$, $C = [I\ 0]$ and $|\phi(x(t),t)| \le \chi|x(t)|$, consider the following estimate $\phi^{\mathrm{T}}(x,t)\phi(x(t),t) \le \chi^2 x_a^{\mathrm{T}}(t) C^{\mathrm{T}} C x_a(t)$ or rewritten in the form:

$$\tau z^{\mathrm{T}}(t)\mathrm{diag}\{\chi^2\, C^{\mathrm{T}} C, -I, 0\} z(t) \ge 0. \tag{23}$$

According to the S-procedure, inequalities (22) and (23) simultaneously hold, if matrix inequality (19) holds. Thus, $z(t)$ is ultimately bounded. Therefore, $x(t)$ and $e(t)$ are ultimately bounded. It follows from comparison principle and $\lim_{t\to\infty}\sup_{t\ge 0}|\psi(t)|^2 \le \kappa_1^2 + (\kappa_2^i)^2$, that $\lambda_{\min}(P)\lim_{t\to\infty}\sup_{t\ge 0}|x(t)|^2 \le \lim_{t\to\infty}\sup_{t\ge 0}(x_a^{\mathrm{T}}(t)Px_a(t)) \le 0.5\alpha^{-1}\beta[\kappa_1^2 + (\kappa_2^i)^2]$. The condition $P - \gamma I > 0$ in (19) leads to maximization of the smallest eigenvalue of P [38], [39], which minimizes the upper bound of $\lim_{t\to\infty}\sup_{t\ge 0}|x(t)|^2$. Theorem 1 is proven. ■

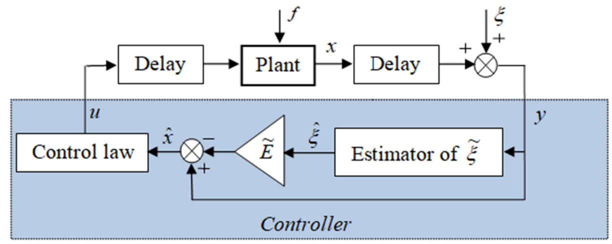

Fig. 2. Control scheme for system with input and output delays.

Remark 3: Let us show the boundedness of all signals in the closed-loop system. Since the signals $x(t)$ and $e(t)$ are ultimately bounded, then the signal $u(t)$ is ultimately bounded from (14). The ultimate boundedness of $\hat{\xi}(t)$ follows from (13). Therefore, the ultimate boundedness of $\tilde{y}(t)$ follows from (8). It follows from (12) that $\int_0^t[\tilde{A}\hat{\xi}(s) - \tilde{A}_1 z(s)]ds$ is bounded. As a result, all signals are bounded in the closed-loop system.

Remark 4: Choose P such that $A_a^{\mathrm{T}} P + PA_a + 2\alpha P < 0$. Then, by the Schur complement, the matrix inequality $\Psi_a < 0$ given by (19) is feasible for large enough τ, β and small enough $\tau\chi^2$.

Remark 5: To find the matrix K that leads to the small ultimate bound δ, we use an iterative procedure (see, e.g., [37]–[39]).

Choose any matrix $K = K^0$ such that A_a given by (17) is Hurwitz. Also choose an arbitrary small number $\epsilon > 0$ and set $j = 0$.

Step 1: Considering (19), maximize γ with $K = K^j$ and obtain (for this K) the solutions P^*, γ^*. Set $P^j = P^*$.

Step 2: Using (19), maximize γ with $P = P^j$ and obtain (for given P) the solution K^*. Set $K^{j+1} = K^*$ and $\gamma_j = \gamma^*$.

Step 3: If $j \ge 1$ and $|\gamma_j^{-1} - \gamma_{j-1}^{-1}| \le \epsilon$, then set $K = K^j$, $\gamma = \gamma_j$ and stop. Otherwise, augment j by 1 and go to Step 1.

Remark 6: The results of Theorem 1 can be extended to polytopic type uncertainties. Denote $\Omega = [A, B, D, G]$ and assume that $\Omega = \sum_{j=1}^N g_j(t)\Omega_j$ for some $0 \le g_j(t) \le 1$, $\sum_{j=1}^N g_j(t) = 1$, where the N vertices of the polytope are described by $\Omega_j = [A_j, B, D_j, G_j]$. Assume that N matrix inequalities (19) in all vertices Ω_j, $j = 1,\ldots,N$, are feasible with the same $\beta > 0$, $\tau > 0$, and $P > 0$. Then, (19) is feasible (see, e.g., [25]).

III. Systems With Uncertain Time-Varying Input and Output Delays

Let a plant (1), (2) contain uncertain input and output delays (see Fig. 2) in the form

$$\dot{x}(t) = Ax(t) + Bu(t - h_1(t))$$
$$+ D\phi(x(t),t) + Gf(t) \tag{24}$$
$$y(t) = x(t - h_2(t)) + \xi(t) \tag{25}$$

where delays $h_1(t) > 0$ and $h_2(t) > 0$ are uniformly bounded: $\bar{h}_1 \ge h_1(t)$ and $\bar{h}_2 \ge h_2(t)$ for all $t \ge 0$, where $\bar{h}_1$ and $\bar{h}_2$ are known bounds. Other parameters of plant (24), (25) are the same as for plant (1), (2) in Section II. The objective is to design a control law guaranteeing (4).

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A. Control Law Design for Time-Delay Systems

In this section, we generalize the results of Section II for systems with delays. Since the control law synthesis is similar to one in Section II, in this section, we write out only the equation containing delays.

Consider control law (5), (6) (see Fig. 2). Design the measurement noise estimator. To this end, rewrite $y(t)$ given by (25) as follows:

$$y(t) = x(t-h_2(t)) + \tilde{E}^{\mathrm{T}}\tilde{\xi}(t) + E_i\xi_i(t). \tag{26}$$

Eliminating the ith equation in (26) and rewriting result w.r.t. $\tilde{\xi}(t)$, we have

$$\tilde{\xi}(t) = \tilde{y}(t) - \tilde{E}x(t-h_2(t)). \tag{27}$$

Integrating (24) in t and employing (27), we obtain

$$\begin{aligned}\tilde{\xi}(t) = \tilde{y}(t) - \tilde{E}\int_0^t &[Ax(s-h_2(s))\\ &+ Bu(s-h_1(s)-h_2(s))\\ &+ D\phi(x(s-h_2(s)), s-h_2(s))\\ &+ Gf(s-h_2(s))]ds.\end{aligned} \tag{28}$$

Taking into account $x(t-h_2(t))$ from (26) and notations (10), rewrite (28) in the form

$$\begin{aligned}\tilde{\xi}(t) = &\int_0^t [\tilde{A}\tilde{\xi}(s) - \tilde{A}_1 y(s)]ds + \tilde{y}(t)\\ &- \int_0^t [\tilde{B}u(s-h_1(s)-h_2(s))\\ &+ \tilde{D}\phi(x(s-h_2(s)), s-h_2(s))\\ &+ \tilde{G}f(s-h_2(s)) - \tilde{A}_2\xi_i(s)]ds.\end{aligned} \tag{29}$$

The second and the third rows in (29) contain unknown functions $\phi(x(t),t)$, $f(t)$, and $\xi_i(t)$. Thus, taking into account the first row in (29), introduce the measurement noise estimator in the form (12). The next section is presented derivation of the closed-loop system and formulation the main result for time-delay systems.

B. Main Result

Taking into account (6) and (26), rewrite control law (5) as

$$u(t) = K[x(t-h_2(t)) + \tilde{E}^{\mathrm{T}}e(t) + E_i\xi_i(t)]. \tag{30}$$

Substituting (30) into (24), we obtain

$$\begin{aligned}\dot{x}(t) = Ax(t) &+ BKx(t-h_1(t)-h_2(t))\\ &+ BK\tilde{E}^{\mathrm{T}}e(t-h_1(t)) + BKE_i\xi_i(t-h_1(t))\\ &+ D\phi(x(t),t) + Gf(t).\end{aligned} \tag{31}$$

Employing (30) and differentiating (13) along the trajectories of (12) and (29), we obtain

$$\begin{aligned}\dot{e}(t) = \tilde{A}e(t) &- \tilde{B}Kx(t-h_1(t)-2h_2(t))\\ &- \tilde{B}KE_i\xi_i(t-h_1(t)-h_2(t))\\ &- \tilde{D}\phi(x(t-h_2(t)), t-h_2(t))\\ &- \tilde{B}K\tilde{E}^{\mathrm{T}}e(t-h_1(t)-h_2(t))\\ &- \tilde{G}f(t-h_2(t)) + \tilde{A}_2\xi_i(t).\end{aligned} \tag{32}$$

Setting

$$\begin{aligned}x_a(t) &= \mathrm{col}\{x(t), e(t)\}\\ \psi(t) &= \mathrm{col}\{\xi_i(t), \xi_i(t-h_1(t))\\ &\qquad \xi_i(t-h_1(t)-h_2(t)), f(t), f(t-h_2(t))\}\\ A_d &= \begin{bmatrix} A & 0\\ 0 & \tilde{A}\end{bmatrix}, \quad D_{a1} = \begin{bmatrix} 0 & BK\tilde{E}^{\mathrm{T}}\\ 0 & 0\end{bmatrix}\\ D_{a2} &= \begin{bmatrix} BK & 0\\ 0 & -\tilde{B}K\tilde{E}^{\mathrm{T}}\end{bmatrix}\\ D_{a3} &= \begin{bmatrix} 0 & 0\\ -\tilde{B}K & 0\end{bmatrix}, \; G_{a1} = \begin{bmatrix} D\\ 0\end{bmatrix}, \; G_{a2} = \begin{bmatrix} 0\\ -\tilde{D}\end{bmatrix}\\ F_d &= \begin{bmatrix} 0 & BKE_i & 0 & G & 0\\ \tilde{A}_2 & 0 & -\tilde{B}KE_i & 0 & -\tilde{G}\end{bmatrix}\end{aligned} \tag{33}$$

rewrite (31) and (32) in the form

$$\begin{aligned}\dot{x}_a(t) = A_d x_a(t) &+ D_{a1}x_a(t-h_1(t))\\ &+ D_{a2}x_a(t-h_1(t)-h_2(t))\\ &+ D_{a3}x_a(t-h_1(t)-2h_2(t))\\ &+ G_{a2}\phi(x(t-h_2(t)), t-h_2(t))\\ &+ G_{a1}\phi(x(t),t) + F_d\psi(t).\end{aligned} \tag{34}$$

Remark 7: According to [25], the closed-loop system (34) is ISS if $A_d + \sum_{i=1}^{3} D_{ai}$ is Hurwitz and $h_1(t)$, $h_2(t)$ are small enough. The conditions of Remark 2 hold here because $A_a = A_d + \sum_{i=1}^{3} D_{ai}$.

Before formulating the main result, consider the following notations:

$$\begin{aligned}\Psi_{11} &= A_d^{\mathrm{T}}P_2 + P_2^{\mathrm{T}}A_d + S_1 + S_2\\ &\quad - e^{-2\alpha\bar{h}}R_1 - e^{-2\alpha\bar{h}_2}R_2 + 2\alpha P\\ \Psi_{12} &= P - P_2^{\mathrm{T}} + A_d^{\mathrm{T}}P_3\\ \Psi_{13} &= P_2^{\mathrm{T}}D_{a1} + e^{-2\alpha\bar{h}}R_1 - e^{-2\alpha\bar{h}}S_{34}^{\mathrm{T}}\\ \Psi_{14} &= P_2^{\mathrm{T}}D_{a2} + e^{-2\alpha\bar{h}}S_{34}^{\mathrm{T}} - e^{-2\alpha\bar{h}}S_{24}^{\mathrm{T}}\\ \Psi_{15} &= P_2^{\mathrm{T}}D_{a3} + e^{-2\alpha\bar{h}}S_{24}^{\mathrm{T}} - e^{-2\alpha\bar{h}}S_{14}^{\mathrm{T}}\\ \Psi_{16} &= e^{-2\alpha\bar{h}}S_{14}^{\mathrm{T}}\\ \Psi_{17} &= e^{-2\alpha\bar{h}_2}R_2 - e^{-2\alpha\bar{h}_2}\tilde{S}_{12}^{\mathrm{T}}\\ \Psi_{18} &= e^{-2\alpha\bar{h}_2}\tilde{S}_{12}^{\mathrm{T}}, \quad \Psi_{19} = P_2^{\mathrm{T}}G_{a1}\\ \Psi_{1,10} &= P_2^{\mathrm{T}}G_{a2}\\ \Psi_{1,11} &= P_2^{\mathrm{T}}F_d, \quad \Psi_{22} = -P_3^{\mathrm{T}} - P_3 + \bar{h}^2R_1 + \bar{h}_2^2R_2\\ \Psi_{23} &= P_3^{\mathrm{T}}D_{a1}, \quad \Psi_{24} = P_3^{\mathrm{T}}D_{a2}, \quad \Psi_{25} = P_3^{\mathrm{T}}D_{a3}\\ \Psi_{29} &= P_3^{\mathrm{T}}G_{a1}, \quad \Psi_{2,10} = P_3^{\mathrm{T}}G_{a2}, \quad \Psi_{2,11} = P_3^{\mathrm{T}}F_d\\ \Psi_{33} &= -2e^{-2\alpha\bar{h}}R_1 + e^{-2\alpha\bar{h}}(S_{34}^{\mathrm{T}} + S_{34})\\ \Psi_{34} &= e^{-2\alpha\bar{h}}R_1 - e^{-2\alpha\bar{h}}S_{34}^{\mathrm{T}} + e^{-2\alpha\bar{h}}S_{24}^{\mathrm{T}} - e^{-2\alpha\bar{h}}S_{23}^{\mathrm{T}}\\ \Psi_{35} &= -e^{-2\alpha\bar{h}}S_{24}^{\mathrm{T}} + e^{-2\alpha\bar{h}}S_{14}^{\mathrm{T}}\\ \Psi_{36} &= -e^{-2\alpha\bar{h}}S_{14}^{\mathrm{T}} + e^{-2\alpha\bar{h}}S_{13}^{\mathrm{T}}\\ \Psi_{44} &= -2e^{-2\alpha\bar{h}}R_1 + e^{-2\alpha\bar{h}}(S_{23}^{\mathrm{T}} + S_{23})\\ \Psi_{45} &= e^{-2\alpha\bar{h}}R_1 - e^{-2\alpha\bar{h}}S_{23}^{\mathrm{T}} + e^{-2\alpha\bar{h}}S_{13}^{\mathrm{T}}\\ \Psi_{46} &= -e^{-2\alpha\bar{h}}S_{23}^{\mathrm{T}} + e^{-2\alpha\bar{h}}S_{12}^{\mathrm{T}}\end{aligned}$$

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$$\Psi_{55} = -2e^{-2\alpha\bar{h}}R_1 + e^{-2\alpha\bar{h}}(S_{12}^{\mathrm{T}} + S_{12})$$
$$\Psi_{56} = e^{-2\alpha\bar{h}}R_1 - e^{-2\alpha\bar{h}}S_{12}$$
$$\Psi_{66} = -e^{-2\alpha\bar{h}}R_1, \quad \Psi_{77} = -e^{-2\alpha\bar{h}_2}R_2$$
$$\Psi_{88} = -e^{-2\alpha\bar{h}_2}R_2 - e^{-2\alpha\bar{h}_2}S_2$$
$$\tilde{\Psi}_{11} = \Psi_{11} + \tau_1\chi^2 C^{\mathrm{T}}C, \quad \tilde{\Psi}_{77} = \Psi_{77} + \tau_2\chi^2 C^{\mathrm{T}}C$$
$$\Psi_{a1} = \begin{bmatrix} \tilde{\Psi}_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} \\ * & * & * & \Psi_{44} & \Psi_{45} & \Psi_{46} \\ * & * & * & * & \Psi_{55} & \Psi_{56} \\ * & * & * & * & * & \Psi_{66} \end{bmatrix}$$
$$\Psi_{a2} = \begin{bmatrix} \Psi_{17} & \Psi_{18} & \Psi_{19} & \Psi_{1,10} & \Psi_{1,11} \\ 0 & 0 & \Psi_{29} & \Psi_{2,10} & \Psi_{2,11} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\Psi_{a3} = \mathrm{diag}\{\tilde{\Psi}_{77}, \Psi_{88}, -\tau_1 I, -\tau_2 I, -\beta I\}$$
$$\bar{h} = \bar{h}_1 + 2\bar{h}_2$$

where $\alpha > 0, \beta > 0, \tau_1 > 0$, and $\tau_2 > 0$ are constants and $P > 0$, P_2, $P_3, S_1 > 0, S_2 > 0, R_1 > 0, R_2 > 0, \tilde{S}_{12}, S_{ij}, i = 1, 2, 3, j = 2, 3, 4$ are $(2n-1) \times (2n-1)$ matrices, $C = [I\ 0,\ \ldots,\ 0]$. The following result is thus in order.

Theorem 2: Let Assumptions 1 and 2 hold. Consider the closed-loop system consisting of plant (1), (2) under control law (5), where $\hat{x}(t)$ is defined by (6) and (12). Given $K, \bar{h}, \bar{h}_2$, and α, if there exist β, τ_1, τ_2, $P, R_1, R_2, P_2, P_3, S_1, S_2, \tilde{S}_{12}, S_{ij}, i = 1, 2, 3, j = 2, 3, 4$ that satisfy the following optimization problem:

$$\text{maximize } \gamma$$
$$\text{subject to}$$
$$\begin{bmatrix} R_1 & S_{ij} \\ * & R_1 \end{bmatrix} \geq 0, \quad i = 1, 2, 3,\ j = 2, 3, 4$$
$$\begin{bmatrix} R_2 & \tilde{S}_{12} \\ * & R_2 \end{bmatrix} \geq 0$$
$$\Psi_d = \begin{bmatrix} \Psi_{a1} & \Psi_{a2} \\ * & \Psi_{a3} \end{bmatrix} < 0 \text{ and } P - \gamma I > 0 \tag{35}$$

then the solutions of the closed-loop system are ultimately bounded and (4) holds with $\delta = \sqrt{\frac{\beta[2\kappa_1^2 + 3(\kappa_2^i)^2]}{2\alpha\gamma}}$. Moreover, all signals are bounded in the closed-loop system.

Proof: According to [24], for the ISS analysis of (34), consider the following Lyapunov–Krasovskii functional:

$$V = \sum_{i=1}^{3} V_i \tag{36}$$

where V_1 given by (20) and

$$V_2 = \int_{t-\bar{h}}^{t} e^{2\alpha(\sigma-t)}x_a^{\mathrm{T}}(\sigma)S_1 x_a(\sigma)d\sigma + \int_{t-\bar{h}_2}^{t} e^{2\alpha(\sigma-t)}x_a^{\mathrm{T}}(\sigma)S_2 x_a(\sigma)d\sigma$$
$$V_3 = \bar{h}\int_{-\bar{h}}^{0}\int_{t+\zeta}^{t} e^{2\alpha(\sigma-t)}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma d\zeta + \bar{h}_2\int_{-\bar{h}_2}^{0}\int_{t+\zeta}^{t} e^{2\alpha(\sigma-t)}\dot{x}_a^{\mathrm{T}}(\sigma)R_2\dot{x}_a(\sigma)d\sigma d\zeta. \tag{37}$$

We use the descriptor method with free matrices P_2 and P_3 (see [25]), where $\dot{x}_a(t)$ is not substituted by the right-hand side of (37). Differentiating (20) and (37) along (34), we have

$$\begin{aligned}\dot{V}_1 + 2\alpha V_1 = {} & 2x_a^{\mathrm{T}}(t)P\dot{x}_a(t) \\ & + 2\alpha x_a^{\mathrm{T}}(t)Px_a(t) + 2[x_a^{\mathrm{T}}(t)P_2^{\mathrm{T}} + \dot{x}_a^{\mathrm{T}}(t)P_3^{\mathrm{T}}] \\ & \times [A_d x_a(t) + D_{a1}x_a(t-h_1(t)) \\ & + D_{a2}x_a(t-h_1(t)-h_2(t)) \\ & + D_{a3}x_a(t-h_1(t)-2h_2(t)) + G_{a1}\phi(x(t),t) \\ & + G_{a2}\phi(x(t-h_2(t)), t-h_2(t)) + F_d\psi(t) - \dot{x}_a(t)]\end{aligned}$$
$$\begin{aligned}\dot{V}_2 + 2\alpha V_2 = {} & x_a^{\mathrm{T}}(t)S_1x_a(t) \\ & - e^{-2\alpha\bar{h}}x_a^{\mathrm{T}}(t-\bar{h})S_1x_a(t-\bar{h}) \\ & + x_a^{\mathrm{T}}(t)S_2x_a(t) - e^{-2\alpha\bar{h}_2}x_a^{\mathrm{T}}(t-\bar{h}_2)S_2x_a(t-\bar{h}_2)\end{aligned}$$
$$\begin{aligned}\dot{V}_3 + 2\alpha V_3 \leq {} & \bar{h}^2\dot{x}_a^{\mathrm{T}}(t)R_1\dot{x}_a(t) \\ & - \bar{h}e^{-2\alpha\bar{h}}\int_{t-\bar{h}}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma, \\ & + \bar{h}_2^2\dot{x}_a^{\mathrm{T}}(t)R_2\dot{x}_a(t) - \bar{h}_2e^{-2\alpha\bar{h}_2} \\ & \times \int_{t-\bar{h}_2}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_2\dot{x}_a(\sigma)d\sigma.\end{aligned} \tag{38}$$

Rewrite integral terms in (38) in the forms

$$\begin{aligned} & -\bar{h}\int_{t-\bar{h}}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma \\ & = -\bar{h}\left(\int_{t-\bar{h}}^{t-h_1(t)-2h_2(t)}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma\right. \\ & + \int_{t-h_1(t)-2h_2(t)}^{t-h_1(t)-h_2(t)}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma \\ & + \int_{t-h_1(t)-h_2(t)}^{t-h_1(t)}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma \\ & \left. + \int_{t-h_1(t)}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma\right) \\ & -\bar{h}_2\int_{t-\bar{h}_2}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_2\dot{x}_a(\sigma)d\sigma \\ & = -\bar{h}_2\left(\int_{t-\bar{h}_2}^{t-h_2(t)}\dot{x}_a^{\mathrm{T}}(\sigma)R_2\dot{x}_a(\sigma)d\sigma\right. \\ & \left. + \int_{t-h_2(t)}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_2\dot{x}_a(\sigma)d\sigma\right).\end{aligned} \tag{39}$$

Denote $e_1(t) = x_a(t-h_1(t)-2h_2(t)) - x_a(t-\bar{h})$, $e_2(t) = x_a(t-h_1(t)-h_2(t)) - x_a(t-h_1(t)-2h_2(t))$, $e_3(t) = x_a(t-h_1(t)) - x_a(t-h_1(t)-h_2(t))$, $e_4(t) = x_a(t) - x_a(t-h_1(t))$, $\tilde{e}_1(t) = x_a(t-h_2(t)) - x_a(t-\bar{h}_2)$, and $\tilde{e}_2(t) = x_a(t) - x_a(t-h_2(t))$.

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Using Jensen's inequality, estimate (39) as follows:

$$\begin{aligned}
&-\bar{h}\int_{t-\bar{h}}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma\\
&\le -\frac{\bar{h}}{\bar{h}-h_1(t)-2h_2(t)}e_1^{\mathrm{T}}R_1e_1-\frac{\bar{h}}{h_2(t)}e_2^{\mathrm{T}}R_1e_2\\
&\quad -\frac{\bar{h}}{h_2(t)}e_3^{\mathrm{T}}R_1e_3-\frac{\bar{h}}{h_1(t)}e_4^{\mathrm{T}}R_1e_4\\
&-\bar{h}_2\int_{t-\bar{h}_2}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_2\dot{x}_a(\sigma)d\sigma\\
&\le -\frac{\bar{h}_2}{\bar{h}_2-h_2(t)}\tilde{e}_1^{\mathrm{T}}R_2\tilde{e}_1-\frac{\bar{h}_2}{h_2(t)}\tilde{e}_2^{\mathrm{T}}R_2\tilde{e}_2.
\end{aligned}\tag{40}$$

Applying [25, Lemma 3.4], rewrite (40) as

$$\begin{aligned}
&-\bar{h}\int_{t-\bar{h}}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_1\dot{x}_a(\sigma)d\sigma\\
&\le -\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}R_1 & S_{12} & S_{13} & S_{14}\\ * & R_1 & S_{23} & S_{24}\\ * & * & R_1 & S_{34}\\ * & * & * & R_1\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\\e_4\end{bmatrix}\\
&-\bar{h}_2\int_{t-\bar{h}_2}^{t}\dot{x}_a^{\mathrm{T}}(\sigma)R_2\dot{x}_a(\sigma)d\sigma\le -\begin{bmatrix}\tilde{e}_1\\\tilde{e}_2\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}R_2 & \tilde{S}_{12}\\ * & R_2\end{bmatrix}\begin{bmatrix}\tilde{e}_1\\\tilde{e}_2\end{bmatrix}.
\end{aligned}$$

Setting

$$\begin{aligned}
z_h(t) = col\{&x_a(t),\dot{x}_a(t),x_a(t-h_1(t))\\
&x_a(t-h_1(t)-h_2(t)),x_a(t-h_1(t)-2h_2(t)),x_a(t-\bar{h})\\
&x_a(t-h_2(t)),x_a(t-\bar{h}_2),\phi(x(t),t)\\
&\phi(x(t-h_2(t)),t-h_2(t)),\psi(t)\}\\
\Psi = \Psi_d&\\
&+\mathrm{diag}\{-\tau_1\chi^2\,C^{\mathrm{T}}C,0,\ldots 0,-\tau_2\chi^2\,C^{\mathrm{T}}C,0,\tau_1 I,\tau_2 I,0\}
\end{aligned}$$

present the following expression:

$$\dot{V}+2\alpha V-\beta\psi^{\mathrm{T}}\psi\le z_h^{\mathrm{T}}\Psi z_h. \tag{41}$$

Taking into account $x = Cx_a$ and $|\phi(x(t),t)|\le\chi|x(t)|$, consider the following estimates:

1) $\phi^2(x(t),t)\le\chi^2x_a^{\mathrm{T}}(t)C^{\mathrm{T}}Cx_a(t)$ or in the form

$$\tau_1 z_h^{\mathrm{T}}(t)\mathrm{diag}\{\chi^2\,C^{\mathrm{T}}C,0,\ldots,0,-I,0,0\}z_h(t)\ge 0. \tag{42}$$

2) $\phi^2(x(t-h_2(t)),t-h_2(t))\le\chi^2x_a^{\mathrm{T}}(t-h_2(t))C^{\mathrm{T}}Cx_a(t-h_2(t))$ or in the form

$$\tau_2 z_h^{\mathrm{T}}(t)\mathrm{diag}\{0,\ldots,0,\chi^2\,C^{\mathrm{T}}C,0,0,-I,0\}z_h(t)\ge 0. \tag{43}$$

According to the S-procedure, inequalities (41)–(43) simultaneously hold, if matrix inequalities in (35) hold. Thus, $z_h(t)$ is ultimately bounded; therefore, $x(t)$ and $e(t)$ are ultimately bounded. The proof of boundendness of other signals in the closed-loop system is the same as in Remark 3. Theorem 2 is proven. ■

Remark 8: The matrix K in (5) can be calculated from (35) similar to Remark 5, where the matrices P_2 and P_3 are used instead of P in optimization algorithm.

Remark 9: The results of Theorem 2 can be extended to unknown time-varying systems with polytopic type uncertainties similar to Remark 6.

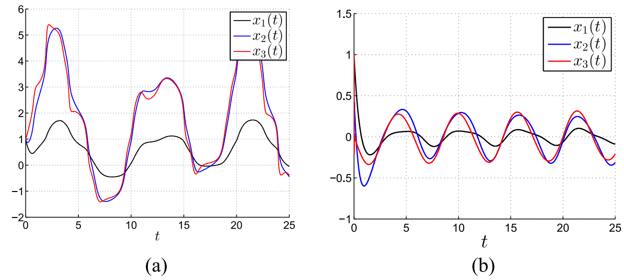

Fig. 3. Transients of $x_1(t)$, $x_2(t)$, and $x_3(t)$ obtained for (a) the control law $u = Ky$ and (b) the control law from [19] and [20].

IV. EXAMPLES

Example 1: Consider unstable system (1) with

$$A=\begin{bmatrix}-3 & 1 & 0\\ -3 & 0 & 1\\ 0.1 & 0 & 0\end{bmatrix},\quad B=\begin{bmatrix}0\\1\\2\end{bmatrix}$$

$$D=[0.1\ 1.5\ 3]^{\mathrm{T}},\quad G=[-0.01\ 1.03\ 1.97]^{\mathrm{T}}.$$

Unknown parameters and signals in (1) and (2) are given in the forms

$$\begin{aligned}
\phi(x(t),t) &= \sin(t)[0.2\sin(x_1)+0.3\sin(2x_2)+\sin(3x_3)]\\
x(0) &= [1\ 1\ 1]^{\mathrm{T}},\ f(t)=1+2\sin(0.7t)\\
\xi_1(t) &= 1+10\sin(3t),\ \xi_2(t)=-2+7\cos(3t)\\
\xi_3(t) &= 0.01\sin(0.8t).
\end{aligned}$$

We will choose further the parameters of the proposed control law. Let $i = 3$ in (8). Then, $\tilde{E}=\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix}$, $\tilde{A}=\begin{bmatrix}-3&1\\-3&0\end{bmatrix}$, and $\tilde{A}_1=\begin{bmatrix}-3&1&0\\-3&0&1\end{bmatrix}$ in (12). In this case, $\|\tilde{B}\|$ is not small enough (i.e., conditions of Remark 2 do not hold), but the matrix $K=-20[1\ 1\ 3]$ ensures Hurwitz of A_a. Considering the optimization algorithm from Remark 5 with $K^0 = K$, the matrix $K^* = [58.2\ \ -71.0\ \ -6.1]$ is obtained for $\epsilon = 10^{-3}$ at the 11th iteration ($j = 11$).

The matrix inequalities in (35) are feasible for $\bar{h}_1 = 3\times\ 10^{-3}$ and $\bar{h}_2 = 2\times\ 10^{-3}$. The simulations in MATLAB/Simulink show that the closed-loop system is practically stable for $\bar{h}_1 = 0.02$ and $\bar{h}_2 = 0.01$.

We will demonstrate the transients for the proposed control law and compare results:

1) with the static output feedback $u = Ky$, where disturbances and noises are not compensated;
2) with the smooth disturbance compensation algorithm [19], [20]

$$\begin{aligned}
\hat{\xi} &= \int_0^t\left[\tilde{A}\hat{\xi}(s)-\tilde{A}_1 z(s)\right]ds+\tilde{z}\\
u &= -\frac{1}{\mu E_i^{\mathrm{T}}B}\left[\hat{x}_i-E_i^{\mathrm{T}}A\int_0^t\hat{x}(s)ds\right],\mu=0.01.
\end{aligned}\tag{44}$$

Let $h_1(t) = 5\times\ 10^{-3}+5\times\ 10^{-3}\sin(10^3t)$ and $h_2(t) = 8\times 10^{-3}+2\times 10^{-3}\sin(10^2t)$. The transients in Fig. 3(a) for the controller $u = Ky$ depend on ξ_1, ξ_2, ξ_3, and f, whereas the transients in Fig. 3(b) for the control law of [19] and [20] depend on ξ_3 and f only. The ultimate bound (approximately 5.3) under the control law $u = Ky$ is at least 11 times larger than the ultimate bound (approximately 0.47) under the control law of [19] and [20]. The transients for the proposed control law also depend on ξ_3 and f only. However, the control law of [19] and [20] has only one designed scalar parameter μ, whereas

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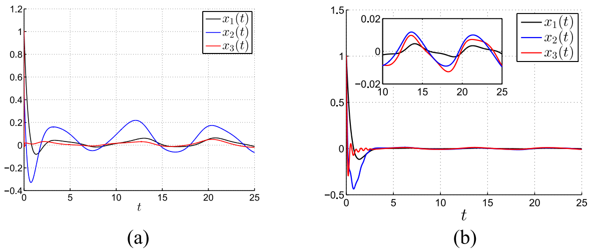

Fig. 4. Transients of $x_1(t)$, $x_2(t)$, and $x_3(t)$ obtained for the proposed control law (a) for $K = K^0$ and (b) for $K = K^*$ from Remark 3.

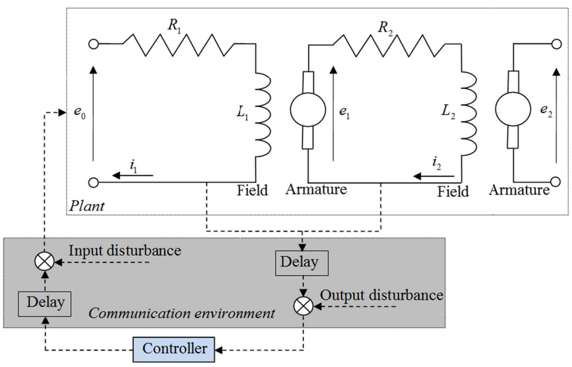

Fig. 5. Simplified representation of an amplidyne control scheme.

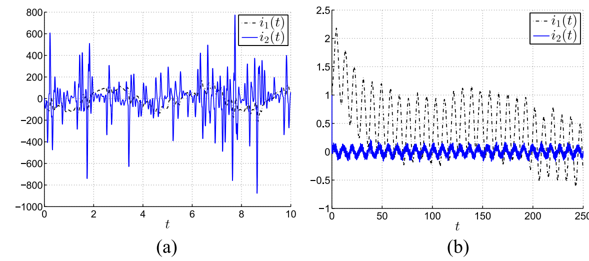

Fig. 6. Transients of $i_1(t)$ and $i_2(t)$ obtained for the control laws (a) $u = Ky$ and (b) $u = [0\ 1]y$.

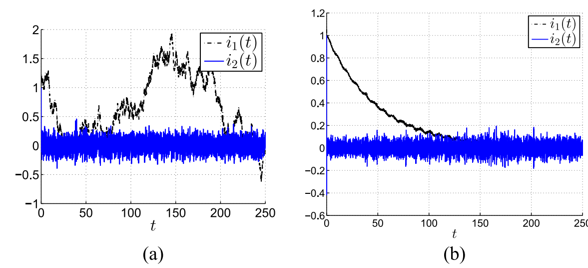

Fig. 7. Transients of $i_1(t)$ and $i_2(t)$ obtained for (a) the control laws from [19] and [20] and (b) the proposed control law.

the proposed control law contains designed matrix K. Fig. 4 shows the transients for the proposed control law with $K = K^0$ and $K = K^*$. The advantage of the proposed control law is clearly seen: the ultimate bound (approximately 0.47) under the control law of [19] and [20] is at least two times larger than the one under the proposed control law (approximately 0.22 for $K = K_0$ and approximately 0.017 for $K = K^*$).

Example 2: Consider the remote control of an amplidyne (see Fig. 5). The amplidyne [40]–[42] is an electric machine used to control a large dc power through a small dc voltage. Amplidynes are applied for electric elevators, point naval guns, antiaircraft artillery radar, control processes in steelworks, remote control rods in nuclear submarine designs, and diesel–electric locomotive control systems. The remote control under uncertain delays widely arises in regulation processes for controlling reversing rolling mills, cold rolling mills, mine hoists, metal cutting, and paper machines [43]–[45]. In these technological processes, the plant output signal is transmitted via communication channels from sensors to an operator point as well as from an operator point to amplidynes. The communication channels always have uncertain time-varying delays. The disturbances and measurement noises are caused by error of measuring devices, noise in data transmission channels, and the influence of external processes. The control can be implemented in operator point through the SCADA system [44], [45].

The amplidyne electrical dynamics is described by

$$\dot{x}(t) = \begin{bmatrix} -\frac{R_2}{L_2} & \frac{k_1}{L_2} \\ -\frac{R_1}{L_1} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{L_1} \end{bmatrix} (u(t) + f(t)) \tag{45}$$

where $x = [i_1, i_2]^{\mathrm{T}}$, i_1 and i_2 are currents in first and second windings accordingly, $u = e_0$ is the input voltage, e_1 and e_2 are the induced voltages given by $e_1 = k_1 i_1$ and $e_2 = k_2 i_2$, L_1 and R_1 denote the inductance and resistance of the first field windings, and L_2 and R_2 are those of the first armature windings together with the second field windings. According to [40], the following numerical values are used: $R_1 = 5\,\Omega$, $L_1 = 0.5\,\mathrm{s}^{-1}$, $k_1 = 20$ V/A, $k_2 = 50$ V/A, $R_2 = 10\,\Omega$, and $L_2 = 10\,\mathrm{s}^{-1}$.

Choosing $i = 2$ in (8), we have $\tilde{E} = [1\ 0]$, $\tilde{A} = -3$, and $\tilde{A}_1 = [-3\ 1]$ in (12). In this case, the conditions of Remark 2 are satisfied. Let $K = [10\ 1]^{\mathrm{T}}$. The matrix inequalities in (35) are feasible for $\bar{h}_1 = \bar{h}_2 = 10^{-3}$. The simulations in MATLAB/Simulink show that the closed-loop system is stable for $\bar{h}_1 = 0.02$ and $\bar{h}_2 = 0.001$.

Compare the following proposed control law:

1) with the static output-feedback control laws $u = Ky$ and $u = [0\ 1]y$ from [40] and [42];
2) with the algorithm [19], [20] given by (44).

Consider the simulations $h_1(t) = 15 \times 10^{-4} + 5 \times 10^{-4} \sin(0.01t)$, $h_2(t) = 10^{-4} + 10^{-5}\sin(0.03t)$, $f = 0.1\sin(0.7t) + d_1(t)$, $y_1(t) = q_1(x_1) + \xi_1(t)$, $y_2(t) = q_2(x_2) + \xi_2(t)$, $\xi_1(t) = 100\sin(1.7t) + d_2(t)$, $\xi_2(t) = 10^{-3}\sin(0.5t) + d_3(t)$, q_1 and q_2 are the quantization functions with the quantization intervals being 0.5 and 0.05, respectively, and the signals $d_1(t)$, $d_2(t)$, and $d_3(t)$ are obtained by the band-limited white noise blocks in MATLAB/Simulink with the following parameters: noise power $1, 3, 10^{-4}$ and sample time 0.1, 0.01, and 0.03 s accordingly. The plots of $i_1(t)$ and $i_2(t)$ are depicted for the control laws $u = Ky$ and $u = [0\ 1]y$ in Fig. 6, for the one from [19] and [20] in Fig. 7(a), and for the proposed one in Fig. 7(b). The transients in Fig. 6(a) depend on the noises ξ_1, ξ_2 and the disturbance f. In Fig. 6(b), the transients do not depend on ξ_1, but influence of the disturbance f is not attenuated. The transients in Fig. 7(a) do not depend on ξ_1 also, but the influence of disturbance f is compensated. Since the control law of [19] and [20] can compensate only smooth disturbances under smooth noises, Fig. 7(a) shows high-frequency picks under nonsmooth ones. As a result, the advantage of the proposed control law is clearly seen: the ultimate bounds under the control laws $u = Ky$ and $u = [0\ 1]y$ and the algorithm from [19] and [20] (approximately 800, 1.2 and 1.8 accordingly) are at least six times larger than the one under the proposed control law (approximately 0.2).

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V. Conclusion

We have considered vector systems, where the full state is measured with the measurement noises. A novel method has been proposed for attenuating the influence of disturbances under measurement noises and uncertain input–output delays. Differently from recent results [19], [20], the proposed design method does not require the smoothness of disturbances and noises. The proposed method provides a better accuracy in the steady state because the closed-loop system depends only on one component of measurement noise vector and on disturbance. The simulations in numerical examples illustrate the efficiency of the presented method and its advantages over alternative methods without disturbance compensation.

Generalization of the presented method to output-feedback control may be a topic for future research.

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