Systems & Control Letters 61 (2012) 666–675

Contents lists available at SciVerse ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Stability and L_2-gain analysis of Networked Control Systems under Round-Robin scheduling: A time-delay approach

Kun Liu [a], Emilia Fridman [a,*], Laurentiu Hetel [b]

[a] School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel

[b] University Lille Nord de France, LAGIS, FRE CNRS 3303, Ecole Centrale de Lille, Cite Scientifique, BP 48, 59651 Villeneuve d'Ascq cedex, France

ARTICLE INFO

Article history:
Received 16 August 2011
Received in revised form
12 December 2011
Accepted 3 March 2012

Keywords:
Networked Control Systems
Scheduling protocols
Lyapunov–Krasovskii method

ABSTRACT

This paper analyzes the exponential stability and the induced L_2-gain of Networked Control Systems (NCS) that are subject to time-varying transmission intervals, time-varying transmission delays and communication constraints. The system sensor nodes are supposed to be distributed over a network. The scheduling of sensor information towards the controller is ruled by the classical Round-Robin protocol. We develop a *time-delay approach* for this problem by presenting the closed-loop system as a switched system with multiple and *ordered time-varying delays*. Linear Matrix Inequalities (LMIs) are derived via appropriate Lyapunov–Krasovskii-based methods. Polytopic uncertainties in the system model can be easily included in the analysis. The efficiency of the method is illustrated on the batch reactor and on the cart-pendulum benchmark problems. Our results essentially improve the hybrid system-based ones and, for the first time, allow treating the case of non-small network-induced delay, which can be greater than the sampling interval.

1. Introduction

Networked Control Systems (NCS) are systems with spatially distributed sensors, actuators and controller nodes which exchange data over a communication data channel. Only one node is allowed to use the communication channel at once. The communication along the data channel is orchestrated by a scheduling rule called protocol. Using such control structures offers several practical advantages: reduced costs, ease of installation and maintenance and increased flexibility. However, from the control theory point of view, it leads to new challenges. Closing the loop over a network introduces undesirable perturbations such as delay, variable sampling intervals, quantization, packet dropouts, scheduling communication constraints, etc. which may affect the system performance and even its stability. It is important in such a configuration to provide a stability certificate that takes into account the network imperfections. For general survey papers we refer to [1–3]. Recent advancements can be found in [4–9] for systems with variable sampling intervals, [10] for dealing with the quantization and [11–13] for control with time delay. Concerning NCS, three main control approaches have been used: discrete-time models (with integration step), input/output time-delay models and impulsive/hybrid models.

In the present paper, we focus on the stability and L_2-gain analysis of NCS with communication constraints. We consider a linear (probably, uncertain) system with distributed sensors. The scheduling of sensor information towards the controller is ruled by the classical Round-Robin protocol. The Round-Robin protocol has been considered in [14,15] (in the framework of hybrid system approach) and in [16,17] (in the framework of discrete-time systems). In [14], stabilization of the nonlinear system based on the impulsive model is studied. However, delays are not included in the analysis. In [15], the authors provide methods for computing the Maximum Allowable Transmission Interval (MATI — i.e. the maximum sampling jitter) and Maximum Allowable Delay (MAD) for which the stability of a nonlinear system is ensured.

In [16], network-based stabilization of Linear Time-Invariant (LTI) with Round-Robin protocol and without delay have been considered (see also [17] for delays less than the sampling interval). The analysis is based on the discretization and the equivalent polytopic model at the transmission instants. For LTI systems, discretization-based results are usually less conservative than the general hybrid system-based results. However, discrete-time models do not take into account the system behavior between two transmissions and are complicated in the case of uncertain systems. Moreover, it is tedious to include large delays in such models and the stability analysis methods may fail when the interval between two transmissions takes small values.

* Corresponding author.
E-mail addresses: liukun@eng.tau.ac.il (K. Liu), emilia@eng.tau.ac.il (E. Fridman), laurentiu.hetel@ec-lille.fr (L. Hetel).

0167-6911/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.sysconle.2012.03.002

In the present paper, for the first time, a *time-delay approach* is developed for the stability and L_2-gain analysis of NCS with Round-Robin scheduling. Discrete-time measurements are considered, where the delay may be larger than the sampling interval. We present the closed-loop system as a *switched continuous-time system with multiple and ordered time-varying delays*. The case of the *ordered time-varying delays* (where one delay is smaller than another) has not been studied yet in the literature. By developing the appropriate Lyapunov–Krasovskii techniques for this case, we derive LMIs for the exponential stability and for L_2-gain analysis. The efficiency and advantages of the presented approach are illustrated by two benchmark examples. Our numerical results essentially improve the hybrid system-based ones [15] and, for the stability analysis, are not far from those obtained via the discrete-time approach [17]. Note that the latter approach is not applicable to the performance analysis. Also, for the first time (under Round-Robin scheduling), the network-induced delay is allowed to be greater than the sampling interval.

Our preliminary results on stability of NCS with constant delay under Round-Robin scheduling have been presented in [18].

Notation: Throughout the paper the superscript 'T' stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space with vector norm $\|\cdot\|$, $\mathcal{R}^{n\times m}$ is the set of all $n\times m$ real matrices, and the notation $P>0$, for $P\in\mathcal{R}^{n\times n}$ means that P is symmetric and positive definite. The symmetric elements of the symmetric matrix will be denoted by $*$. The space of functions $\phi:[a,b]\to\mathcal{R}^n$, which are absolutely continuous on $[a,b)$, have a finite $\lim_{\theta\to b^-}\phi(\theta)$ and have square integrable first order derivatives is denoted by $W[a,b]$ with the norm $\|\phi\|_W=\max_{\theta\in[a,b]}|\phi(\theta)|+\left[\int_a^b|\dot\phi(s)|^2ds\right]^{\frac{1}{2}}$. $\mathcal{N}$ denotes the set $\{0,1,2,3,\ldots\}$.

2. Problem formulation and the switched system model

Consider the following system controlled through a network (see Fig. 1):

$$\begin{aligned}\dot x(t)&=Ax(t)+Bu(t)+B_1w(t),\\ z(t)&=C_0x(t)+D_{12}u(t),\end{aligned}\qquad(1)$$

where $x(t)\in\mathcal{R}^n$ is the state vector, $u(t)\in\mathcal{R}^m$ is the control input, $w(t)\in\mathcal{R}^{n_w}$ is the disturbance, $z(t)\in\mathcal{R}^{n_z}$ is controlled output, A, B, B_1, C_0 and D_{12} are system matrices with appropriate dimensions. These matrices can be uncertain with polytopic type uncertainty. The system has several nodes (distributed sensors, a controller node and an actuator node) which are connected via two networks: a sensor network (relaying the sensors to the controller node) and a control network (from the controller node to the actuator). For the sake of simplicity, we consider two sensor nodes $y^i(t)=C^ix(t)$, $i=1,2$ and we denote $C=\begin{bmatrix}C^1\\C^2\end{bmatrix}$, $y(t)=\begin{bmatrix}y^1(t)\\y^2(t)\end{bmatrix}\in\mathcal{R}^{n_y}$. The results can be easily extended to any finite number of sensors. We let s_k denote the unbounded monotonously increasing sequence of sampling instants, i.e.

$$0=s_0<s_1<\cdots<s_k<\cdots,\quad k\in\mathcal{N},\quad \lim_{k\to\infty}s_k=\infty.\qquad(2)$$

At each sampling instant s_k, one of the outputs $y^i(t)$ is sampled and transmitted via the network. The choice of the active output node is ruled by a Round-Robin scheduling protocol: the outputs are transmitted one after another, i.e. $y^i(t)$ is transmitted only at the sampling instant $t=s_{2p+i-1}$, $p\in\mathcal{N}$. After each transmission and reception, the values in $y^i(t)$ are updated with the newly received values, while the other values in $y(t)$ remain the same, as

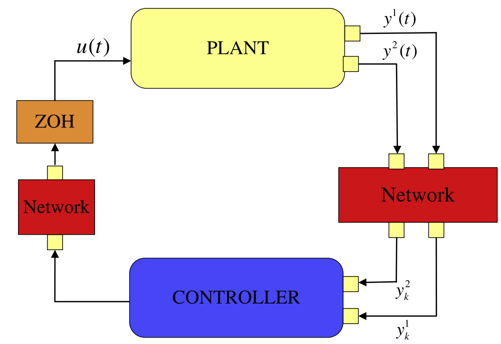

Fig. 1. System architecture.

no additional information is received. This leads to the constrained data exchange expressed as

$$y_k^i=\begin{cases}y^i(s_k)=C^ix(s_k)+F^iv(s_k), & k=2p+i-1,\\ y_{k-1}^i, & k\neq 2p+i-1,\end{cases}$$
$$p\in\mathcal{N},\qquad(3)$$

where v is a measurement noise signal and F^i is the matrix with appropriate dimension, $i=1,2$.

We suppose that data loss is not possible and that the transmission of the information over the two networks (between the sensor and the actuator) is subject to a variable delay $h_k=h_k^{sc}+h_k^{ca}$, where h_k^{sc} and h_k^{ca} are the network-induced delays (from the sensor to the controller and from the controller to the actuator respectively). Then $t_k=s_k+h_k$ is the updating instant time of the Zero-Order Hold (ZOH).

Differently from [15,17], we do not restrict the network delays to be small with $t_k=s_k+h_k<s_{k+1}$, i.e. $h_k<s_{k+1}-s_k$. As in [19] we allow the delay to be non-small provided that the old sample cannot get to the destination (to the controller or to the actuator) after the most recent one

$$s_k+h_k^{sc}<s_{k+1}+h_{k+1}^{sc},\qquad s_k+h_k<s_{k+1}+h_{k+1},\qquad(4)$$

i.e. $h_k<t_{k+1}-s_k$. Assumption (4) is a *necessary condition for making scheduling reasonable*. A sufficient condition for (4) is that the delays are bounded $h_k^{sc}\in[h_m^{sc},MAD^{sc}]$, $h_k\in[h_m,MAD]$, where h_m, h_m^{sc}, MAD, MAD^{sc} are known bounds with $MAD^{sc}-h_m^{sc}\leq MAD-h_m$, and the delay range is less than the sampling interval: $h_k-h_{k+1}\leq MAD-h_m<s_{k+1}-s_k$.

Assume that the network-induced delay h_k and the time span between the updating and the most recent sampling instants are bounded:

$$t_{k+1}-t_k+h_k\leq\tau_M,\quad 0\leq h_m\leq h_k,\ k\in\mathcal{N},\qquad(5)$$

where τ_M and h_m are known bounds. Note that $\tau_M=MATI+MAD$. Then

$$\begin{aligned}&t_{k+1}-t_k\leq\tau_M-h_m,\\ &t_{k+1}-t_{k-1}+h_{k-1}\leq 2\tau_M-h_m\triangleq\bar\tau_M.\end{aligned}\qquad(6)$$

We suppose that the controller and the actuator act in an event-driven manner. The general dynamic output feedback controller is assumed to be given in the following form:

$$\begin{aligned}\dot x_c(t)&=A_cx_c(t)+B_cy_k,\\ u(t)&=C_cx_c(t)+D_cy_k,\quad t\in[t_k,t_{k+1}),\ k\in\mathcal{N},\end{aligned}\qquad(7)$$

where $x_c(t) \in \mathcal{R}^{n_c}$ is the state of the controller, $y_k = \begin{bmatrix} y_k^1 \\ y_k^2 \end{bmatrix} \in \mathcal{R}^{n_y} (k = 1, 2, \dots)$ is the most recently received output of the plant, which satisfies (3) and $y_0 = \begin{bmatrix} y_0^1 \\ 0 \end{bmatrix}$. A_c, B_c, C_c and D_c are the matrices with appropriate dimensions.

2.1. Static output feedback control

Consider first the particular case of (7) with $C_c = 0, D_c = K$, where we suppose that there exists a matrix $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$, $K_1 \in \mathcal{R}^{m \times n_1}$, $K_2 \in \mathcal{R}^{m \times (n_y - n_1)}$ such that $A + BKC$ is Hurwitz. Consider the static output feedback of the form:

$$u_k = K_1 y_k^1 + K_2 y_k^2, \quad k = 1, 2, \dots$$

and $u_0 = K_1 y_0^1$. Then the control law is piecewise constant with

$$u(t) = u_k, \quad \forall t \in [t_k, t_{k+1}) .$$

The closed-loop system can be presented in the form of the switched system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1 x(t_k - h_k) + A_2 x(t_{k-1} - h_{k-1}) \\ &\quad + B_1 w(t) + D_{21} v(t_k - h_k) + D_{22} v(t_{k-1} - h_{k-1}), \\ z(t) &= C_0 x(t) + D_1 x(t_k - h_k) + D_2 x(t_{k-1} - h_{k-1}), \\ &\quad + E_{21} v(t_k - h_k) + E_{22} v(t_{k-1} - h_{k-1}), \quad t \in [t_k, t_{k+1}), \\ \dot{x}(t) &= Ax(t) + A_1 x(t_k - h_k) + A_2 x(t_{k+1} - h_{k+1}) \\ &\quad + B_1 w(t) + D_{21} v(t_k - h_k) + D_{22} v(t_{k+1} - h_{k+1}), \\ z(t) &= C_0 x(t) + D_1 x(t_k - h_k) + D_2 x(t_{k+1} - h_{k+1}), \\ &\quad + E_{21} v(t_k - h_k) + E_{22} v(t_{k+1} - h_{k+1}), \quad t \in [t_{k+1}, t_{k+2}), \end{aligned} \tag{8}$$

where

$$\begin{aligned} &k = 2p, \quad p \in \mathcal{N}, \qquad A_i = BK_i C^i, \qquad D_{2i} = BK_i F^i, \\ &D_i = D_{12} K_i C^i, \qquad E_{2i} = D_{12} K_i F^i, \quad i = 1, 2. \end{aligned} \tag{9}$$

From (5), (6) we have

$$\begin{aligned} t \in [t_k, t_{k+1}) \;&\Rightarrow\; t - t_k + h_k \in [h_m, \tau_M], \\ &\qquad t - t_{k-1} + h_{k-1} \in [h_m, \bar{\tau}_M], \\ t \in [t_{k+1}, t_{k+2}) \;&\Rightarrow\; t - t_k + h_k \in [h_m, \bar{\tau}_M], \\ &\qquad t - t_{k+1} + h_{k+1} \in [h_m, \tau_M]. \end{aligned} \tag{10}$$

Moreover,

$$\begin{aligned} &t - t_k + h_k < t - t_{k-1} + h_{k-1}, \quad t \in [t_k, t_{k+1}), \\ &t - t_k + h_k > t - t_{k+1} + h_{k+1}, \quad t \in [t_{k+1}, t_{k+2}). \end{aligned}$$

For $t \in [t_k, t_{k+1})$ we can represent $t_k - h_k = t - \tau_1(t),\ t_{k-1} - h_{k-1} = t - \tau_2(t)$, where

$$\begin{aligned} &\tau_1(t) = t - t_k + h_k < \tau_2(t) = t - t_{k-1} + h_{k-1}, \\ &\tau_1(t) \in [h_m, \tau_M], \qquad \tau_2(t) \in [h_m, \bar{\tau}_M], \quad t \in [t_k, t_{k+1}). \end{aligned} \tag{11}$$

Therefore, (8) for $t \in [t_k, t_{k+1})$ *can be considered as a system with two time-varying interval delays*, where $\tau_1(t) < \tau_2(t)$. Similarly, for $t \in [t_{k+1}, t_{k+2})$ (8) is a system with two time-varying delays, one of which is less than another. This case of ordered time-varying interval delays has not been studied yet in the literature.

Assume that $v(t) = 0,\ t \le t_0$. Denote for $k = 2p,\ p \in \mathcal{N}$

$$\begin{aligned} &v_1(t) = v(t_k - h_k), \qquad v_2(t) = v(t_{k-1} - h_{k-1}), \quad t \in [t_k, t_{k+1}), \\ &v_1(t) = v(t_{k+1} - h_{k+1}), \qquad v_2(t) = v(t_k - h_k), \\ &\quad t \in [t_{k+1}, t_{k+2}). \end{aligned}$$

The switched continuous-time system (8) has three disturbances $w \in L_2[t_0, \infty),\ v_i \in L_2[t_0, \infty),\ i = 1, 2$ with

$$\begin{aligned} \|v_1\|_{L_2}^2 &= \int_{t_0}^{\infty} |v_1(t)|^2 dt = \sum_{p=0}^{\infty} (t_{2p+1} - t_{2p}) |v(t_{2p} - h_{2p})|^2 \\ &\quad + \sum_{p=0}^{\infty} (t_{2p+2} - t_{2p+1}) |v(t_{2p+1} - h_{2p+1})|^2 \\ &= \sum_{k=0}^{\infty} (t_{k+1} - t_k) |v(t_k - h_k)|^2, \\ \|v_2\|_{L_2}^2 &= \int_{t_0}^{\infty} |v_2(t)|^2 dt = \sum_{k=0}^{\infty} (t_{k+1} - t_k) |v(t_{k-1} - h_{k-1})|^2. \end{aligned}$$

For a given scalar $\gamma > 0$, we thus define the following performance index:

$$\begin{aligned} J &= \|z\|_{L_2}^2 - \gamma^2 [\|w\|_{L_2}^2 + \|v_1\|_{L_2}^2 + \|v_2\|_{L_2}^2] \\ &= \int_{t_0}^{\infty} [z^T(t) z(t) - \gamma^2 w^T(t) w(t)] dt \\ &\quad - \gamma^2 \sum_{k=0}^{\infty} (t_{k+1} - t_k) [v^T(t_k - h_k) v(t_k - h_k) \\ &\quad + v^T(t_{k-1} - h_{k-1}) v(t_{k-1} - h_{k-1})]. \end{aligned} \tag{12}$$

The above H_∞-performance extends the indexes of [13,20] to the case of Round-Robin scheduling. It takes into account the updating rates of the measurement and is related to the energy of the measurement noise. The system (8) is said to have an L_2-gain less than γ if $J < 0$ along (8) for the zero initial function and for all non-zero $w \in L_2$, $v \in l_2$.

2.2. Dynamic output feedback

Consider now (1) under the dynamic output feedback controller (7), where we assume that the controller is directly connected to the actuator, i.e. $h_k = h_k^{sc}$. The closed-loop system (1), (7) can be presented in the form of (8), where the system state and the matrices are changed by the ones with the bars as follows:

$$\bar{x} = [x^T \ x_c^T]^T, \qquad \bar{A} = \begin{bmatrix} A & BC_c \\ 0 & A_c \end{bmatrix}, \qquad \bar{A}_1 = \begin{bmatrix} BD_c \begin{bmatrix} C^1 \\ 0 \end{bmatrix} & 0 \\ B_c \begin{bmatrix} C^1 \\ 0 \end{bmatrix} & 0 \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} BD_c \begin{bmatrix} 0 \\ C^2 \end{bmatrix} & 0 \\ B_c \begin{bmatrix} 0 \\ C^2 \end{bmatrix} & 0 \end{bmatrix}, \qquad \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$

$$\bar{D}_{21} = \begin{bmatrix} BD_c \begin{bmatrix} F^1 \\ 0 \end{bmatrix} \\ B_c \begin{bmatrix} F^1 \\ 0 \end{bmatrix} \end{bmatrix}, \qquad \bar{D}_{22} = \begin{bmatrix} BD_c \begin{bmatrix} 0 \\ F^2 \end{bmatrix} \\ B_c \begin{bmatrix} 0 \\ F^2 \end{bmatrix} \end{bmatrix},$$

$$\bar{C}_0 = [C_0 \ \ D_{12} C_c], \qquad \bar{D}_1 = \begin{bmatrix} D_{12} D_c \begin{bmatrix} C^1 \\ 0 \end{bmatrix} & 0 \end{bmatrix},$$

$$\bar{D}_2 = \begin{bmatrix} D_{12} D_c \begin{bmatrix} 0 \\ C^2 \end{bmatrix} & 0 \end{bmatrix}, \qquad \bar{E}_{21} = D_{12} D_c \begin{bmatrix} F^1 \\ 0 \end{bmatrix},$$

$$\bar{E}_{22} = D_{12} D_c \begin{bmatrix} 0 \\ F^2 \end{bmatrix}.$$

In the present paper, we will derive LMI conditions for the exponential stability of the disturbance-free switched model (8) and for the L_2-gain-analysis of (8) via direct Lyapunov method. The results under variable network-induced delay is studied in Section 3.2. Note that in some NCS (such as Control Area Networks, where transmission over the network is almost instantaneous compared to the plant dynamic) the network-induced transmission delay may be usually neglected (see e.g. [14]). However, a constant delay may appear due to measurement or due to control computation. Therefore, Section 3.3 considers Round-Robin scheduling under variable sampling with constant delay. Finally, the efficiency of the new criteria is illustrated via benchmark examples of batch reactor and cart-pendulum.

Remark 1. In the above reasoning, we assumed that packet loss does not occur. However, for small delays $h_k < s_{k+1} - s_k$ we could accommodate for packet dropouts by modeling them as prolongations of the transmission interval similar to [15,17].

3. Stability and L_2-gain analysis

3.1. Useful lemmas

We will apply the following lemmas:

Lemma 1. *Let there exist positive numbers β, δ and a functional $V : \mathcal{R} \times W \times L_2[-\bar{\tau}_M, 0] \to [t_0, \infty)$ such that*

$$\beta|\phi(0)|^2 \le V(t, \phi, \dot{\phi}) \le \delta\|\phi\|_W^2. \tag{13}$$

Let the function $\bar{V}(t) = V(t, x_t, \dot{x}_t)$ be continuous from the right for $x(t)$ satisfying (8), absolutely continuous for $t \neq t_k$ and satisfies

$$\lim_{t \to t_k^-} \bar{V}(t) \ge \bar{V}(t_k). \tag{14}$$

(i) *If along (8) with $w = 0$ and $v = 0$*

$$\dot{\bar{V}}(t) \le -\tilde{\beta}|x(t)|^2 \quad \text{for } t \neq t_k \text{ and for some scalar } \tilde{\beta} > 0,$$

then (8) with $w = 0$ and $v = 0$ is asymptotically stable.

(ii) *If along (8) with $w = 0$ and $v = 0$ for some $\alpha > 0$*

$$\dot{\bar{V}}(t) + 2\alpha\bar{V}(t) \le 0 \quad \text{for } t \neq t_k,$$

then (8) with $w = 0$ and $v = 0$ is exponentially stable with the decay rate α.

(iii) *For a given $\gamma > 0$, if along (8)*

$$\dot{\bar{V}}(t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) - \gamma^2[v^T(t_k - h_k)v(t_k - h_k) + v^T(t_{k-1} - h_{k-1})v(t_{k-1} - h_{k-1})] < 0, \quad t \in [t_k, t_{k+1}),\ k \in \mathcal{N}, \tag{15}$$

then the performance index (12) achieves $J < 0$ for all non-zero $w \in L_2$, $v \in l_2$ and for the zero initial function.

Proof. (i) and (ii) follow from the standard arguments for the switched and the time-delay systems (see e.g. [21,7]).

(iii) Given $N \gg 1$, we integrate inequality (15) from t_0 till t_N. We have for $w \neq 0$, $v \neq 0$

$$\bar{V}(t_N) - \bar{V}(t_{N-1}) + \bar{V}(t_{N-1}^-) - \bar{V}(t_{N-2}) \cdots + \bar{V}(t_1^-) - \bar{V}(t_0) + \int_{t_0}^{t_N}[z^T(s)z(s) - \gamma^2 w^T(s)w(s)]ds - \gamma^2\sum_{k=0}^{N-1}(t_{k+1} - t_k)[v^T(t_k - h_k)v(t_k - h_k) + v^T(t_{k-1} - h_{k-1})v(t_{k-1} - h_{k-1})] < 0.$$

Taking into account that $\bar{V}(t_N) \ge 0$, $\bar{V}(t_k^-) - \bar{V}(t_k) \ge 0$ for $k = 1, \ldots, N-1$ and $\bar{V}(t_0) = 0$, we arrive for $N \to \infty$ to $J < 0$. □

Lemma 2 ([22]). *Let $f_1, f_2, \ldots, f_N : \mathcal{R}^n \to \mathcal{R}$ have positive values in an open subset $\mathcal{D}$ of $\mathcal{R}^n$. Then, the reciprocally convex combination of f_i over $\mathcal{D}$ satisfies*

$$\min_{\left\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\right\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j} : \mathcal{R}^n \to \mathcal{R}, g_{j,i}(t) \triangleq g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \ge 0 \right\}.$$

3.2. Stability and L_2-gain analysis of NCS: variable h_k

Consider the switched system (8) as a system with two ordered time-varying delays $\tau_1(t)$ and $\tau_2(t)$ either from $[h_m, \tau_M]$ or from $[h_m, \bar{\tau}_M]$. Our stability and L_2-gain analysis will be based on the common (for both subsystems of the switched system) time-independent Lyapunov–Krasovskii Functional (LKF) for the exponential stability of systems with time-varying delay from the maximum delay interval $[h_m, \bar{\tau}_M]$ [23,24]:

$$V(x_t, \dot{x}_t) = \bar{V}(t) = V_0(x_t, \dot{x}_t) + V_1(x_t, \dot{x}_t), \tag{16}$$

where

$$\begin{aligned} V_0(x_t, \dot{x}_t) &= x^T(t)Px(t) + \int_{t-h_m}^{t} e^{2\alpha(s-t)}x^T(s)S_0x(s)ds \\ &\quad + h_m\int_{-h_m}^{0}\int_{t+\theta}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)R_0\dot{x}(s)dsd\theta, \\ V_1(x_t, \dot{x}_t) &= \int_{t-\bar{\tau}_M}^{t-h_m} e^{2\alpha(s-t)}x^T(s)S_1x(s)ds \\ &\quad + (\bar{\tau}_M - h_m)\int_{-\bar{\tau}_M}^{-h_m}\int_{t+\theta}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)R_1\dot{x}(s)dsd\theta, \\ &\quad P > 0,\ S_0 > 0, R_0 > 0,\ S_1 > 0,\ R_1 > 0,\ \alpha > 0. \end{aligned} \tag{17}$$

By taking into account the order of the delays τ_1, τ_2 and applying convexity arguments of [22] we prove the following:

Theorem 1. (i) *Given $0 \le h_m < \tau_M, \alpha > 0$ if there exist $n \times n$ matrices $P > 0, S_i > 0, R_i > 0 (i = 0, 1)$, $G_1^i, G_2^i, G_3^i (i = 1, 2)$ such that the following four LMIs are feasible:*

$$\Omega_i = \begin{bmatrix} R_1 & G_1^i & G_2^i \\ * & R_1 & G_3^i \\ * & * & R_1 \end{bmatrix} \ge 0, \tag{18}$$

$$\Xi_i = \begin{bmatrix} \Phi_{11} & R_0e^{-2\alpha h_m} & PA_i & PA_{3-i} & 0 & A^TH \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & G_2^ie^{-2\alpha\bar{\tau}_M} & 0 \\ * & * & \Phi_{33} & \Phi_{34} & (G_3^i - G_2^i)e^{-2\alpha\bar{\tau}_M} & A_i^TH \\ * & * & * & \Phi_{44} & (R_1 - G_3^i)e^{-2\alpha\bar{\tau}_M} & A_{3-i}^TH \\ * & * & * & * & -(S_1 + R_1)e^{-2\alpha\bar{\tau}_M} & 0 \\ * & * & * & * & * & -H \end{bmatrix} < 0, \tag{19}$$

where

$$\begin{aligned} \Phi_{11} &= PA + A^TP + S_0 - R_0e^{-2\alpha h_m} + 2\alpha P, \\ \Phi_{22} &= (-S_0 + S_1 - R_0)e^{-2\alpha h_m} - R_1e^{-2\alpha\bar{\tau}_M}, \\ \Phi_{23} &= (R_1 - G_1^i)e^{-2\alpha\bar{\tau}_M}, \\ \Phi_{24} &= (G_1^i - G_2^i)e^{-2\alpha\bar{\tau}_M}, \\ \Phi_{33} &= (-2R_1 + G_1^i + G_1^{iT})e^{-2\alpha\bar{\tau}_M}, \\ \Phi_{34} &= (R_1 - G_1^i + G_2^i - G_3^i)e^{-2\alpha\bar{\tau}_M}, \\ \Phi_{44} &= (-2R_1 + G_3^i + G_3^{iT})e^{-2\alpha\bar{\tau}_M}, \\ H &= h_m^2R_0 + (\bar{\tau}_M - h_m)^2R_1, \quad i = 1, 2. \end{aligned} \tag{20}$$

Then system (8) with $w = 0$ and $v = 0$ is exponentially stable with the decay rate α.

(ii) *Given* $\gamma > 0$, *if* (18) *and the following LMIs*

$$\begin{bmatrix} & | & PB_1 & PD_{21} & PD_{22} & C_0^T \\ & | & 0 & 0 & 0 & 0 \\ \Xi_i|_{\alpha=0} & | & 0 & 0 & 0 & D_i^T \\ & | & 0 & 0 & 0 & D_{3-i}^T \\ & | & 0 & 0 & 0 & 0 \\ & | & HB_1 & HD_{21} & HD_{22} & 0 \\ - & - & - & - & - & - \\ * & | & -\gamma^2 I & 0 & 0 & 0 \\ * & | & * & -\gamma^2 I & 0 & E_{21}^T \\ * & | & * & * & -\gamma^2 I & E_{22}^T \\ * & | & * & * & * & -I \end{bmatrix} < 0, \qquad (21)$$

$i = 1, 2$ *with notations given in* (20) *are feasible. Then* (8) *is internally exponentially stable and has* L_2*-gain less than* γ.

Proof. (i) Differentiating $\bar{V}(t)$ along (8) with $w = 0$ and $v = 0$, we have

$$\begin{aligned}\dot{\bar{V}}(t) + 2\alpha\bar{V}(t) \le\ & 2x^T(t)P\dot{x}(t) + x^T(t)[S_0 + 2\alpha P]x(t) \\ & - x^T(t-h_m)[S_0 - S_1]e^{-2\alpha h_m}x(t-h_m) \\ & + \dot{x}^T(t)[h_m^2 R_0 + (\bar{\tau}_M - h_m)^2 R_1]\dot{x}(t) \\ & - x^T(t-\bar{\tau}_M)S_1 e^{-2\alpha\bar{\tau}_M}x(t-\bar{\tau}_M) \\ & - h_m e^{-2\alpha h_m}\int_{t-h_m}^{t}\dot{x}^T(s)R_0\dot{x}(s)ds \\ & - (\bar{\tau}_M - h_m)e^{-2\alpha\bar{\tau}_M}\int_{t-\bar{\tau}_M}^{t-h_m}\dot{x}^T(s)R_1\dot{x}(s)ds.\end{aligned}$$

Consider $t \in [t_k, t_{k+1})$. By Jensen's inequality [25], we have

$$\begin{aligned} & h_m\int_{t-h_m}^{t}\dot{x}^T(s)R_0\dot{x}(s)ds \ge \int_{t-h_m}^{t}\dot{x}^T(s)dsR_0\int_{t-h_m}^{t}\dot{x}(s)ds \\ & \quad = [x(t) - x(t-h_m)]^T R_0[x(t) - x(t-h_m)].\end{aligned}$$

Taking into account that $t_{k-1} - h_{k-1} < t_k - h_k$ (i.e. that the delays satisfy the relation (11)) and applying further Jensen's inequality we obtain

$$\begin{aligned} & -(\bar{\tau}_M - h_m)\int_{t-\bar{\tau}_M}^{t-h_m}\dot{x}^T(s)R_1\dot{x}(s)ds \\ & = -(\bar{\tau}_M - h_m)\Bigg[\int_{t_k-h_k}^{t-h_m}\dot{x}^T(s)R_1\dot{x}(s)ds \\ & \quad + \int_{t_{k-1}-h_{k-1}}^{t_k-h_k}\dot{x}^T(s)R_1\dot{x}(s)ds + \int_{t-\bar{\tau}_M}^{t_{k-1}-h_{k-1}}\dot{x}^T(s)R_1\dot{x}(s)ds\Bigg] \\ & \le -\frac{1}{\alpha_1}f_1(t) - \frac{1}{\alpha_2}f_2(t) - \frac{1}{\alpha_3}f_3(t), \end{aligned} \qquad (22)$$

where

$$\alpha_1 = \frac{t - h_m - t_k + h_k}{\bar{\tau}_M - h_m}, \qquad \alpha_2 = \frac{t_k - h_k - t_{k-1} + h_{k-1}}{\bar{\tau}_M - h_m},$$

$$\alpha_3 = \frac{\bar{\tau}_M - t + t_{k-1} - h_{k-1}}{\bar{\tau}_M - h_m},$$

$$f_1(t) = [x(t-h_m) - x(t_k-h_k)]^T R_1[x(t-h_m) - x(t_k-h_k)],$$

$$\begin{aligned} f_2(t) &= [x(t_k-h_k) - x(t_{k-1}-h_{k-1})]^T \\ &\quad \times R_1[x(t_k-h_k) - x(t_{k-1}-h_{k-1})], \\ f_3(t) &= [x(t_{k-1}-h_{k-1}) - x(t-\bar{\tau}_M)]^T \\ &\quad \times R_1[x(t_{k-1}-h_{k-1}) - x(t-\bar{\tau}_M)].\end{aligned}$$

Denote

$$\begin{aligned} g_{1,2}(t) &= [x(t-h_m) - x(t_k-h_k)]^T \\ &\quad \times G_1^1[x(t_k-h_k) - x(t_{k-1}-h_{k-1})], \\ g_{1,3}(t) &= [x(t-h_m) - x(t_k-h_k)]^T \\ &\quad \times G_2^1[x(t_{k-1}-h_{k-1}) - x(t-\bar{\tau}_M)], \\ g_{2,3}(t) &= [x(t_k-h_k) - x(t_{k-1}-h_{k-1})]^T \\ &\quad \times G_3^1[x(t_{k-1}-h_{k-1}) - x(t-\bar{\tau}_M)].\end{aligned}$$

Note that (18) with $i = 1$ guarantees $\begin{bmatrix} R_1 & G_j^1 \\ * & R_1 \end{bmatrix} \ge 0$ $(j = 1, 2, 3)$, and, thus,

$$\begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \ge 0, \quad i \ne j,\ i = 1, 2,\ j = 2, 3.$$

Then by Lemma 2, we arrive to

$$\begin{aligned} & -(\bar{\tau}_M - h_m)\int_{t-\bar{\tau}_M}^{t-h_m}\dot{x}^T(s)R_1\dot{x}(s)ds \\ & \quad \le -\frac{1}{\alpha_1}f_1(t) - \frac{1}{\alpha_2}f_2(t) - \frac{1}{\alpha_3}f_3(t) \\ & \quad \le -f_1(t) - f_2(t) - f_2(t) - 2g_{1,2}(t) - 2g_{1,3}(t) - 2g_{2,3}(t) \\ & \quad = -\lambda^T(t)\Omega_1\lambda(t), \end{aligned}$$

where $\lambda(t) = col\{x(t-h_m) - x(t_k-h_k), x(t_k-h_k) - x(t_{k-1} - h_{k-1}), x(t_{k-1}-h_{k-1}) - x(t-\bar{\tau}_M)\}$ and Ω_1 is given by (18) with $i = 1$.

Hence, setting $\xi(t) = col\{x(t), x(t-h_m), x(t_k-h_k), x(t_{k-1} - h_{k-1}), x(t-\bar{\tau}_M)\}$, we find that

$$\begin{aligned} & \dot{\bar{V}}(t) + 2\alpha\bar{V}(t) \\ & \le \xi^T(t)\begin{bmatrix} \Phi_{11} & R_0e^{-2\alpha h_m} & PA_1 & PA_2 & 0 \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & G_2^1e^{-2\alpha\bar{\tau}_M} \\ * & * & \Phi_{33} & \Phi_{34} & (-G_2^1 + G_3^1)e^{-2\alpha\bar{\tau}_M} \\ * & * & * & \Phi_{44} & (R_1 - G_3^1)e^{-2\alpha\bar{\tau}_M} \\ * & * & * & * & (-S_1 - R_1)e^{-2\alpha\bar{\tau}_M} \end{bmatrix}\xi(t) \\ & \quad + [Ax(t) + A_1x(t_k-h_k) + A_2x(t_{k-1}-h_{k-1})]^T H \\ & \quad \times [Ax(t) + A_1x(t_k-h_k) + A_2x(t_{k-1}-h_{k-1})], \end{aligned}$$

where notations are given by (20) with $i = 1$. Hence, by Schur complements, (19) with $i = 1$ guarantees that $\dot{\bar{V}}(t) + 2\alpha\bar{V}(t) \le 0$ for $t \in [t_k, t_{k+1})$.

Similarly, when $t \in [t_{k+1}, t_{k+2})$, we prove that (18) and (19) with $i = 2$ can guarantee $\dot{\bar{V}}(t) + 2\alpha\bar{V}(t) < 0$. Thus by (ii) of Lemma 1, (8) with $w = 0$ and $v = 0$ is exponentially stable with the decay rate α.

(ii) By using arguments of (i), we find that (15) holds along (8) if LMIs (18) and (21) are feasible, which completes the proof. □

Remark 2. The switched system (8) can be alternatively analyzed by the standard arguments for systems with two (independent) time-varying delays. However, this leads to the overlapping delay intervals $\tau_1(t) \in [h_m, \tau_M]$ and $\tau_2(t) \in [h_m, \bar{\tau}_M]$, which may be conservative since the relation $\tau_1(t) < \tau_2(t)$ is ignored. We give now the standard LKF for two independent delays: $V(x_t, \dot{x}_t) = \sum_{i=0}^{2}V_i(x_t, \dot{x}_t)$, where $V_i(x_t, \dot{x}_t) (i = 0, 1)$ are given by (17) with $\bar{\tau}_M$ changed by τ_M and

$$\begin{aligned} & V_2(x_t, \dot{x}_t) \\ & \quad = \int_{t-\bar{\tau}_M}^{t-h_m} e^{2\alpha(s-t)}x^T(s)S_2x(s)ds \\ & \qquad + (\bar{\tau}_M - h_m)\int_{-\bar{\tau}_M}^{-h_m}\int_{t+\theta}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)R_2\dot{x}(s)dsd\theta, \\ & \qquad S_2 > 0, R_2 > 0, \alpha > 0. \end{aligned}$$

Extending arguments of [22] to the exponential stability of systems with two delays, we obtain the following LMIs:

$$\begin{bmatrix} R_i & G_i^1 & 0 & 0 \\ * & R_i & 0 & 0 \\ * & * & R_i & G_i^2 \\ * & * & * & R_i \end{bmatrix} \geq 0, \tag{23}$$

$$\tilde{\Xi}_i = \begin{bmatrix} \Phi_{11} & R_0 e^{-2\alpha h_m} & PA_i & PA_{3-i} & 0 & 0 & A^T\tilde{H} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & G_1^i e^{-2\alpha\tau_M} & G_2^i e^{-2\alpha\bar{\tau}_M} & 0 \\ * & * & \Sigma_{33} & 0 & \Sigma_{23} & 0 & A_i^T\tilde{H} \\ * & * & * & \Sigma_{44} & 0 & \Sigma_{24} & A_{3-i}^T\tilde{H} \\ * & * & * & * & \Sigma_{55} & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 \\ * & * & * & * & * & * & -\tilde{H} \end{bmatrix} < 0, \tag{24}$$

where Φ_{11} is given in (20) and

$$\begin{aligned} \Sigma_{22} &= (-S_0 + S_1 + S_2 - R_0)e^{-2\alpha h_m} - R_1 e^{-2\alpha\tau_M} - R_2 e^{-2\alpha\bar{\tau}_M}, \\ \Sigma_{23} &= (R_1 - G_1^i)e^{-2\alpha\tau_M}, \\ \Sigma_{24} &= (R_2 - G_2^i)e^{-2\alpha\bar{\tau}_M}, \\ \Sigma_{33} &= (-2R_1 + G_1^i + G_1^{iT})e^{-2\alpha\tau_M}, \\ \Sigma_{44} &= (-2R_2 + G_2^i + G_2^{iT})e^{-2\alpha\bar{\tau}_M}, \\ \Sigma_{55} &= -(S_1 + R_1)e^{-2\alpha\tau_M}, \\ \Sigma_{66} &= -(S_2 + R_2)e^{-2\alpha\bar{\tau}_M}, \\ \tilde{H} &= h_m^2 R_0 + (\tau_M - h_m)^2 R_1 + (\bar{\tau}_M - h_m)^2 R_2, \quad i = 1, 2. \end{aligned} \tag{25}$$

The LMIs for L_2-gain analysis are given by (21) with Ξ and H changed by $\tilde{\Xi}$ and $\tilde{H}$ respectively. Note that LMIs (23), (24) for independent delays possess the same number of decision variables as LMIs (18), (19) of Theorem 1 (up to the symmetry of R_2, S_2), but *have a higher-order*:

2 LMIs of $7n \times 7n$ and 2 LMIs of $4n \times 4n$ in (23), (24),
2 LMIs of $6n \times 6n$ and 2 LMIs of $3n \times 3n$ in (18), (19).

The examples below illustrate the improvement (in the maximum value of τ_M which preserves the stability and in the computational time) by taking into account the order of the delays.

Remark 3. Our method can be extended to the dynamic output feedback with two networks, where

$$u(t) = C_c x_c(t_k - h_k^{ca}) + D_c y_k, \quad t \in [t_k, t_{k+1}).$$

In this case the closed-loop system can be considered as the system with three delays: two ordered $\tau_1(t) = t - t_k + h_k < \tau_2(t) = t - t_{k-1} + h_{k-1}$ and the independent one $\tau_3(t) = t - t_k + h_k^{ca}$. Such a system can be analyzed by combining the standard Lyapunov-based method (for $\tau_3(t)$) with the Theorem 1 (for the ordered delays $\tau_1(t)$ and $\tau_2(t)$).

3.3. Stability and L_2-gain analysis of NCS: constant h

In this subsection we consider the case of negligible network-induced delay, where $h \geq 0$ is the constant measurement delay. We note that, till recently, the conventional time-independent LKF of (16) (for systems with interval time-varying delays) were applied to NCS (see e.g. [4,11]). These functionals did not take advantage of the sawtooth evolution of the delays induced by sample-and-hold. The latter drawback was removed in [5,7], where time-dependent Lyapunov functionals for sampled-data systems were introduced. We will adapt to the Round-Robin scheduling a time-dependent Lyapunov functional construction of [26], which is based on the extension of Wirtinger's inequality [27] to the vector case:

Lemma 3 ([28]). *Let $z(t) \in W[a, b)$ and $z(a) = 0$. Then for any $n \times n$-matrix $R > 0$ the following inequality holds:*

$$\int_a^b z^T(\xi)Rz(\xi)d\xi \leq \frac{4(b-a)^2}{\pi^2}\int_a^b \dot{z}^T(\xi)R\dot{z}(\xi)d\xi. \tag{26}$$

We introduce the following *discontinuous in time* Lyapunov functional:

$$\begin{aligned} V(t, x_t, \dot{x}_t) &= \bar{V}_1(t) \\ &= V_0(x_t, \dot{x}_t) + V_1(t, x_t, \dot{x}_t) + V_2(t, x_t, \dot{x}_t), \end{aligned} \tag{27}$$

where $V_0(x_t, \dot{x}_t)$ is given by (17) with $h_m = h, \alpha = 0$ and

$$\begin{aligned} V_1(t, x_t, \dot{x}_t) &= 4(\tau_M - h)^2 \int_{t_k-h}^{t} \dot{x}^T(s)W_1\dot{x}(s)ds \\ &\quad - \frac{\pi^2}{4}\int_{t_k-h}^{t-h}[x(s) - x(t_k - h)]^T W_1[x(s) - x(t_k - h)]ds, \\ &\quad t \in [t_k, t_{k+2}), \end{aligned}$$

$$V_2(t, x_t, \dot{x}_t) = \begin{cases} 4(\tau_M - h)^2 \int_{t_{k-1}-h}^{t} \dot{x}^T(s)W_2\dot{x}(s)ds \\ \quad - \frac{\pi^2}{4}\int_{t_{k-1}-h}^{t-h}[x(s) - x(t_{k-1} - h)]^T W_2 \\ \quad \times [x(s) - x(t_{k-1} - h)]ds, \quad t \in [t_k, t_{k+1}), \\ 4(\tau_M - h)^2 \int_{t_{k+1}-h}^{t} \dot{x}^T(s)W_2\dot{x}(s)ds \\ \quad - \frac{\pi^2}{4}\int_{t_{k+1}-h}^{t-h}[x(s) - x(t_{k+1} - h)]^T W_2 \\ \quad \times [x(s) - x(t_{k+1} - h)]ds, \quad t \in [t_{k+1}, t_{k+2}), \end{cases}$$

$$W_1 > 0, \; W_2 > 0, \; k = 2p.$$

Note that V_0 is a "nominal" Lyapunov functional for the "nominal" system with a constant delay

$$\dot{x}(t) = Ax(t) + (A_1 + A_2)x(t - h). \tag{28}$$

The terms V_1, V_2 are extensions of the discontinuous constructions of [26]. We note that V_1 can be represented as a sum of the continuous in time term $4(\tau_M - h)^2 \int_{t-h}^{t} \dot{x}^T(s)W_1\dot{x}(s)ds \geq 0, \; t \in [t_k, t_{k+2})$, with the discontinuous (for $t = t_k$) one

$$\begin{aligned} V_{W1} &\triangleq 4(\tau_M - h)^2 \int_{t_k-h}^{t-h} \dot{x}^T(s)W_1\dot{x}(s)ds \\ &\quad - \frac{\pi^2}{4}\int_{t_k-h}^{t-h}[x(s) - x(t_k - h)]^T W_1[x(s) - x(t_k - h)]ds, \\ &\quad t \in [t_k, t_{k+2}). \end{aligned}$$

Note that $V_{W1|t=t_k} = 0$ and, by the extended Wirtinger's inequality (26), $V_{W1} \geq 0$ for all $t \geq t_0$. Therefore, V_1 does not grow in the jumps.

In a similar way, V_2 can be represented as a sum of the continuous in time term $4(\tau_M - h)^2 \int_{t-h}^{t} \dot{x}^T(s)W_2\dot{x}(s)ds \geq 0$, with the discontinuous for $t = t_{k+1}$ term

$$V_{W2} \triangleq \begin{cases} -\frac{\pi^2}{4}\int_{t_{k-1}-h}^{t-h}[x(s) - x(t_{k-1} - h)]^T W_2[x(s) - x(t_{k-1} - h)]ds \\ \quad + 4(\tau_M - h)^2 \int_{t_{k-1}-h}^{t-h} \dot{x}^T(s)W_2\dot{x}(s)ds, \quad t \in [t_k, t_{k+1}), \\ -\frac{\pi^2}{4}\int_{t_{k+1}-h}^{t-h}[x(s) - x(t_{k+1} - h)]^T W_2[x(s) - x(t_{k+1} - h)]ds \\ \quad + 4(\tau_M - h)^2 \int_{t_{k+1}-h}^{t-h} \dot{x}^T(s)W_2\dot{x}(s)ds, \quad t \in [t_{k+1}, t_{k+2}). \end{cases}$$

We have $V_{W2|t=t_{k+1}} = 0$ and, by the extended Wirtinger's inequality (26), $V_{W2} \geq 0$ for all $t \geq t_0$, i.e. V_2 does not grow in the jumps. Therefore, $\bar{V}_1$ does not grow in the jumps: $\lim_{t\to t_k^-} \bar{V}_1(t) \geq \bar{V}_1(t_k)$ and $\lim_{t\to t_{k+1}^-} \bar{V}_1(t) \geq \bar{V}_1(t_{k+1})$ hold.

Theorem 2. *(i) Given $\tau_M > h \geq 0$, the system (8) with $h_k \equiv h$, $w = 0$ and $v = 0$ is asymptotically stable, if there exist $n\times n$ matrices $P > 0, R_0 > 0, S_0 > 0, W_i > 0 (i = 1, 2)$, such that the following LMI is feasible:*

$$\Xi = \begin{bmatrix} \Psi_1 & R_0 & PA_1 & PA_2 & A^TW \\ * & \Psi_2 & \frac{\pi^2}{4}W_1 & \frac{\pi^2}{4}W_2 & 0 \\ * & * & -\frac{\pi^2}{4}W_1 & 0 & A_1^TW \\ * & * & * & -\frac{\pi^2}{4}W_2 & A_2^TW \\ * & * & * & * & -W \end{bmatrix} < 0, \tag{29}$$

where

$$\begin{aligned} &\Psi_1 = PA + A^TP + S_0 - R_0, \\ &\Psi_2 = -\frac{\pi^2}{4}W_1 - \frac{\pi^2}{4}W_2 - S_0 - R_0, \\ &W = h^2R_0 + 4(\tau_M - h)^2(W_1 + W_2). \end{aligned} \tag{30}$$

(2) Given $\gamma > 0$, if the following LMI

$$\left[\begin{array}{c|cccc} & PB_1 & PD_{21} & PD_{22} & C_0^T \\ & 0 & 0 & 0 & 0 \\ \Xi & 0 & 0 & 0 & D_1^T \\ & 0 & 0 & 0 & D_2^T \\ & WB_1 & WD_{21} & WD_{22} & 0 \\ \hline * & -\gamma^2I & 0 & 0 & 0 \\ * & * & -\gamma^2I & 0 & E_{21}^T \\ * & * & 0 & -\gamma^2I & E_{22}^T \\ * & * & * & * & -I \end{array}\right] < 0 \tag{31}$$

with notations given by (30) is feasible. Then (8) with $h_k \equiv h$ is internally stable and has L_2-gain less than γ.

Proof. (i) Differentiating $V_0(x_t, \dot{x}_t)$ and applying Jensen's inequality [25], we have

$$\begin{aligned} \frac{d}{dt}V_0(x_t, \dot{x}_t) \leq\ & 2x^T(t)P\dot{x}(t) + x^T(t)S_0x(t) \\ & - x^T(t-h)S_0x(t-h) + h^2\dot{x}^T(t)R_0\dot{x}(t) \\ & - [x(t) - x(t-h)]^TR_0[x(t) - x(t-h)]. \end{aligned}$$

Consider $t \in [t_k, t_{k+1})$. Along (8) with $h_k \equiv h$, $w = 0$ and $v = 0$ we have

$$\begin{aligned} &\frac{d}{dt}\sum_{i=1}^{2} V_i(t, x_t, \dot{x}_t) \\ &= \dot{x}^T(t)[4(\tau_M - h)^2(W_1 + W_2)]\dot{x}(t) \\ &\quad -\frac{\pi^2}{4}[x(t-h) - x(t_k - h)]^TW_1[x(t-h) - x(t_k - h)] \\ &\quad -\frac{\pi^2}{4}[x(t-h) - x(t_{k-1} - h)]^TW_2[x(t-h) - x(t_{k-1} - h)]. \end{aligned}$$

Then substitution of $Ax(t) + A_1x(t_k - h) + A_2x(t_{k-1} - h)$ for $\dot{x}(t)$ leads to

$$\begin{aligned} \dot{\bar{V}}_1(t) \leq\ & \zeta^T(t)\begin{bmatrix} \Psi_1 & R_0 & PA_1 & PA_2 \\ * & \Psi_2 & \frac{\pi^2}{4}W_1 & \frac{\pi^2}{4}W_2 \\ * & * & -\frac{\pi^2}{4}W_1 & 0 \\ * & * & * & -\frac{\pi^2}{4}W_2 \end{bmatrix}\zeta(t) \\ & + [Ax(t) + A_1x(t_k - h) + A_2x(t_{k-1} - h)]^TW \\ & \times [Ax(t) + A_1x(t_k - h) + A_2x(t_{k-1} - h)], \end{aligned}$$

where $\zeta(t) = col\{x(t), x(t-h), x(t_k - h), x(t_{k-1} - h)\}$ and W is given in (30). Hence, by Schur complements, (29) guarantees that $\dot{\bar{V}}_1(t) \leq -\tilde{\beta}|x(t)|^2$ for some $\tilde{\beta} > 0$.

Similarly, when $t \in [t_{k+1}, t_{k+2})$, $k = 2p$, we prove that (29) guarantees $\dot{\bar{V}}_1(t) \leq -\tilde{\beta}|x(t)|^2$ for some $\tilde{\beta} > 0$, which proves the asymptotic stability (see Lemma 1(i)).

(ii) By using arguments of (i), we find that (15) with $\dot{\bar{V}}(t) = \dot{\bar{V}}_1(t)$ holds along (8) with $h_k \equiv h$ if LMI (31) is feasible, which completes the proof. □

Remark 4. Compared to the stability LMI conditions of Theorem 1, of Remark 2 and of [18], the LMI of Theorem 2 is essentially simpler (single LMI of $5n \times 5n$ with fewer decision variables) and is less conservative (see Examples below).

Remark 5. Similar to [26], the decay rate of the exponential stability for (8) can be found by changing the variable $\bar{x}(t) = x(t)e^{\alpha t}$ and by applying LMI (29) to the resulting system with polytopic type uncertainty.

For the case of constant delay h, we can combine the methods of Theorems 1 and 2. Thus the following Corollary is obtained:

Corollary 1. *Given $\tau_M > h \geq 0$, the system (8) with $h_k \equiv h$, $w = 0$ and $v = 0$ is asymptotically stable, if there exist $n \times n$ matrices $P > 0, S_i > 0, R_i > 0 (i = 0, 1), G_1^i, G_2^i, G_3^i, W_i > 0 (i = 1, 2)$, such that (18) and $\Xi_i|_{\alpha=0} (i = 1, 2)$ hold, where $\Xi_i (i = 1, 2)$ are defined in (19) with h_m changed by h and $\Phi_{22}, \Phi_{23}, \Phi_{24}, \Phi_{33}, \Phi_{44}, H$ changed by*

$$\begin{aligned} &\Phi_{22} = -S_0 + S_1 - R_0 - R_1 - \frac{\pi^2}{4}W_1 - \frac{\pi^2}{4}W_2, \\ &\Phi_{23} = R_1 - G_1^i + \frac{\pi^2}{4}W_i, \\ &\Phi_{24} = G_1^i - G_2^i + \frac{\pi^2}{4}W_{3-i}, \\ &\Phi_{33} = -2R_1 + G_1^i + G_1^{iT} - \frac{\pi^2}{4}W_i, \\ &\Phi_{44} = -2R_1 + G_3^i + G_3^{iT} - \frac{\pi^2}{4}W_{3-i}, \\ &H = h^2R_0 + 4(\tau_M - h)^2(R_1 + W_1 + W_2), \quad i = 1, 2. \end{aligned} \tag{32}$$

We note that the LMIs of Corollary 1 with $W_i = 0 (i = 1, 2)$ or with $R_1 = S_1 = G_j^i = 0 (i = 1, 2, j = 1, 2, 3)$ are reduced to the ones of Theorem 1 or of Theorem 2, respectively.

Remark 6. When there is no measurement delay, i.e. $h_k \equiv 0$, the problem for NCS is reduced to the one for sampled-data systems with scheduling (see e.g. [14]), where the closed-loop system has a form of (8) with $h_k \equiv 0, k \in \mathcal{N}$. As we will see in the example below, for $h \to 0$ the conditions of Theorem 2 become conservative. Less conservative conditions can be derived in this case via different from (27) *continuous in time* Lyapunov functionals.

For the constant sampling, where $t_{k+1}-t_k=\tau_M,\ k\in\mathcal{N}$, choose Lyapunov functional of the form

$$V(t,x_t,\dot{x}_t)=\bar{V}_2(t)=x^T(t)Px(t)+\sum_{i=1}^{2}V_i(t,\dot{x}_t)$$
$$+\sum_{i=1}^{2}V_{Xi}(t,x_t),\quad P>0,\ t\in[t_k,t_{k+2}),\ k=2p,\ p\in\mathcal{N},\quad (33)$$

where

$$V_1(t,\dot{x}_t)=(t_{k+2}-t)\int_{t_k}^{t}e^{2\alpha(s-t)}\dot{x}^T(s)U_1\dot{x}(s)ds,$$

$$V_2(t,\dot{x}_t)=\begin{cases}(t_{k+1}-t)\int_{t_{k-1}}^{t}e^{2\alpha(s-t)}\dot{x}^T(s)U_2\dot{x}(s)ds,\\ \quad t\in[t_k,t_{k+1}),\\ (t_{k+3}-t)\int_{t_{k+1}}^{t}e^{2\alpha(s-t)}\dot{x}^T(s)U_2\dot{x}(s)ds,\\ \quad t\in[t_{k+1},t_{k+2}),\end{cases}$$

$$V_{X1}(t,x_t)=(t_{k+2}-t)\xi_0^T(t)\begin{bmatrix}\frac{X+X^T}{2} & -X+X_1\\ * & -\bar{X}_1\end{bmatrix}\xi_0(t),$$

$$V_{X2}(t,x_t)=\begin{cases}(t_{k+1}-t)\xi_{-1}^T(t)\begin{bmatrix}\frac{X_2+X_2^T}{2} & -X_2+X_3\\ * & -\bar{X}_3\end{bmatrix}\xi_{-1}(t),\\ \quad t\in[t_k,t_{k+1}),\\ (t_{k+3}-t)\xi_1^T(t)\begin{bmatrix}\frac{X_2+X_2^T}{2} & -X_2+X_3\\ * & -\bar{X}_3\end{bmatrix}\xi_1(t),\\ \quad t\in[t_{k+1},t_{k+2}),\end{cases}$$

with $\xi_i(t)=col\{x(t),x(t_{k+i})\}(i=0,\pm1),\bar{X}_1=X_1+X_1^T-\frac{X+X^T}{2},\ \bar{X}_3=X_3+X_3^T-\frac{X_2+X_2^T}{2},U_1>0,\ U_2>0,\ k=2p$.

The terms V_i and $V_{Xi}(i=1,2)$ extend the constructions of [7] to the case of multiple sampling intervals. These terms are continuous in time along (8) with $h_k=0$ since

$$V_{1|t=t_k^-}=V_{1|t=t_k}=0,\qquad V_{1|t=t_{k+1}^-}=V_{1|t=t_{k+1}}\geq 0,$$
$$V_{2|t=t_k^-}=V_{2|t=t_k}\geq 0,\qquad V_{2|t=t_{k+1}^-}=V_{2|t=t_{k+1}}=0,$$
$$V_{X1|t=t_k^-}=V_{X1|t=t_k}=0,\qquad V_{X1|t=t_{k+1}^-}=V_{X1|t=t_{k+1}}\geq 0,$$
$$V_{X2|t=t_k^-}=V_{X2|t=t_k}\geq 0,\qquad V_{X2|t=t_{k+1}^-}=V_{X2|t=t_{k+1}}=0.$$

The condition $V(t,x_t,\dot{x}_t)\geq\bar{\beta}|x(t)|^2$ holds for $t\in[t_k,t_{k+1}),\ k=2p$, if

$$\begin{bmatrix}P+\tau_M(X+X^T)+\tau_M\frac{X_2+X_2^T}{2} & 2\tau_M(-X+X_1) & \tau_M(-X_2+X_3)\\ * & -2\tau_M\bar{X}_1 & 0\\ * & * & -\tau_M\bar{X}_3\end{bmatrix}>0,\quad (34)$$

$$\begin{bmatrix}P+\tau_M\frac{X+X^T}{2} & \tau_M(-X+X_1)\\ * & -\tau_M\bar{X}_1\end{bmatrix}>0,\quad (35)$$

and for $t\in[t_{k+1},t_{k+2}),\ k=2p$, if

$$\begin{bmatrix}P+\tau_M(X_2+X_2^T)+\tau_M\frac{X+X^T}{2} & \tau_M(-X+X_1) & 2\tau_M(-X_2+X_3)\\ * & -\tau_M\bar{X}_1 & 0\\ * & * & -2\tau_M\bar{X}_3\end{bmatrix}>0,\quad (36)$$

$$\begin{bmatrix}P+\tau_M\frac{X_2+X_2^T}{2} & \tau_M(-X_2+X_3)\\ * & -\tau_M\bar{X}_3\end{bmatrix}>0.\quad (37)$$

Table 1
Example 1: max. value of τ_M for $h_k\equiv 0$.

Method	$\tau_M=MATI$
[29]	0.0082
[15]	0.0088
[17]	0.0645
Remark 6: $t_{k+1}-t_k\equiv\tau_M$	0.0655
Remark 6: $t_{k+1}-t_k\leq\tau_M$	0.0622
Theorem 2 with $h=0$: $t_{k+1}-t_k\leq\tau_M$	0.049

Lyapunov functional V of (33) with $X=X_i=0\ (i=1,2,3)$ is applicable to systems with variable sampling $t_{k+1}-t_k\leq\tau_M$. The resulting LMI conditions can be found in [18].

Remark 7. LMIs of Theorems 1 and 2, of Corollary 1 and of [18] are affine in A. Therefore, if A resides in the uncertain polytope

$$A=\sum_{j=1}^{M}\mu_j(t)A^{(j)},\quad 0\leq\mu_j(t)\leq 1,\ \sum_{j=1}^{M}\mu_j(t)=1,$$

one have to solve these LMIs simultaneously for all the M vertices $A^{(j)}$, applying the same decision matrices.

4. Examples

4.1. Example 1: batch reactor

We illustrate the efficiency of the given conditions on the benchmark example of a batch reactor under the dynamic output feedback with $h_k=h_k^{sc}$ [29,15,17], where

$$A=\begin{bmatrix}1.380 & -0.208 & 6.715 & -5.676\\ -0.581 & -4.2902 & 0 & 0.675\\ 1.067 & 4.273 & -6.654 & 5.893\\ 0.048 & 4.273 & 1.343 & -2.104\end{bmatrix},$$

$$B=\begin{bmatrix}0 & 0\\ 5.679 & 0\\ 1.136 & -3.146\\ 1.136 & 0\end{bmatrix},\qquad B_1=\begin{bmatrix}10 & 0\\ 0 & 5\\ 10 & 0\\ 0 & 5\end{bmatrix},$$

$$C_0=\begin{bmatrix}1 & 0 & 1 & -1\\ 0 & 1 & 0 & 0\end{bmatrix},\qquad D_{12}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},$$

$$\left[\begin{array}{c|c}A_c & B_c\\ \hline C_c & D_c\end{array}\right]=\left[\begin{array}{cc|cc}0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ \hline -2 & 0 & 0 & -2\\ 0 & 8 & 5 & 0\end{array}\right].$$

As in [15], the controlled output z is chosen to be equal to the measured output $y=\begin{bmatrix}y^1\\ y^2\end{bmatrix}$. Thus,

$$C=\begin{bmatrix}C^1\\ \hline C^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 1 & -1\\ \hline 0 & 1 & 0 & 0\end{bmatrix},\qquad F^1=F^2=0.\quad (38)$$

We start with the stability analysis in the disturbance-free case, where $w=0$. When there is no communication delay, i.e. $h_k\equiv 0$, by applying the method of Remark 6 with $\alpha=0$ we find the maximum values of $\tau_M=MATI+MAD$ that preserve the asymptotic stability (see Table 1). Our results are close to the discretization-based results of [17], whereas the latter results are not applicable to the performance analysis. Further, for the values of h_m given in Table 2, by applying Theorem 1 and Remark 2 with $\alpha=0$, Theorem 2 and Corollary 1 with constant delay $h=h_m$, we obtain the maximum values of τ_M that preserve the stability (see Table 2). From Table 2, it is seen that the results of Theorems 1 and 2 essentially improve the hybrid system-based results [15]. Moreover, our results are applicable when the delay is larger than the sampling interval. For $h_m=0.04$ and the corresponding

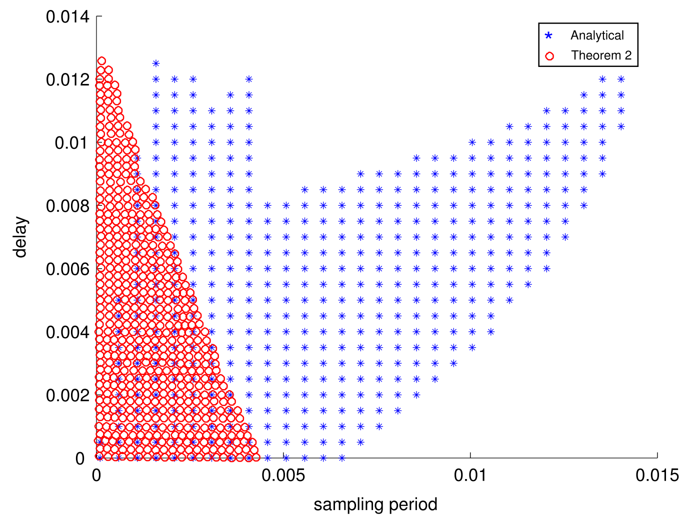

Fig. 2. Estimation of stability domain for Round-Robin scheduling with constant sampling and constant delay based on discretization.

Table 2
Example 1: max. value of $\tau_M = MATI + MAD$ for different h_m.

$\tau_M \setminus h_m$	0	0.004	0.02	0.03	0.04
[15] ($MAD = 0.004$)	0.0088	0.0088	–	–	–
[17] ($MAD = 0.03$)	0.068	0.068	0.068	0.068	–
Remark 2 (var h_k)	0.036	0.038	0.047	0.053	0.059
Theorem 1 (var h_k)	0.042	0.044	0.053	0.058	0.063
Theorem 2 (con h_m)	0.049	0.051	0.057	0.061	0.065
Corollary 1 (con h_m)	0.051	0.054	0.060	0.063	0.067

Table 3
Example 1: the computational time for maximum τ_M.

$h_m = 0.04$	Remark 2	Theorem 1	Theorem 2	Corollary 1
τ_M	0.059	0.063	0.065	0.067
Time	8.77	6.55	0.97	13.48

Table 4
Example 1: min. γ for different $\tau_M = MATI + MAD$ and h_m.

h_m	0	0	0.02	0.03	0.04
τ_M	0.0056	0.0149	0.03	0.04	0.05
[15] $\left(MAD = \frac{\tau_M}{2}\right)$	2.50	200	–	–	–
Remark 2 (var delay)	2.07	2.32	2.51	2.90	3.97
Theorem 1 (var delay)	2.06	2.25	2.43	2.74	3.48
Theorem 2 (con delay)	2.02	2.13	2.30	2.52	2.97

maximum τ_M we give also the computational time (in seconds) for different methods (see Table 3). It is seen that the improvement (till 15% increase of the maximum τ_M and till 25% decrease of computational time) is achieved by taking into account order of the delays in Theorem 1 (for variable delay h_k). Theorem 2 essentially decreases the computational time (for constant delay $h_k \equiv h_m$).

Consider next the perturbed model of the batch reactor, i.e. $w \neq 0$. As in [15], we assume that $y^i(s_k) = C^i x(s_k),\ i = 1, 2$, where C^i is given in (38) and consider $J = \int_{t_0}^{\infty} [z^T(t)z(t) - \gamma^2 w^T(t)w(t)]dt$. For the values of h_m given in Table 4, by applying Theorem 1 we find the minimum values of γ for different values of h_m (see Table 4). From Table 4 it is seen that our results are favorably compared with [15]. As previously, Theorem 1 is applicable when the delay is larger than the sampling interval.

4.2. Example 2: cart-pendulum

Consider the following linearized model of the inverted pendulum on a cart:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{-mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{(M+m)g}{Ml} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{a}{M} \\ 0 \\ \dfrac{-a}{Ml} \end{bmatrix} u + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} w,$$

$$z = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}^T + 0.1u$$

with $M = 3.9249$ kg, $m = 0.2047$ kg, $l = 0.2302$ m, $g = 9.81$ N/kg, $a = 25.3$ N/V. In the model, x and θ represent cart position coordinate and pendulum angle from vertical, respectively.

We start with the disturbance-free case, where $w = 0$. The pendulum can be stabilized by a state feedback

$$u(t) = [5.825\ 5.883\ 24.941\ 5.140] \begin{bmatrix} x(t) & \dot{x}(t) & \theta(t) & \dot{\theta}(t) \end{bmatrix}^T,$$

which leads to the closed-loop system eigenvalues $\{-100, -2 + 2i, \ -2 - 2i, \ -2\}$. In practice the variables $\theta, \dot{\theta}$ and $x, \dot{x}$ are not accessible simultaneously. We consider

$$C^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \qquad C^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The applied control is obtained from the following blocks of K

$$K_1 = \begin{bmatrix} 5.825 & 5.883 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 24.941 & 5.140 \end{bmatrix}.$$

Using the classical discretization-based model for the case of constant sampling and for the values of constant delay h, Fig. 2 shows the stability domain for the inverted pendulum example with Round-Robin scheduling protocol (in constant sampling interval / delay plane). By applying further Theorem 1 with $\alpha = 0$ for the values of h_m given in Table 5 and Theorem 2 with $h \equiv h_m$, we find the maximum values of τ_M that preserve the asymptotic stability (see Table 5).

When there is no communication delay, i.e. $h_k \equiv 0$, by applying Remark 6 with $\alpha = 0$, we find the maximum values of τ_M that preserve the asymptotic stability (see Table 6). It is seen from Table 6 that our results for the constant sampling are close to the analytical one and are less conservative than for the variable sampling.

Choose further $\tau_M = 4.0 \times 10^{-3}$, $h_m = 2.0 \times 10^{-3}$, by applying Theorem 1, we find that the system is exponentially stable with

Table 5
Example 2: max. value of τ_M for different h_m.

$h_m \setminus \tau_M$	Theorem 1		[18]	Theorem 2	Analytical
1.0×10^{-3}	4.7×10^{-3}	$h \equiv h_m$	3.7×10^{-3}	4.9×10^{-3}	8.5×10^{-3}
2.0×10^{-3}	5.4×10^{-3}		4.5×10^{-3}	5.5×10^{-3}	1.05×10^{-2}
3.0×10^{-3}	6.1×10^{-3}		5.2×10^{-3}	6.1×10^{-3}	1.25×10^{-2}
4.0×10^{-3}	6.8×10^{-3}		6.0×10^{-3}	6.8×10^{-3}	1.45×10^{-2}

Table 6
Example 2: max. value of τ_M for $h_k \equiv 0$.

Method	τ_M
Analytical: $t_{k+1} - t_k \equiv \tau_M$	6.8×10^{-3}
Remark 6: $t_{k+1} - t_k \equiv \tau_M$	6.4×10^{-3}
Remark 6: $t_{k+1} - t_k \leq \tau_M$	5.3×10^{-3}
Theorem 2 with $h = 0$: $t_{k+1} - t_k \leq \tau_M$	4.3×10^{-3}

the decay rate $\alpha = 1.94$. Consider next the perturbed model of pendulum and the noisy measurements, i.e. $w \neq 0$ and $v \neq 0$. We assume that $y^i(s_k) = C^i x(s_k) + 0.1v(s_k),\ i = 1, 2$. By applying Theorem 1, we find that for $\tau_M = 4.0 \times 10^{-3}$, $h_m = 2.0 \times 10^{-3}$ the system has an L_2-gain less than $\gamma = 1.24$, whereas for $\tau_M \to 0$ the resulting $\gamma = 1.19$.

5. Conclusions

In this paper, a time-delay approach has been introduced for the exponential stability and L_2-gain analysis of NCS with Round-Robin scheduling, variable communication delay and variable sampling intervals. The closed-loop system is modeled as a switched system with multiple and *ordered* time-varying delays. By developing appropriate Lyapunov–Krasovskii-based methods, sufficient conditions are derived in terms of LMIs. The batch reactor example illustrates the advantages of the new method over the existing ones: essential improvement of the results comparatively to the hybrid system approach, performance analysis comparatively to the discrete-time approach, non-small network-induced delay (which is not smaller than the sampling interval) comparatively to both existing approaches.

Future work will involve analysis and design for NCS under other scheduling protocols and under quantization effects.

Acknowledgments

This work was partially supported by Israel Science Foundation (grant No 754/10) and by China Scholarship Council.

References

[1] W. Zhang, M.S. Branicky, S.M. Phillips, Stability of networked control systems, IEEE Control Systems Magazine 21 (1) (2001) 84–99.
[2] P. Antsaklis, J. Baillieul, Special issue on technology of networked control systems, Proceedings of the IEEE 95 (1) (2007) 5–8.
[3] J.-P. Richard, T. Divoux, Systemes commandes en reseau, Hermes–Lavoisier, IC2, Systemes Automatises, 2007.
[4] E. Fridman, A. Seuret, J.-P. Richard, Robust sampled-data stabilization of linear systems: an input delay approach, Automatica 40 (2004) 1441–1446.
[5] P. Naghshtabrizi, J. Hespanha, A.R. Teel, Exponential stability of impulsive systems with application to uncertain sampled-data systems, Systems & Control Letters 57 (2008) 378–385.
[6] H. Fujioka, A discrete-time approach to stability analysis of systems with aperiodic sample-and-hold devices, IEEE Transactions on Automatic Control 54 (10) (2009) 2440–2445.
[7] E. Fridman, A refined input delay approach to sampled-data control, Automatica 46 (2010) 421–427.
[8] L. Hetel, J. Daafouz, C. Iung, Analysis and control of LTI and switched systems in digital loops via an event-based modeling, International Journal of Control 81 (7) (2008) 1125–1138.
[9] A. Seuret, A novel stability analysis of linear systems under asynchronous samplings, Automatica 48 (1) (2012) 177–182.
[10] D. Liberzon, Quantization, time delays, and nonlinear stabilization, IEEE Transactions on Automatic Control 51 (7) (2006) 1190–1195.
[11] H. Gao, T. Chen, J. Lam, A new system approach to network-based control, Automatica 44 (1) (2008) 39–52.
[12] M.B.G. Cloosterman, L. Hetel, N. van de Wouw, W.P.M.H. Heemels, J. Daafouz, H. Nijmeijer, Controller synthesis for networked control systems, Automatica 46 (10) (2010) 1584–1594.
[13] K. Liu, E. Fridman, Networked-based stabilization via discontinuous Lyapunov functionals, International Journal of Robust and Nonlinear Control 22 (4) (2012) 420–436.
[14] D. Nesic, A.R. Teel, Input–output stability properties of networked control systems, IEEE Transactions on Automatic Control 49 (10) (2004) 1650–1667.
[15] W.P.M.H. Heemels, A.R. Teel, N. van de Wouw, D. Nesic, Networked control systems with communication constraints: tradeoffs between transmission intervals, delays and performance, IEEE Transactions on Automatic Control 55 (8) (2010) 1781–1796.
[16] M.C.F. Donkers, L. Hetel, W.P.M.H. Heemels, N. van de Wouw, M. Steinbuch, Stability analysis of networked control systems using a switched linear systems approach, in: Rupak Majumdar, Paulo Tabuada (Eds.), Proceedings of the 12th International Conference on Hybrid Systems: Computation and Control, in: Lecture Notes in Computer Science, Springer, 2009, pp. 150–164. Book Chapter.
[17] M.C.F. Donkers, W.P.M.H. Heemels, N. van de Wouw, L. Hetel, Stability analysis of networked control systems using a switched linear systems approach, IEEE Transactions on Automatic Control 56 (9) (2011) 2101–2115.
[18] K. Liu, E. Fridman, L. Hetel, J.-P. Richard, Sampled-data stabilization via round-robin scheduling: a direct Lyapunov–Krasovskii approach, in: Proceedings of the 18th World Congress of the International Federation of Automatic Control, Milano, Italy, 2011.
[19] P. Naghshtabrizi, J. Hespanha, A. Teel, Stability of delay impulsive systems with application to networked control systems, Transactions of the Institute of Measurement and Control 32 (5) (2010) 511–528. Special issue on hybrid and switched systems.
[20] V. Suplin, E. Fridman, U. Shaked, Sampled-data H_∞control and filtering: nonuniform uncertain sampling, Automatica 43 (2007) 1072–1083.
[21] D. Liberzon, Switching in systems and control, in: Systems and Control: Foundation and Applications, Birkhauser, 2003.
[22] P. Park, J. Ko, C. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, Automatica 47 (2011) 235–238.
[23] E. Fridman, A new Lyapunov technique for robust control of systems with uncertain non-small delays, IMA Journal of Mathematical Control and Information 23 (2006) 165–179.
[24] Y. He, Q.-G. Wang, C. Lin, M. Wu, Delay-range-dependent stability for systems with time-varying delay, Automatica 43 (2007) 371–376.
[25] K. Gu, V. Kharitonov, J. Chen, Stability of Time-delay Systems, Birkhauser, Boston, 2003.
[26] K. Liu, E. Fridman, Wirtinger's inequality and Lyapunov-based sampled-data stabilization, Automatica 48 (1) (2012) 102–108.
[27] G. Hardy, J. Littlewood, G. Polya, Inequalities, Cambridge University Press, Cambridge, 1934.
[28] K. Liu, V. Suplin, E. Fridman, Stability of linear systems with general sawtooth delay, IMA Journal of Mathematical Control and Information 27 (2010) 419–436.
[29] D. Carnevale, A.R. Teel, D. Nesic, A Lyapunov proof of improved maximum allowable transfer interval for networked control systems, IEEE Transactions on Automatic Control 55 (2007) 892–897.