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Brief paper

Sub-predictors for network-based control under uncertain large delays[☆]

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ABSTRACT

This paper, for the first time, provides sub-predictors for networked control systems (NCSs) under uncertain large communication delays. We use a time-delay approach to NCS and employ sub-predictors to partially compensate large uncertain transmission delays in the sensor-to-controller and controller-to-actuator channels by dividing the long delay into small pieces. We consider systems with norm-bounded uncertainties, and take into account Round-Robin scheduling protocol in sensor-to-controller channel. In comparison with the traditional reduction-based classical predictor involving distributed input, the sub-predictor-based feedback is more friendly in the presence of norm-bounded uncertainties and is simpler for implementation. The sub-predictor-based feedback is further extended to decentralized control of interconnected systems provided that the couplings are not strong. The stability analysis of the closed-loop system is based on the Lyapunov–Krasovskii method and the stability conditions are given in terms of linear matrix inequalities.

1. Introduction

Modern control makes use of digital and communication technology for implementation. Networked control systems (NCSs), where the plant and the controller exchange data via communication network, become quite popular in practice. The actuator and sensor delays due to the transmission are among the most common dynamic phenomena in NCSs, and have adverse impact on stability and transient performance when disregarded.

In the state-of-the-art, the approaches to handle delays may be generally classified into two categories:

- The first method is the predictor-free feedback where the robustness with respect to small enough delays is studied (Freirich & Fridman, 2016; Fridman, 2014; Liu, Selivanov, & Fridman, 2019). The benefit of this method is that the control algorithm would be quite simple, whereas the potential shortcoming is that the delay value tolerated by the control system may be small.
- The second method is the predictor-based feedback via delay compensation to handle large delays.

There are two main approaches to the predictor control. The first one is the reduction-based predictor feedback which was initiated in Artstein (1982) and then extended to time-varying delays (Bekiaris-Liberis & Krstic, 2013; Karafyllis & Krstic, 2013; Yue & Han, 2005; Zhou, Lin, & Duan, 2012), sampled-data control (Selivanov & Fridman, 2016, 2016a, 2016b; Zhang, Branicky, & Phillips, 2001; Zhu & Fridman, 2020a, 2020b) and delay-adaptive control (Zhu & Krstic, 2020; Zhu, Krstic, & Su, 2017). In such a predictor-based framework, the controller employs the prediction of the future values of the state. The resulting closed-loop system evolves like a reduced system (as emphasized by the name *reduction*) as if there were no delay at all after an initial transient period of the delay time units. The predictor feedback is able to deal with large delays, but it may be non-trivial to numerically compute the integration of the distributed input signals over a historical time interval in the control laws (Furtat, Fridman, & Fradkov, 2018; Karafyllis & Krstic, 2017; Mondie & Michiels, 2003).

The second one is the sub-predictors via a chain of observers. The latter was initiated to handle constant output delays in Ahmed-Ali, Cherrier, and Lamnabhi-Lagarrigue (2012) and Germani, Manes, and Pepe (2001, 2002), and

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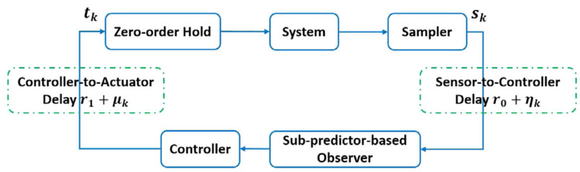

Fig. 1. Sub-predictors feedback for NCSs with large Delays.

further developed to address constant input delays by Najafia, Hosseinnia, Sheikholeslam, and Karimadini (2013). See also some recent extensions: parabolic systems with constant output delays (Ahmed-Ali et al., 2020), linear stochastic systems with time-varying input delays (Cacace, Germani, Manes, & Papi, 2020), deterministic systems with time-varying input delays (Mazenc & Malisoff, 2017; Sanz, Garcia, Fridman, & Albertos, 2018, 2020), and linear time-varying systems with both constant input and output delays (Mazenc & Malisoff, 2020). By dividing a large delay into small pieces, the basic idea of the sub-predictors is to use a chain of observers to sequentially estimate the future state in terms of the time units of the divided delay. The references Mazenc and Malisoff (2017) and Sanz et al. (2018, 2020) studied the continuous-time control with time-varying delays. As clarified in Remark 2 of Mazenc and Malisoff (2017), the results for time-varying delays cannot be trivially applicable to arbitrary sawtooth shaped delays arising from sampling. The sub-predictors feedback was extended to sampled-data control with discrete-time output in Mazenc and Malisoff (2020). However, the measurement sampling in Mazenc and Malisoff (2020) was periodic, the actuator delay was constant, and the continuous-discrete sequential observers were employed. Extension of such method to additional time-varying delays and system uncertainties may be problematic.

In this paper, building on the chain-observer-based continuous-time control concept in Ahmed-Ali et al. (2012), Germani et al. (2002) and Najafia et al. (2013), we provide a sub-predictor-based control for NCSs with uncertain large time-varying communication delays. Here, for the first time, we extend the sequential sub-predictors to the stabilization of NCSs with norm-bounded uncertainty, multi-sensors under Round-Robin scheduling protocol, and to the interconnected systems when the interactions among subsystems are not strong (i.e. Euclidean norms of coupling matrices are small enough as explained in Remarks 2 and 3 of Zhu & Fridman, 2020a, 2020b). The analysis in our paper is based on the *time-delay* approach to NCSs and the results are formulated in terms of *linear matrix inequalities* (LMIs) (Fridman, 2014; Liu, Fridman, & Hetel, 2012; Liu et al., 2019). Moreover, the samplings in input and output of this paper are aperiodic and the control system is robust to a time-varying input delay uncertainty. In comparison with the classical reduction-based predictor (Selivanov & Fridman, 2016, 2016a, 2016b; Zhang et al., 2001) employing an integral formula of distributed input, the sub-predictor-based feedback is more friendly and simpler for implementation in the presence of norm-bounded uncertainties and for interconnected systems. Some preliminary results of the paper (confined to the single plant) will be presented in Zhu and Fridman (2020c).

2. Sub-predictors feedback for NCSs under delay and norm-bounded uncertainty

Consider linear systems as follows:

$$\begin{cases}\dot{x}(t)=(A+\Delta A)x(t)+(B+\Delta B)u(t),\\ y(t)=(C+\Delta C)x(t),\end{cases}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the unmeasurable plant state, $u(t)\in\mathbb{R}^m$ is the control input, and $y(t)\in\mathbb{R}^q$ is the measured output. The pair (A,B) is stabilizable and (A,C) is detectable. The norm-bounded uncertainties are of the forms

$$\Delta A=H_A\Delta(t)E_A,\quad \Delta B=H_B\Delta(t)E_B,\quad \Delta C=H_C\Delta(t)E_C$$

in which the matrices H_A, E_A, H_B, E_B, H_C, E_C are constant and known, whereas $\Delta(t)\in\mathbb{R}^{l_1\times l_2}$ is a time-varying uncertain matrix satisfying the following inequality:

$$\Delta(t)^T\Delta(t)\le I\tag{2}$$

As shown in Fig. 1, we consider network-based control of (1) in the presence of two networks: from sensors to controller and from controller to actuators. We denote by $\{s_k\}$ sampling instants on the sensors side and $\{t_k\}$ updating instants on the actuators side with $k\in\mathbb{Z}_0^+$, respectively, and they satisfy

$$\begin{aligned}&0=s_0<s_1<s_2<\cdots,\quad \lim_{k\to\infty}s_k=\infty,\quad s_{k+1}-s_k\le h,\\ &r=r_0+r_1,\quad t_k=s_k+r+\eta_k+\mu_k,\quad t_k\le t_{k+1},\\ &0\le\eta_k\le\eta_M,\quad 0\le\mu_k\le\mu_M,\end{aligned}\tag{3}$$

where h is the maximum sampling interval, i.e., the Maximum Allowable Transmission Interval (MATI), $r_0+r_1+\eta_k+\mu_k$ is the time-varying transmission delay from sensors to actuators, in which $r=r_0+r_1>0$ is the known constant delay which may be much larger than the sampling interval, whereas η_k is the time-varying sensor-to-controller delay upper bounded by η_M as well as μ_k is the time-varying controller-to-actuator delay upper bounded by μ_M. In the controller design, the time stamp s_k will be transmitted with the sampled-data together, thus η_k could be calculated by the controller. The delay uncertainty μ_k is unknown.

Under the above networked control (3), (1) becomes

$$\begin{aligned}&\dot{x}(t)=(A+\Delta A)x(t)+(B+\Delta B)u(s_k),\quad t\in[t_k,t_{k+1})\\ &y(s_k)=(C+\Delta C)x(s_k)\end{aligned}\tag{4}$$

Note that the notation $u(s_k)$ is defined to be the control input employing the information measured at s_k. Using the time-stamp, we assume both of $y(s_k)$ and s_k are available on the controller side.

In order to deal with the large delay r, we divide r into M ($M\in\mathbb{Z}^+$) pieces like $\frac{r}{M}$ and employ a chain of sub-predictors to sequentially achieve the future state prediction such that $\hat{x}_1(t-r)\to\hat{x}_2\left(t-\frac{M-1}{M}r\right)\to\cdots\to\hat{x}_i\left(t-\frac{M-i+1}{M}r\right)\to\hat{x}_{i+1}\left(t-\frac{M-i}{M}r\right)\to\cdots\to\hat{x}_{M-1}\left(t-\frac{2}{M}r\right)\to\hat{x}_M\left(t-\frac{1}{M}r\right)\to x(t)$. Define the estimation error

$$\begin{cases}e_1(t)=\hat{x}_1(t-r)-\hat{x}_2\left(t-\frac{M-1}{M}r\right)\\ e_2(t)=\hat{x}_2\left(t-\frac{M-1}{M}r\right)-\hat{x}_3\left(t-\frac{M-2}{M}r\right)\\ \quad\vdots\\ e_{M-1}(t)=\hat{x}_{M-1}\left(t-\frac{2}{M}r\right)-\hat{x}_M\left(t-\frac{1}{M}r\right)\\ e_M(t)=\hat{x}_M\left(t-\frac{1}{M}r\right)-x(t)\end{cases}\tag{5}$$

A chain of sub-predictor-based observers is designed as

$$\begin{cases}\dot{\hat{x}}_1(t)=A\hat{x}_1(t)+LC\left(\hat{x}_1\left(t-\frac{r}{M}\right)-\hat{x}_2(t)\right)+Bu(t)\\ \dot{\hat{x}}_2(t)=A\hat{x}_2(t)+LC\left(\hat{x}_2\left(t-\frac{r}{M}\right)-\hat{x}_3(t)\right)+Bu\left(t-\frac{1}{M}r\right)\\ \quad\vdots\\ \dot{\hat{x}}_{M-1}(t)=A\hat{x}_{M-1}(t)+LC\left(\hat{x}_{M-1}\left(t-\frac{r}{M}\right)-\hat{x}_M(t)\right)\\ \quad+Bu\left(t-\frac{M-2}{M}r\right)\\ \dot{\hat{x}}_M(t)=A\hat{x}_M(t)+LC\hat{x}_M\left(s_k-\frac{r}{M}\right)-L(C+\Delta C)x(s_k)\\ \quad+Bu\left(t-\frac{M-1}{M}r\right),\quad t\in[s_k,s_{k+1})\end{cases}\tag{6}$$

with $\hat{x}_i(t) = 0$ for $t \leq 0$, and the controller is chosen as

$$u(t) = K\hat{x}_1(t) \tag{7}$$

where L and K are selected to make $A + LC$ and $A + BK$ Hurwitz, respectively. Note that, for conceptional and notational simplicity, we employ an identical L in each observer. An improved selection is to apply different observer gains L_i with $i = 1, \ldots, M$ to different observers. The signal $\hat{x}_1(s_k)$ can be calculated by solving (6) at the time instant s_k on the controller side (Selivanov & Fridman, 2016b). Note that in our design (6)–(7), the sampled-data measurement is employed only in the last chain observer of (6) (and not in every observer), which allows to avoid redundant sampling-induced delayed terms in the closed-loop system.

For analysis, we have the following equations:

$$\begin{cases} \dot{\hat{x}}_1(t-r) = A\hat{x}_1(t-r) + LC\left(\hat{x}_1\left(t - \frac{M+1}{M}r\right) - \hat{x}_2(t-r)\right) \\ \quad +Bu(t-r) \\ \dot{\hat{x}}_2\left(t - \frac{M-1}{M}r\right) = A\hat{x}_2\left(t - \frac{M-1}{M}r\right) + LC\left(\hat{x}_2(t-r)\right. \\ \quad \left. -\hat{x}_3\left(t - \frac{M-1}{M}r\right)\right) + Bu(t-r) \\ \vdots \\ \dot{\hat{x}}_{M-1}\left(t - \frac{2}{M}r\right) = A\hat{x}_{M-1}\left(t - \frac{2}{M}r\right) + LC\left(\hat{x}_{M-1}\left(t - \frac{3}{M}r\right)\right. \\ \quad \left. -\hat{x}_M\left(t - \frac{2}{M}r\right)\right) + Bu(t-r) \\ \dot{\hat{x}}_M\left(t - \frac{r}{M}\right) = A\hat{x}_M\left(t - \frac{r}{M}\right) + LC\left(\hat{x}_M\left(s_k - \frac{r}{M}\right) - x(s_k)\right) \\ \quad -L\Delta Cx(s_k) + Bu(t-r), \quad t \in \left[s_k + \frac{r}{M}, s_{k+1} + \frac{r}{M}\right) \end{cases} \tag{8}$$

Then the first equation of (8) and the dynamics of the estimation error (5) take the following forms:

$$\begin{cases} \dot{\hat{x}}_1(t-r) = (A+BK)\hat{x}_1(t-r) + LCe_1\left(t - \frac{r}{M}\right) \\ \dot{e}_1(t) = Ae_1(t) + LCe_1\left(t - \frac{r}{M}\right) - LCe_2\left(t - \frac{r}{M}\right) \\ \vdots \\ \dot{e}_{M-2}(t) = Ae_{M-2}(t) + LCe_{M-2}\left(t - \frac{r}{M}\right) - LCe_{M-1}\left(t - \frac{r}{M}\right) \end{cases}$$

$$\begin{cases} \dot{e}_{M-1}(t) = Ae_{M-1}(t) + LCe_{M-1}\left(t - \frac{r}{M}\right) - LCe_M\left(t - \frac{r}{M}\right) \\ \quad -LCv_{e_M}(t) + L\Delta C\left(\hat{x}_1\left(t - r - \frac{r}{M}\right) - \sum_{i=1}^M e_i\left(t - \frac{r}{M}\right)\right) \\ \quad +L\Delta C\left(v_{\hat{x}_1}(t) - \sum_{i=1}^M v_{e_i}(t)\right) \\ \dot{e}_M(t) = Ae_M(t) + LCe_M\left(t - \frac{r}{M}\right) + LCv_{e_M}(t) \\ \quad -\Delta A\left(\hat{x}_1(t-r) - \sum_{i=1}^M e_i(t)\right) \\ \quad -L\Delta C\left(\hat{x}_1\left(t - r - \frac{r}{M}\right) - \sum_{i=1}^M e_i\left(t - \frac{r}{M}\right)\right) \\ \quad -L\Delta C\left(v_{\hat{x}_1}(t) - \sum_{i=1}^M v_{e_i}(t)\right) \\ \quad +BK\left(\hat{x}_1(t-r) - \hat{x}_1(t-r-\tau(t))\right) - \Delta BK\hat{x}_1(t-r-\tau(t)) \end{cases} \tag{9}$$

where $\tau(t) = t - r - s_k$, $t \in [t_k, t_{k+1})$, $0 \leq \tau(t) \leq h + \eta_M + \mu_M = \tau_M$, and $v_{\hat{x}_1}(t) = \hat{x}_1(s_k - r) - \hat{x}_1\left(t - r - \frac{r}{M}\right)$, $v_{e_i}(t) = e_i(s_k) - e_i\left(t - \frac{r}{M}\right)$, $t \in \left[s_k + \frac{r}{M}, s_{k+1} + \frac{r}{M}\right)$.

Defining $\xi(t) = \left(\hat{x}_1(t-r)^T \ e_1(t)^T \ \ldots \ e_M(t)^T\right)^T$, the closed-loop (9) is rewritten as follows:

$$\begin{aligned} \dot{\xi}(t) &= \bar{A}\xi(t) + \bar{L}\xi\left(t - \tfrac{r}{M}\right) + H_1 v_\xi(t) + H_2\xi(t-\tau(t)) \\ &\quad + \Delta\bar{A}\xi(t) + \Delta\bar{B}\xi(t-\tau(t)) + \Delta\bar{C}\xi\left(t - \tfrac{r}{M}\right) + \Delta\bar{C}v_\xi(t) \\ &= \bar{A}\xi(t) + \bar{L}\xi\left(t - \tfrac{r}{M}\right) + H_1 v_\xi(t) + H_2\xi(t-\tau(t)) \\ &\quad + \bar{H}_A\bar{\Delta}_A(t)\bar{E}_A\xi(t) + \bar{H}_B\bar{\Delta}_B(t)\bar{E}_B\xi(t-\tau(t)) \\ &\quad + \bar{H}_C\bar{\Delta}_C(t)\bar{E}_C\xi\left(t - \tfrac{r}{M}\right) + \bar{H}_C\bar{\Delta}_C(t)\bar{E}_C v_\xi(t) \end{aligned} \tag{10}$$

where $v_\xi(t) = \xi(s_k) - \xi\left(t - \frac{r}{M}\right)$, $\quad t \in \left[s_k + \frac{r}{M}, s_{k+1} + \frac{r}{M}\right)$, $\bar{A} = \begin{pmatrix} A+BK & 0 & \cdots & 0 & 0 \\ 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & A & 0 \\ BK & 0 & \cdots & 0 & A \end{pmatrix}$, $\bar{L} = \begin{pmatrix} 0 & LC & 0 & \cdots & 0 & 0 \\ 0 & LC & -LC & \cdots & 0 & 0 \\ 0 & 0 & LC & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & LC & -LC \\ 0 & 0 & 0 & \cdots & 0 & LC \end{pmatrix}$, $H_1 = \begin{pmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & -LC \\ 0 & \cdots & 0 & LC \end{pmatrix}$,

$H_2 = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ -BK & 0 & \cdots & 0 & 0 \end{pmatrix}$, $\Delta\bar{A} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ -\Delta A & \Delta A & \cdots & \Delta A & \Delta A \end{pmatrix}$,

$\Delta\bar{B} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ -\Delta BK & 0 & \cdots & 0 & 0 \end{pmatrix}$, $\Delta\bar{C} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ L\Delta C & -L\Delta C & \cdots & -L\Delta C & -L\Delta C \\ -L\Delta C & L\Delta C & \cdots & L\Delta C & L\Delta C \end{pmatrix}$,

$\bar{H}_A = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & H_A \end{pmatrix}$, $\bar{H}_B = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ H_B & 0 & \cdots & 0 & 0 \end{pmatrix}$, $\bar{H}_C = \begin{pmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & -LH_C \\ 0 & \cdots & 0 & LH_C \end{pmatrix}$,

$\bar{\Delta}_A(t) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \Delta(t) \end{pmatrix}$, $\bar{\Delta}_B(t) = \begin{pmatrix} \Delta(t) \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $\bar{\Delta}_C(t) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \Delta(t) \end{pmatrix}$, $\bar{E}_A = \left(-E_A \ E_A \ \cdots \ E_A\right)$, $\bar{E}_B = \left(-E_BK \ 0 \ \cdots \ 0\right)$, $\bar{E}_C = \left(-E_C \ E_C \ \cdots \ E_C\right)$.

Theorem 1. *Consider the closed-loop system consisting of the plant (4) and sub-predictor-based controller (6)–(7). Given positive tuning parameters $r, M, h, \tau_M = r + \mu_M + \eta_M, \alpha$, let $(M+1)n \times (M+1)n$ matrices $P, S_0, R_0, W, S_1, R_1 > 0$, $(M+1)n \times (M+1)n$ matrix G_1, and a parameter $\lambda > 0$, satisfy the LMIs:*

$$\begin{aligned} &\begin{pmatrix} \Phi & \Psi \\ * & -\Xi \end{pmatrix} < 0, \quad \begin{pmatrix} R_1 & G_1 \\ * & R_1 \end{pmatrix} > 0, \\ &\Psi = \left(\Xi^T\bar{A}, \ \Xi^T\bar{L}, \ \Xi^T H_1, \ \Xi^T H_2, \ 0, \ \Xi^T\bar{H}_A, \ \Xi^T\bar{H}_B, \ \Xi^T\bar{H}_C, \ \Xi^T\bar{H}_C\right)^T, \\ &\Xi = \frac{r^2}{M^2}R_0 + h^2e^{2\alpha h}W + \tau_M^2 R_1 \end{aligned} \tag{11}$$

where Φ is a symmetric matrix composed of

$$\begin{aligned} &\Phi_{11} = \bar{A}^TP + P\bar{A} + 2\alpha P + S_0 + S_1 - e^{-2\alpha\frac{r}{M}}R_0 - e^{-2\alpha\tau_M}R_1 \\ &\quad + \lambda\bar{E}_A^T\bar{E}_A, \quad \Phi_{12} = P\bar{L} + e^{-2\alpha\frac{r}{M}}R_0, \quad \Phi_{13} = PH_1, \\ &\Phi_{14} = PH_2 + e^{-2\alpha\tau_M}(R_1 - G_1), \quad \Phi_{15} = e^{-2\alpha\tau_M}G_1, \\ &\Phi_{16} = P\bar{H}_A, \quad \Phi_{17} = P\bar{H}_B, \quad \Phi_{18} = \Phi_{19} = P\bar{H}_C, \\ &\Phi_{22} = -e^{-2\alpha\frac{r}{M}}(S_0 + R_0) + \lambda\bar{E}_C^T\bar{E}_C, \\ &\Phi_{33} = -\frac{\pi^2}{4}e^{-2\alpha\frac{r}{M}}W + \lambda\bar{E}_C^T\bar{E}_C, \\ &\Phi_{44} = e^{-2\alpha\tau_M}(G_1^T + G_1 - 2R_1) + \lambda\bar{E}_B^T\bar{E}_B, \\ &\Phi_{45} = e^{-2\alpha\tau_M}(R_1 - G_1), \quad \Phi_{55} = -e^{-2\alpha\tau_M}(S_1 + R_1), \\ &\Phi_{66} = \Phi_{77} = \Phi_{88} = \Phi_{99} = -\lambda I. \end{aligned}$$

Then the closed-loop system (10) is exponentially stable with a decay rate α. ■

Proof. Consider the Lyapunov–Krasovskii functional (LKF) $V(t) = V_P(t) + V_{S_0}(t) + V_{R_0}(t) + V_W(t) + V_{S_1}(t) + V_{R_1}(t)$, where

$$\begin{aligned} &V_P(t) = \xi(t)^TP\xi(t), \\ &V_{S_0}(t) = \int_{t-\frac{r}{M}}^t e^{2\alpha(s-t)}\xi(s)^TS_0\xi(s)ds, \\ &V_{R_0}(t) = \frac{r}{M}\int_{-\frac{r}{M}}^0\int_{t+\theta}^t e^{2\alpha(s-t)}\dot{\xi}(s)^TR_0\dot{\xi}(s)dsd\theta, \\ &V_W(t) = h^2e^{2\alpha h}\int_{s_k}^t e^{2\alpha(s-t)}\dot{\xi}(s)^TW\dot{\xi}(s)ds \\ &\quad -\frac{\pi^2}{4}\int_{s_k}^{t-\frac{r}{M}} e^{2\alpha(s-t)}\left(\xi(s_k) - \xi(s)\right)^TW\left(\xi(s_k) - \xi(s)\right)ds, \\ &\quad t \in \left[s_k + \frac{r}{M}, s_{k+1} + \frac{r}{M}\right), \\ &V_{S_1}(t) = \int_{t-\tau_M}^t e^{2\alpha(s-t)}\xi(s)^TS_1\xi(s)ds, \\ &V_{R_1}(t) = \tau_M\int_{-\tau_M}^0\int_{t+\theta}^t e^{2\alpha(s-t)}\dot{\xi}(s)^TR_1\dot{\xi}(s)dsd\theta. \end{aligned} \tag{12}$$

Note that the terms V_{S_0} and V_{R_0} in (12), which stem from Fridman (2014, Ch. 3.6.2, p. 90), are used to deal with the constant delay term $\xi\left(t-\frac{r}{M}\right)$ in (10). The terms V_{S_1} and V_{R_1} in (12), which borrowed from Fridman (2014, Ch. 3.6.2, p. 91), are employed to address the time-varying delay term $\xi\,(t-\tau(t))$ in (10). The term V_W in (12), which came from Fridman (2014, Ch. 7.4.2, pp. 302–304), is utilized to handle the term $\upsilon_\xi(t)$ in (10). We note that some advanced result by augmented LKF via Legendre polynomial and Bessel–Legendre inequality was presented in Seuret and Gouaisbaut (2015) to improve the upper bound of the allowable constant delay. Here, for conceptional clearness, we just employ the simplest LKF regardless of conservativeness. Employing Jensen's inequality (see Fridman, 2014, Ch. 3.5.5, p. 87) and reciprocally convex approach (see Fridman, 2014, Ch. 3.6.3, pp. 95–98), we have

$$
\begin{aligned}
&\dot V_P(t)+2\alpha V_P(t)=\xi(t)^T\left(2P\bar A+2\alpha P\right)\xi(t)\\
&\quad+2\xi(t)^TP\left(\bar L\xi\left(t-\tfrac{r}{M}\right)+H_1\upsilon_\xi(t)+H_2\xi(t-\tau(t))\right.\\
&\quad+\bar H_A\bar\Delta_A(t)\bar E_A\xi(t)+\bar H_B\bar\Delta_B(t)\bar E_B\xi(t-\tau(t))\\
&\quad\left.+\bar H_C\bar\Delta_C(t)\bar E_C\xi\left(t-\tfrac{r}{M}\right)+\bar H_C\bar\Delta_C(t)\bar E_C\upsilon_\xi\,(t)\right)\\
&\dot V_{S_0}(t)+2\alpha V_{S_0}(t)=\xi(t)^TS_0\xi(t)\\
&\quad-e^{-2\alpha\frac{r}{M}}\xi\left(t-\tfrac{r}{M}\right)^TS_0\xi\left(t-\tfrac{r}{M}\right)\\
&\dot V_{R_0}(t)+2\alpha V_{R_0}(t)\le\tfrac{r^2}{M^2}\dot\xi(t)^TR_0\dot\xi(t)\\
&\quad-e^{-2\alpha\frac{r}{M}}\left(\xi(t)-\xi\left(t-\tfrac{r}{M}\right)\right)^TR_0\left(\xi(t)-\xi\left(t-\tfrac{r}{M}\right)\right)\\
&\dot V_W(t)+2\alpha V_W(t)=h^2e^{2\alpha h}\dot\xi\,(t)^T\,W\dot\xi\,(t)\\
&\quad-\tfrac{\pi^2}{4}e^{-2\alpha\frac{r}{M}}\upsilon_\xi\,(t)^T\,W\upsilon_\xi\,(t)\\
&\dot V_{S_1}(t)+2\alpha V_{S_1}(t)=\xi(t)^TS_1\xi(t)\\
&\quad-e^{-2\alpha\tau_M}\xi\,(t-\tau_M)^T\,S_1\xi\,(t-\tau_M)\\
&\dot V_{R_1}(t)+2\alpha V_{R_1}(t)\le\tau_M^2\dot\xi(t)^TR_1\dot\xi(t)\\
&\quad-e^{-2\alpha\tau_M}\begin{pmatrix}\xi\,(t)-\xi(t-\tau(t))\\ \xi\,(t-\tau(t))-\xi(t-\tau_M)\end{pmatrix}^T\begin{pmatrix}R_1&G_1\\ *&R_1\end{pmatrix}\\
&\quad\times\begin{pmatrix}\xi\,(t)-\xi(t-\tau(t))\\ \xi\,(t-\tau(t))-\xi(t-\tau_M)\end{pmatrix}
\end{aligned}\tag{13}
$$

From (2), we have the following inequalities:

$$
\begin{aligned}
&\xi(t)^T\bar E_A^T\bar\Delta_A(t)^T\bar\Delta_A(t)\bar E_A\xi(t)\le\xi(t)^T\bar E_A^T\bar E_A\xi(t)\\
&\xi(t-\tau(t))^T\bar E_B^T\bar\Delta_B(t)^T\bar\Delta_B(t)\bar E_B\xi(t-\tau(t))\\
&\quad\le\xi(t-\tau(t))^T\bar E_B^T\bar E_B\xi(t-\tau(t))\\
&\xi\left(t-\tfrac{r}{M}\right)^T\bar E_C^T\bar\Delta_C(t)^T\bar\Delta_C(t)\bar E_C\xi\left(t-\tfrac{r}{M}\right)\\
&\quad\le\xi\left(t-\tfrac{r}{M}\right)^T\bar E_C^T\bar E_C\xi\left(t-\tfrac{r}{M}\right)\\
&\upsilon_\xi\,(t)^T\,\bar E_C^T\bar\Delta_C(t)^T\bar\Delta_C(t)\bar E_C\upsilon_\xi\,(t)\\
&\quad\le\upsilon_\xi\,(t)^T\,\bar E_C^T\bar E_C\upsilon_\xi\,(t)
\end{aligned}\tag{14}
$$

Applying S-procedure, we get

$$
\begin{aligned}
&\dot V(t)+2\alpha V(t)\\
&\quad+\lambda\left(\xi(t)^T\bar E_A^T\bar E_A\xi(t)-\xi(t)^T\bar E_A^T\bar\Delta_A(t)^T\bar\Delta_A(t)\bar E_A\xi(t)\right)\\
&\quad+\lambda\left(\xi(t-\tau(t))^T\bar E_B^T\bar E_B\xi(t-\tau(t))\right.\\
&\quad\left.-\xi(t-\tau(t))^T\bar E_B^T\bar\Delta_B(t)^T\bar\Delta_B(t)\bar E_B\xi(t-\tau(t))\right)\\
&\quad+\lambda\left(\xi\left(t-\tfrac{r}{M}\right)^T\bar E_C^T\bar E_C\xi\left(t-\tfrac{r}{M}\right)\right.\\
&\quad\left.-\xi\left(t-\tfrac{r}{M}\right)^T\bar E_C^T\bar\Delta_C(t)^T\bar\Delta_C(t)\bar E_C\xi\left(t-\tfrac{r}{M}\right)\right)\\
&\quad+\lambda\left(\upsilon_\xi\,(t)^T\,\bar E_C^T\bar E_C\upsilon_\xi\,(t)-\upsilon_\xi\,(t)^T\,\bar E_C^T\bar\Delta_C(t)^T\bar\Delta_C(t)\bar E_C\upsilon_\xi\,(t)\right)\\
&\le\eta^T(t)\Phi\eta(t)+\eta^T(t)\Psi\,\Xi^{-1}\Psi^T\eta(t)\le0
\end{aligned}\tag{15}
$$

where $\eta(t)=\mathrm{col}\{\xi(t),\xi\left(t-\frac{r}{M}\right),\upsilon_\xi(t),\xi(t-\tau(t)),\xi(t-\tau_M),$ $\bar\Delta_A(t)\bar E_A\xi(t),\ \bar\Delta_B(t)\bar E_B\xi(t-\tau(t)),\ \bar\Delta_C(t)\bar E_C\xi\left(t-\frac{r}{M}\right),$ $\bar\Delta_C(t)\bar E_C\upsilon_\xi\,(t)\}$. Applying Schur complement, the inequality (15) implies (11). ∎

Remark 1. Note that the analysis LMIs (11) contain the control gain K and observer gain L which are specified by the designers. The analysis LMIs would be nonlinear when K and L are unknown. Finding K and L is related to "linearization" of LMIs (e.g. via the descriptor method in Fridman, 2014, Ch. 5.2, p. 209). Since the design LMIs can be suggested based on our analysis LMIs, we believe providing the analysis LMIs is the main problem of interest. ∎

Remark 2. If the delay uncertainty $\tau(t)$ and norm-bounded uncertainty $\Delta(t)$ in (9) are zero, the closed-loop system (9) satisfies the separation principle: the stability of $e_M(t)$ guarantees the stability of $e_{M-1}(t)$ till $e_1(t)$, and $\hat x_1(t-r)$. Due to the existences of $\tau(t)$ and $\Delta(t)$, there is no separation principle for (9). ∎

Remark 3. An alternative way to compensate large delays in the uncertain NCS (4) is the classical predictor (Artstein, 1982), where the observer is designed as

$$
\begin{aligned}
&\dot{\hat x}(t)=A\hat x(t)+LC\hat x(s_k)-L(C+\Delta C)x(s_k)+Bu(t-r),\\
&\quad t\in[s_k,s_{k+1})
\end{aligned}\tag{16}
$$

and the predictor controller is selected as

$$
u(t)=K\hat z(t),\quad \hat z(t)=e^{Ar}\hat x(t)+\textstyle\int_{t-r}^{t}e^{A(t-s)}Bu(s)ds.\tag{17}
$$

Taking the time-derivative of $\hat z(t)$ along (16), we get the z-dynamics such that

$$
\begin{aligned}
&\dot{\hat z}(t)=A\hat z(t)+Bu(t)-e^{Ar}LC(x(s_k)-\hat x(s_k))-e^{Ar}L\Delta Cx(s_k)\\
&=(A+BK)\hat z(t)-e^{Ar}LCe(s_k)-e^{Ar}LH_C\Delta(t)E_C\\
&\quad\times\left(e(s_k)+e^{-Ar}\hat z(s_k)-\zeta(s_k)\right),\quad t\in[s_k,s_{k+1})
\end{aligned}\tag{18}
$$

where $e(t)=x(t)-\hat x(t)$ is the estimation error and $\zeta(t)=\int_{t-r}^{t}e^{A(t-s-r)}Bu(s)ds=\int_{t-r}^{t}e^{A(t-s-r)}BK\hat z(s)ds$. Subtracting (16) from (4), we obtain the estimation error system as

$$
\begin{aligned}
&\dot e(t)=Ae(t)+LCe(s_k)+\Delta Ax(t)+L\Delta Cx(s_k)\\
&\quad+B(u(s_k)-u(t-r))+\Delta Bu(s_k),\quad t\in[t_k,t_{k+1})\cap[s_k,s_{k+1})\\
&=Ae(t)+LCe(s_k)+H_A\Delta(t)E_A\left(e(t)+e^{-Ar}\hat z(t)-\zeta(t)\right)\\
&\quad+LH_C\Delta(t)E_C\left(e(s_k)+e^{-Ar}\hat z(s_k)-\zeta(s_k)\right)\\
&\quad+BK(\hat z(t-r-\tau(t))-\hat z(t-r))\\
&\quad+H_B\Delta(t)E_BK\hat z(t-r-\tau(t)),\quad t\in[s_k,s_{k+1})
\end{aligned}\tag{19}
$$

where $\tau(t)=t-r-s_k,\ t\in[t_k,t_{k+1}),\ 0\le\tau(t)\le h+\mu_M+\eta_M=\tau_M$.

The distributed delay terms $\zeta(t)$ and $\zeta(s_k)$ that stem from the inverse Artstein's transformation appear in (18)–(19), which greatly complicates the stability analysis. ∎

3. Sub-predictors feedback for NCSs under Round-Robin Scheduling

In this section, we extend the sub-predictors feedback to NCSs with multi-sensor nodes. For conceptional and notational simplicity, we assume there is no norm-bounded uncertainty here. As illustrated in Fig. 2 and under network-based control (3), we consider the system equipped with distributed multiple sensors, a controller and an actuator such that

$$
\begin{aligned}
&\dot x(t)=Ax(t)+Bu(s_k),\quad t\in[t_k,t_{k+1})\\
&y_j(s_k)=C_jx(s_k),\quad j=1,\dots,N
\end{aligned}\tag{20}
$$

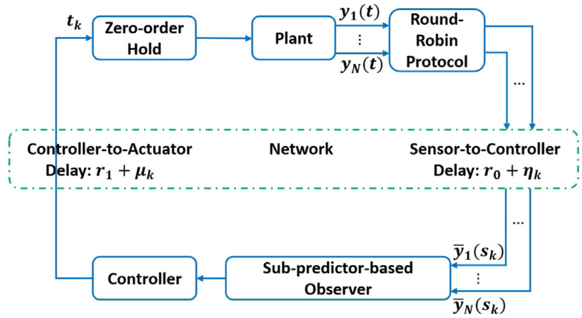

Fig. 2. Sub-predictors feedback for NCSs under scheduling.

where $y_j(s_k) \in \mathbb{R}^{q_j}$, $\sum_{j=1}^N q_j = q$, $y(s_k) = \left(y_1(s_k)^T \dots y_N(s_k)^T \right)^T = \left(C_1^T \dots C_N^T \right)^T x(s_k) = Cx(s_k)$.

Accordingly, we denote by $\bar{y}(s_k) = \left(\bar{y}_1(s_k)^T \dots \bar{y}_N(s_k)^T \right)^T$ the output of the scheduling protocol. At each sampling instant s_k, one of the sensor-to-controller channels $j \in \{1, \dots, N\}$ is active, that is only one of $\bar{y}_j(s_k)$ values is updated with the recent measurement $y_j(s_k)$. Let $j_k^* \in \{1, \dots, N\}$ denote the active channel at s_k, which will be chosen by the scheduling protocol. Then $\bar{y}_j(s_k) = \begin{cases} y_j(s_k), & j = j_k^* \\ \bar{y}_j(s_{k-1}), & j \neq j_k^* \end{cases}$. In this paper, we consider the popular Round-Robin protocol where the channel is activated in a periodic order: $\begin{cases} j_k^* = j_{k+N}^*, & \forall k \in \mathbb{Z}_0^+ \\ j_m^* \neq j_n^*, & 0 \le m < n \le N-1 \end{cases}$ and N is the protocol period. Under Round-Robin scheduling, when $t \ge s_{N-1}$ (i.e., when all the measurements are transmitted at least once), the measurements available to the controller satisfy $\bar{y}_j(s_k) = y_j(s_{k-\Delta_k^j}) = C_j x(s_{k-\Delta_k^j})$ where $\Delta_k^j \in \{0, 1, \dots, N-1\}$.

The chain of sub-predictor-based observers and the controller under scheduling is designed the same as (6) and (7), except for the last equation in (6) changed by:

$$\begin{cases} \dot{\hat{x}}_M(t) = A\hat{x}_M(t) + \sum_{j=1}^N L_j C_j \left(\hat{x}_M \left(s_{k-\Delta_k^j} - \frac{r}{M} \right) - x(s_{k-\Delta_k^j}) \right) \\ \quad + Bu\left(t - \frac{M-1}{M} r \right), \quad t \in [s_k, s_{k+1}) \end{cases} \tag{21}$$

where L_j for $j = 1, \dots, N$ is selected to let $A + \sum_{j=1}^N L_j C_j$ Hurwitz.

The estimation error are defined the same as (5). Doing the similar calculation to (8), we get the closed-loop system below,

$$\begin{aligned} \dot{\xi}(t) = {} & \bar{A}\xi(t) + \bar{L}\xi\left(t - \tfrac{r}{M}\right) + \sum_{j=1}^N H_{1,j}\xi\left(t - \tau_{1,j}(t)\right), \\ & + H_2\xi\left(t - \tau_2(t)\right), \end{aligned} \tag{22}$$

where $\xi(t) = \left(\hat{x}_1(t-r)^T \; e_1(t)^T \dots e_M(t)^T \right)^T$, $\bar{A} = \begin{pmatrix} A+BK & 0 & \cdots & 0 & 0 \\ 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & A & 0 \\ BK & 0 & \cdots & 0 & A \end{pmatrix}$, $\bar{L} = \begin{pmatrix} 0 & LC & 0 & \cdots & 0 & 0 \\ 0 & LC & -LC & \cdots & 0 & 0 \\ 0 & 0 & LC & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & LC & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$, $H_{1,j} = \begin{pmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & -L_jC_j \\ 0 & \cdots & 0 & L_jC_j \end{pmatrix}$, $H_2 = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ -BK & 0 & \cdots & 0 & 0 \end{pmatrix}$,

and $\tau_{1,j}(t) = t - s_{k-\Delta_k^j}$, $t \in \left[s_k + \frac{r}{M}, s_{k+1} + \frac{r}{M}\right)$, $\frac{r}{M} \le \tau_{1,j}(t) \le s_{k+1} - s_{k-\Delta_k^j} + \frac{r}{M} \le (\Delta_k^j + 1)\cdot h + \frac{r}{M} \le N \cdot h + \frac{r}{M} = \bar{\tau}$, $\tau_2(t) = t - r - s_k$, $t \in [t_k, t_{k+1})$, $0 \le \tau_2(t) \le h + \eta_M + \mu_M = \tau_M$.

Theorem 2. *Consider the closed-loop system consisting of the plant (20) and sub-predictor-based controller (6)–(7) and (21). Given positive tuning parameters $r, M, h, \bar{\tau}, \tau_M = h + \eta_M + \mu_M, \alpha$, let $(M+1)n \times (M+1)n$ matrices $P, S_0, R_0, S_{1,j}, R_{1,j}, S_2, R_2 > 0$, and $(M+1)n \times (M+1)n$ matrices $G_{1,j}, G_2$ for $j = 1, \dots, N$, satisfy the LMIs:*

$$\begin{aligned} & \begin{pmatrix} \Phi & \Psi \\ * & -\Xi \end{pmatrix} < 0, \quad \begin{pmatrix} R_{1,j} & G_{1,j} \\ * & R_{1,j} \end{pmatrix} > 0, \quad \begin{pmatrix} R_2 & G_2 \\ * & R_2 \end{pmatrix} > 0, \\ & \Psi = \left(\Xi^T \bar{A}, \; \Xi^T \bar{L}, \; \mathrm{row}_{\{j=1,\dots,N\}} \Xi^T H_{1,j}, \; 0, \; \Xi^T H_2, \; 0 \right)^T, \\ & \Xi = \frac{r^2}{M^2} R_0 + N^2 h^2 \sum_{j=1}^N R_{1,j} + \tau_M^2 R_2, \end{aligned} \tag{23}$$

where Φ is a symmetric matrix composed of

$$\begin{aligned} & \Phi_{11} = \bar{A}^T P + P\bar{A} + 2\alpha P + S_0 + S_2 - e^{-2\alpha\frac{r}{M}} R_0 - e^{-2\alpha\tau_M} R_2, \\ & \Phi_{12} = P\bar{L} + e^{-2\alpha\frac{r}{M}} R_0, \quad \Phi_{13} = \mathrm{row}_{\{j=1,\dots,N\}}\{PH_{1,j}\}, \\ & \Phi_{15} = PH_2 + e^{-2\alpha\tau_M}(R_2 - G_2), \quad \Phi_{16} = e^{-2\alpha\tau_M} G_2, \\ & \Phi_{22} = -e^{-2\alpha\frac{r}{M}}(S_0 + R_0) - \sum_{j=1}^N \left(e^{-2\alpha\bar{\tau}} R_{1,j} - e^{-2\alpha\frac{r}{M}} S_{1,j} \right), \\ & \Phi_{23} = \mathrm{row}_{\{j=1,\dots,N\}} e^{-2\alpha\bar{\tau}}(R_{1,j} - G_{1,j}), \\ & \Phi_{24} = \sum_{j=1}^N e^{-2\alpha\bar{\tau}} G_{1,j}, \\ & \Phi_{33} = \mathrm{diag}_{\{j=1,\dots,N\}} e^{-2\alpha\bar{\tau}}(G_{1,j} + G_{1,j}^T - 2R_{1,j}), \\ & \Phi_{34} = \mathrm{col}_{\{j=1,\dots,N\}} e^{-2\alpha\bar{\tau}}(R_{1,j} - G_{1,j}), \\ & \Phi_{44} = -\sum_{j=1}^N e^{-2\alpha\bar{\tau}}(S_{1,j} + R_{1,j}), \; \Phi_{55} = e^{-2\alpha\tau_M}(G_2^T + G_2 - 2R_2), \\ & \Phi_{56} = e^{-2\alpha\tau_M}(R_2 - G_2), \quad \Phi_{66} = -e^{-2\alpha\tau_M}(S_2 + R_2). \end{aligned}$$

Then the closed-loop system (22) is exponentially stable with a decay rate α. ■

Proof. Consider the Lyapunov candidate $V(t) = V_P(t) + V_{S_0}(t) + V_{R_0}(t) + V_{S_1}(t) + V_{R_1}(t) + V_{S_2}(t) + V_{R_2}(t)$, where

$$\begin{aligned} & V_P(t) = \xi(t)^T P \xi(t), \\ & V_{S_0}(t) = \int_{t-\frac{r}{M}}^t e^{2\alpha(s-t)} \xi(s)^T S_0 \xi(s) ds, \\ & V_{R_0}(t) = \frac{r}{M} \int_{-\frac{r}{M}}^0 \int_{t+\theta}^t e^{2\alpha(s-t)} \dot{\xi}(s)^T R_0 \dot{\xi}(s) ds d\theta, \\ & V_{S_1}(t) = \sum_{j=1}^N \left[\int_{t-\bar{\tau}}^{t-\frac{r}{M}} e^{2\alpha(s-t)} \xi(s)^T S_{1,j} \xi(s) ds \right], \\ & V_{R_1}(t) = \sum_{j=1}^N \left[Nh \int_{-\bar{\tau}}^{-\frac{r}{M}} \int_{t+\theta}^t e^{2\alpha(s-t)} \dot{\xi}(s)^T R_{1,j} \dot{\xi}(s) ds d\theta \right], \\ & V_{S_2}(t) = \int_{t-\tau_M}^t e^{2\alpha(s-t)} \xi(s)^T S_2 \xi(s) ds, \\ & V_{R_2}(t) = \tau_M \int_{-\tau_M}^0 \int_{t+\theta}^t e^{2\alpha(s-t)} \dot{\xi}(s)^T R_2 \dot{\xi}(s) ds d\theta. \end{aligned} \tag{24}$$

Thus we have

$$
\begin{aligned}
&\dot V_P(t)+2\alpha V_P(t)=\xi(t)^T\left(2P\bar A+2\alpha P\right)\xi(t)\\
&\quad+2\xi(t)^TP\left(\bar L\xi\left(t-\tfrac{r}{M}\right)+\sum_{j=1}^{N}H_{1,j}\xi(t-\tau_{1,j}(t))\right.\\
&\quad\left.+H_2\xi(t-\tau_2(t))\right)\\
&\dot V_{S_0}(t)+2\alpha V_{S_0}(t)=\xi(t)^TS_0\xi(t)\\
&\quad-e^{-2\alpha\frac{r}{M}}\xi\left(t-\tfrac{r}{M}\right)^TS_0\xi\left(t-\tfrac{r}{M}\right)\\
&\dot V_{R_0}(t)+2\alpha V_{R_0}(t)\le\tfrac{r^2}{M^2}\dot\xi(t)^TR_0\dot\xi(t)\\
&\quad-e^{-2\alpha\frac{r}{M}}\left(\xi(t)-\xi\left(t-\tfrac{r}{M}\right)\right)^TR_0\left(\xi(t)-\xi\left(t-\tfrac{r}{M}\right)\right)\\
&\dot V_{S_1}(t)+2\alpha V_{S_1}(t)=\sum_{j=1}^{N}\left[e^{-2\alpha\frac{r}{M}}\xi\left(t-\tfrac{r}{M}\right)^TS_{1,j}\xi\left(t-\tfrac{r}{M}\right)\right.\\
&\quad\left.-e^{-2\alpha\bar\tau}\xi\left(t-\bar\tau\right)^TS_{1,j}\xi\left(t-\bar\tau\right)\right]\\
&\dot V_{S_2}(t)+2\alpha V_{S_2}(t)=\xi(t)^TS_2\xi(t)\\
&\quad-e^{-2\alpha\tau_M}\xi\left(t-\tau_M\right)^TS_2\xi\left(t-\tau_M\right)\\
&\dot V_{R_1}(t)+2\alpha V_{R_1}(t)\le\sum_{j=1}^{N}\left[N^2h^2\dot\xi(t)^TR_{1,j}\dot\xi(t)\right.\\
&\quad-e^{-2\alpha\bar\tau}\begin{pmatrix}\xi\left(t-\frac{r}{M}\right)-\xi(t-\tau_{1,j}(t))\\ \xi\left(t-\tau_{1,j}(t)\right)-\xi(t-\bar\tau)\end{pmatrix}^T\begin{pmatrix}R_{1,j}&G_{1,j}\\ *&R_{1,j}\end{pmatrix}\\
&\quad\left.\times\begin{pmatrix}\xi\left(t-\frac{r}{M}\right)-\xi(t-\tau_{1,j}(t))\\ \xi\left(t-\tau_{1,j}(t)\right)-\xi(t-\bar\tau)\end{pmatrix}\right]\\
&\dot V_{R_2}(t)+2\alpha V_{R_2}(t)\le\tau_M^2\dot\xi(t)^TR_2\dot\xi(t)\\
&\quad-e^{-2\alpha\tau_M}\begin{pmatrix}\xi\left(t\right)-\xi(t-\tau_2(t))\\ \xi\left(t-\tau_2(t)\right)-\xi(t-\tau_M)\end{pmatrix}^T\begin{pmatrix}R_2&G_2\\ *&R_2\end{pmatrix}\\
&\quad\times\begin{pmatrix}\xi\left(t\right)-\xi(t-\tau_2(t))\\ \xi\left(t-\tau_2(t)\right)-\xi(t-\tau_M)\end{pmatrix}
\end{aligned}
\tag{25}
$$

On the basis of (25), we get

$$
\dot V(t)+2\alpha V(t)\le\lambda^T(t)\Phi\lambda(t)+\lambda^T(t)\Psi\Xi^{-1}\Psi^T\lambda(t)\le 0 \tag{26}
$$

where $\lambda(t)=\mathrm{col}\{\xi(t),\xi\left(t-\frac{r}{M}\right),\mathrm{col}_{\{j=1,\dots,N\}}\{\xi(t-\tau_{1,j}(t))\},\xi(t-\bar\tau),\xi(t-\tau_2(t)),\xi(t-\tau_M)\}$. Applying Schur complement, the inequality (26) implies (23). ■

4. Decentralized sub-predictors feedback for large-scale interconnected systems

If the uncertainty in (1) arises from the disturbance from exo-system, the sub-predictors feedback for a single plant with norm-bounded uncertainty can be straightforwardly extended to decentralized control for large-scale interconnected systems. For simplicity, assuming $\Delta A=0$, $\Delta B=0$, $\Delta C=0$ in (1), we consider large-scale coupled systems such that

$$
\begin{aligned}
\dot x^j(t)&=A^jx^j(t)+B^ju^j(t-r^j)+\sum_{l\neq j}F^{lj}x^l(t),\\
y^j(t)&=C^jx^j(t),
\end{aligned}
\tag{27}
$$

where $j=1,\dots,N$ is the subsystem index, $x^j(t)\in\mathbb{R}^{n^j}$, $u^j(t)\in\mathbb{R}^{m^j}$ and $y^j(t)\in\mathbb{R}^{q^j}$ is the unmeasurable state, the control input with a constant delay $r^j>0$, and the measured output of the jth subsystem, respectively, whereas $x^l(t)\in\mathbb{R}^{n^l}$ is the unmeasurable state of the lth subsystem, as well as F^{lj} denotes the interaction between plants j and l, the pair (A^j,B^j) is stabilizable and (A^j,C^j) is detectable. For conceptional clearness, we consider continuous-time control. Under decentralized sub-predictor feedback, to compensate large delays in interconnected systems, the coupling matrix F^{lj} in (27) should have a small enough Euclidean norm (see the analysis in Remarks 2 and 3 of Zhu & Fridman, 2020a, 2020b).

The estimation errors of the jth subsystem are defined by (5), where e_i, $\hat x_i\,(i=1,\dots,M^j)$ and x have the upper script j. The chain of sub-predictor-based observers is designed as

$$
\begin{cases}
\dot{\hat x}^j_1(t)=A^j\hat x^j_1(t)+L^jC^j\left(\hat x^j_1\left(t-\frac{r^j}{M^j}\right)-\hat x^j_2(t)\right)+B^ju^j(t)\\
\dot{\hat x}^j_2(t)=A^j\hat x^j_2(t)+L^jC^j\left(\hat x^j_2\left(t-\frac{r^j}{M^j}\right)-\hat x^j_3(t)\right)\\
\quad+B^ju^j\left(t-\frac{1}{M^j}r^j\right)\\
\quad\vdots\\
\dot{\hat x}^j_{M^j-1}(t)=A^j\hat x^j_{M^j-1}(t)+L^jC^j\left(\hat x^j_{M^j-1}\left(t-\frac{r^j}{M^j}\right)-\hat x^j_{M^j}(t)\right)\\
\quad+B^ju^j\left(t-\frac{M^j-2}{M^j}r^j\right)\\
\dot{\hat x}^j_{M^j}(t)=A^j\hat x^j_{M^j}(t)+L^jC^j\left(\hat x^j_{M^j}\left(t-\frac{r^j}{M^j}\right)-x^j(t)\right)\\
\quad+B^ju^j\left(t-\frac{M^j-1}{M^j}r^j\right)
\end{cases}
\tag{28}
$$

with $\hat x^j_i(t)=0$, $i=1,\dots,M^j$ for $t\le 0$, and the controller is chosen as

$$
u^j(t)=K^j\hat x^j_1(t) \tag{29}
$$

Taking the sum of all equations in (5) for the lth plant, we have $-x^l(t)=-\hat x^l_1(t-r^l)+e^l_1(t)+\cdots+e^l_{M^l}(t)$. Employing a similar calculation to (8), we arrive at the closed-loop system of the form

$$
\dot\xi^j(t)=\bar A^j\xi^j(t)+\bar L^j\xi^j\left(t-\tfrac{r^j}{M^j}\right)+\textstyle\sum_{l\neq j}\bar F^{lj}\xi^l(t), \tag{30}
$$

where $\xi^j(t)=\left(\hat x^j_1(t-r^j)^T\ e^j_1(t)^T\ \dots\ e^j_{M^j}(t)^T\right)^T$,

$$
\bar A^j=\begin{pmatrix}A^j+B^jK^j&0&\cdots&0&0\\0&A^j&\cdots&0&0\\ \vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&A^j&0\\0&0&\cdots&0&A^j\end{pmatrix},\ \bar L^j=\begin{pmatrix}0&L^jC^j&0&\cdots&0&0\\0&L^jC^j&-L^jC^j&\cdots&0&0\\0&0&L^jC^j&\cdots&0&0\\ \vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&L^jC^j&-L^jC^j\\0&0&0&\cdots&0&L^jC^j\end{pmatrix},\ \bar F^{lj}=
\begin{pmatrix}0&0&\cdots&0&0\\ \vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&0&0\\-F^{lj}&F^{lj}&\cdots&F^{lj}&F^{lj}\end{pmatrix}.
$$

Theorem 3. *Consider the closed-loop system consisting of the plant (27) and sub-predictor-based controller (28)–(29). Given positive tuning parameters $r^j, M^j, \alpha, \epsilon$ with $\alpha>\epsilon$, let $(M^j+1)n^j\times(M^j+1)n^j$ matrices $P^j, S^j, R^j>0$, and $(M^l+1)n^l\times(M^l+1)n^l$ matrices $P^l>0$ for $l=1,\dots,N$ and $l\neq j$, satisfy the LMIs:*

$$
\begin{aligned}
&\begin{pmatrix}\Phi^j&\Psi^j\\ *&-\Xi^j\end{pmatrix}<0,\quad \Xi^j=\frac{{r^j}^2}{{M^j}^2}R^j,\\
&\Psi^j=\left({\Xi^j}^T\bar A^j,\ {\Xi^j}^T\bar L^j,\ \mathrm{row}_{l=1,\dots,N}\{{\Xi^j}^T\bar F^{lj},l\neq j\}\right)^T,
\end{aligned}
\tag{31}
$$

where Φ^j is a symmetric matrix such that

$$
\begin{aligned}
&\Phi^j_{11}=(\bar A^j)^TP^j+P^j\bar A^j+2\alpha P^j+S^j-e^{-2\alpha\frac{r^j}{M^j}}R^j,\\
&\Phi^j_{12}=P^j\bar L^j+e^{-2\alpha\frac{r^j}{M^j}}R^j,\quad \Phi^j_{13}=\mathrm{row}_{l=1,\dots,N}\{P^j\bar F^{lj},l\neq j\},\\
&\Phi^j_{22}=-e^{-2\alpha\frac{r^j}{M^j}}(S^j+R^j),\\
&\Phi^j_{33}=\mathrm{diag}_{l=1,\dots,N}\left\{-\tfrac{2\epsilon}{N-1}P^l,l\neq j\right\}.
\end{aligned}
$$

Then the closed-loop system (30) is exponentially stable with a decay rate $\alpha-\epsilon$. ■

Table 1
Sub-predictors vs Classical Predictor.

	$h = 0.138$, $\alpha = 0.001$	
	Delay r	Delay uncertainty
Classical predictor (Selivanov & Fridman, 2016)	2	0.2
Sub-predictors M = 5	0.45×5	0.28
Sub-predictors M = 7	0.40×7	0.26

Proof. The Lyapunov candidate of the jth plant and the calculation of its time-derivative are similar to those in the proof of Theorem 1. We get

$$\begin{aligned}&\dot{V}^j(t) + 2\alpha V^j(t) - \tfrac{2\epsilon}{N-1} V^l(t)\\ &\leq \lambda^j(t)^T \Phi^j \lambda^j(t) + \lambda^j(t)^T \Psi^j {\Xi^j}^{-1} {\Psi^j}^T \lambda^j(t) \leq 0\end{aligned} \tag{32}$$

where $\lambda^j(t) = \text{col}\{\xi^j(t), \xi^j\left(t - \frac{r^j}{M^j}\right), \text{col}_{l=1,\ldots,N}\{\xi^l(t), l \neq j\}\}$. Applying Schur complement, the inequality (32) implies (31). With (32), we have

$$\dot{V}(t) + 2(\alpha - \epsilon)V(t) \leq 0 \tag{33}$$

where $V(t) = \sum_{j=1}^{N} V^j(t)$, which ensures the exponential stability of the whole system. ■

Remark 4. Similar to Remark 3, applying the classical predictor approach to the interconnected systems (Zhu & Fridman, 2020a, 2020b), the inverse Artstein's transformation from the neighbors $\sum_{l \neq j} F^{lj}\Big(e^l(t) + e^{-A^l r^l}\hat{z}^l(t) - \int_{t-r^l}^{t} e^{A^l(t-r^l-s)} B^l u^l(s)ds\Big)$ with $e^l(t) = x^l(t) - \hat{x}^l(t)$ appears in the jth closed-loop system (see (45) in Zhu & Fridman, 2020b), in which some special terms (see (49) in Zhu & Fridman, 2020b) should be added to the LKF to compensate the distributed delay term of the coupling subsystems, and this complicates the analysis. ■

5. Examples

5.1. Example 1

For the case of a single NCS, we consider the example from Selivanov and Fridman (2016) and Zhang et al. (2001), which is of the form (1) where $A = \begin{pmatrix} 0 & 1 \\ 0 & -0.1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0.1 \end{pmatrix}^T$, $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$, $\Delta A = \Delta B = \Delta C = 0$, the controller and observer gains are selected as $K = \begin{pmatrix} -3.75 & -11.5 \end{pmatrix}$, $L = \begin{pmatrix} -1.4 & -0.36 \end{pmatrix}^T$. The simulation of the sampled-data feedback with the sub-predictors is shown in Table 1. It is seen that starting from $M = 5$, sub-predictor allows larger constant delay r and delay uncertainty. Different from the classical predictor, the sub-predictors are also applicable to norm-bounded uncertainty. The results of Table 1 in the case of sub-predictors also hold for norm-bounded uncertainty $\Delta A = \begin{pmatrix} 0 & 0 \\ g & -g \end{pmatrix}$ with $|g| \leq 0.000031$. The maximum delay value r could be improved by increasing the number of sub-predictors, namely, dividing the large delay into smaller pieces.

5.2. Example 2

Consider Example 2 in Liu et al. (2012), which is of the form (20) where $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -0.512 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 44.838 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 6.446 & 0 & -28.002 \end{pmatrix}^T$, $C_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$, $C_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, and the controller gain is chosen as $K = \begin{pmatrix} 5.825 & 5.883 & 24.941 & 5.140 \end{pmatrix}$. The LMI-simulation of the sub-predictors under scheduling protocol is shown in Table 2. It is seen that starting from $M = 2$, sub-predictor allows larger delay r than the controller without predictor.

Table 2
Sub-predictors with scheduling protocol.

	$h = 0.0028$, $\alpha = 0.001$	
	Delay r	Delay uncertainty
Output feedback (Liu et al., 2012)	0.004	–
Sub-predictors M = 2	0.0034×2	0.0001
Sub-predictors M = 3	0.0031×3	0.0001

Table 3
Sub-predictors for coupled systems.

	$\alpha - \epsilon = 0.0001$	
	Delay r	Delay uncertainty
Classical predictor (PDE) (Zhu & Fridman, 2020b)	0.2	–
Classical predictor (ODE) (Zhu & Fridman, 2020b)	0.1	–
Sub-predictors M = 3	0.069×3	0.001
Sub-predictors M = 4	0.052×4	0.0005
Sub-predictors M = 4	0.027×4	0.016

5.3. Example 3

For the coupled systems, we consider an example of two coupled inverted pendulums on two carts from Dolk, Borgers, and Heemels (2017) and Freirich and Fridman (2016), which is of the form (27) where $N = 2$, $A^1 = A^2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 2.9156 & 0 & -0.0005 & 0 \\ 0 & 0 & 0 & 1 \\ -1.6663 & 0 & 0.0002 & 0 \end{pmatrix}$, $B^1 = B^2 = \begin{pmatrix} 0 & -0.0042 & 0 & 0.0167 \end{pmatrix}^T$, $C^1 = C^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, $F^{21} = F^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0011 & 0 & 0.0005 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0003 & 0 & -0.0002 & 0 \end{pmatrix}$. The controller gains are selected as $K^1 = K^2 = \begin{pmatrix} 11396 & 7196.2 & 573.96 & 1199.0 \end{pmatrix}$. The observer gains are selected as $L^1 = L^2 = \begin{pmatrix} -11.7 & -37 & 1.2 & 7.9 \\ 1.2 & 8.9 & -11 & -36 \end{pmatrix}^T$. As shown in Table 3, starting from $M = 3$, the sub-predictors feedback in the decentralized manner allows a relatively larger delay than the classical predictor.

6. Conclusion

In this paper, we developed a sub-predictor for NCSs with uncertain large delays. Furthermore, we extended the sub-predictors to Round-Robin scheduling from sensors to controller and norm-bounded uncertainty, as well as interconnected systems. The sampling intervals may be variable, and the network-induced delays are allowed to be time-varying. The design and analysis of our paper are based on the time-delay approach to NCSs with the LMIs technique. Comparative to the classical reduction-based predictor involving an integral formula of distributed input, the sub-predictor-based feedback is more friendly in the presence of norm-bounded uncertainties and for interconnected systems, and is simpler for implementation.

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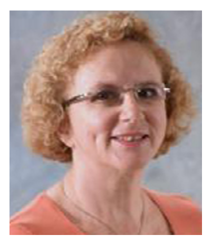

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