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Brief paper

Predictor methods for decentralized control of large-scale systems with input delays☆

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ABSTRACT

This paper develops a decentralized predictor-based control for large-scale systems with large input delays under the premise that the interconnections between subsystems are not strong. The local controller operates independently. Given any large delays, the predictor which exponentially stabilizes each uncoupled system, will stabilize the coupled one provided that the coupling is not solid. We propose two methods for the delay compensation: the backstepping-based partial differential equation (PDE) approach and the reduction-based ordinary differential equation (ODE) approach. We present decentralized Lyapunov-based analysis under the two predictor methods. It appears that the first predictor method leads to simpler conditions and manages with larger delays, whereas the second is easily applied to decentralized asynchronous sampled-data implementation, both under continuous and under discrete-time measurements. Through a benchmark example of two coupled cart–pendulum systems, the proposed methods are demonstrated to be effective when the input delays are too large for the system to be stabilized without a predictor.

1. Introduction

Over the past 60 years, the predictor feedback has been demonstrated to be quite effective in compensating large delays, and major breakthroughs have been successively reported, such as the framework of "Smith predictor" and the "reduction" approach in Artstein (1982), Smith (1959), Yue and Han (2005) and Zhou, Lin, and Duan (2012). However, most of results about predictors are limited to a single plant with a centralized controller (Karafyllis & Krstic, 2012, 2017; Mazenc & Normand-Cyrot, 2013; Selivanov & Fridman, 2016b). Utilizing the concept of a PDE representation of delayed input (Krstic, 2009), the recent paper (Liu, Sun, & Krstic, 2018) considers predictor-based stabilization for two interconnected systems and the results are restricted to continuous-time control.

With rapid development of computer, modern control usually employs digital technology for implementation. Networked control systems (NCSs), where sensors and actuators exchange data through communication network, are quite popular in many practical applications. Among the imperfections induced by network, the presence of time-delay in communication is a non-negligible factor to degrade the performance of the control loop and even lead to instability of NCSs. A majority of existing literature on NCSs focus on robust stability with respect to small communication delays, namely, they study the maximum upper bound on delay that preserves the performance and formulate the results in terms of linear matrix inequality (LMI) (Freirich & Fridman, 2016; Fridman, 2014; Gao, Chen, & Lam, 2008; Gu, Kharitonov, & Chen, 2003; Liu & Fridman, 2012; Park, Ko, & Jeong, 2011).

This paper extends the predictor feedback to decentralized control for large-scale systems with large input delays, by both continuous-time and sampled-data control. By large delays we understand such input delays that do not preserve the stability of the closed-loop system (which is stable without the delays), and need compensation. Otherwise, the delays are called small. The local control network of each subsystem is designed in a decentralized manner without using information from other neighbors, provided that the interactions in large-scale systems are not strong. The input delays and sampling instants of each subsystem may be distinct from each other. Note that decentralized control of large-scale interconnected systems with local independent controllers was studied in the presence of small delays in Freirich and Fridman (2016), Heemels, Borgers, Wouw, Nesic, and Teel (2013) and Dolk, Borgers, and Heemels (2017).

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We employ two predictor methods for the delay compensation: the backstepping-based PDE approach (Krstic, 2009) and the reduction-based ODE approach (Artstein, 1982). For the single system (Artstein, 1982), two predictor approaches in the continuous-time lead to equivalent results. Our main objective is comparison of two main predictor methods in application to large-scale systems. As initiated in Freirich and Fridman (2016) for the case of small delays, we present a decentralized Lyapunov–Krasovskii method for the exponential stability analysis of the whole system under decentralized predictors. Note that the decentralized Lyapunov method leads to simpler reduced-order LMIs comparatively to Lyapunov method that is applied to the whole system (see Section 2.1 and Example in Section 5). For large-scale systems in the continuous-time, the PDE-based method leads to simpler LMI conditions and manages with larger delays, so that the PDE predictor is less conservative. In contrast, the ODE-based method is easily applied to asynchronous sampled-data implementation under both, continuous-time and sampled-data measurements. Finally, a benchmark example of two coupled cart–pendulum systems (Dolk et al., 2017; Freirich & Fridman, 2016; Heemels et al., 2013) is provided to verify the proposed control scheme when the input delays are large enough so that the predictor-free control (Dolk et al., 2017; Freirich & Fridman, 2016; Heemels et al., 2013) is unable to handle.

A conference version of the paper was presented in Zhu and Fridman (2019), where the sampled-data case is limited to continuous-time measurements.

2. PDE-based continuous-time control

Consider large-scale interconnected linear systems with input delays

$$\dot{x}_j(t) = A_j x_j(t) + B_j u_j(t - r_j) + \sum_{l \neq j} F_{lj} x_l(t), \quad t \geq 0 \tag{1}$$

where $j = 1, 2, \ldots, M$ is the subsystem index, $x_j(t) \in \mathbb{R}^{n^j}$ is the state of the jth plant, $u_j(t) \in \mathbb{R}^{m^j}$ is the local control input of the jth plant which is subject to a large constant and known input delay $r_j > 0$, $x_l(t) \in \mathbb{R}^{n^l}$ are coupling terms, A_j, B_j and F_{lj} are matrices of appropriate dimensions. We assume that the pair (A_j, B_j) is stabilizable. Let K_j be a matrix that leads to Hurwitz $A_j + B_j K_j$.

The system (1) could be interpreted as large-scale systems with large input delays shown in Fig. 1. As all signals are continuous, we use the PDE-based feedback to address delays.

Based on Krstic (2009), the system with input delays (1) could be represented by the ODE–PDE cascade as follows:

$$\dot{x}_j(t) = A_j x_j(t) + B_j v_j(0, t) + \sum_{l \neq j} F_{lj} x_l(t), \quad t \geq 0 \tag{2}$$

$$\partial_t v_j(\sigma, t) = \partial_\sigma v_j(\sigma, t), \quad \sigma \in [0, r_j] \tag{3}$$

$$v_j(r_j, t) = u_j(t) \tag{4}$$

with the solution of transport PDE (3)–(4) being

$$v_j(\sigma, t) = u_j(t + \sigma - r_j), \quad \sigma \in [0, r_j]. \tag{5}$$

The prediction-based boundary controller is designed as

$$u_j(t) = v_j(r_j, t) = K_j\left(e^{A_j r_j} x_j(t) + \int_0^{r_j} e^{A_j(r_j - \delta)} B_j v_j(\delta, t) d\delta\right) \tag{6}$$

For stability analysis, we bring in the invertible backstepping transformation

$$w_j(\sigma, t) = v_j(\sigma, t) - K_j\left(e^{A_j \sigma} x_j(t) + \int_0^{\sigma} e^{A_j(\sigma - \delta)} B_j v_j(\delta, t) d\delta\right) \tag{7}$$

$$v_j(\sigma, t) = w_j(\sigma, t) + K_j\left(e^{(A_j + B_j K_j)\sigma} x_j(t) + \int_0^{\sigma} e^{(A_j + B_j K_j)(\sigma - \delta)} B_j w_j(\delta, t) d\delta\right) \tag{8}$$

through which the ODE–PDE cascade (2)–(4) is converted into the target system as follows:

$$\dot{x}_j(t) = (A_j + B_j K_j) x_j(t) + B_j w_j(0, t) + \sum_{l \neq j} F_{lj} x_l(t), \tag{9}$$

$$\partial_t w_j(\sigma, t) = \partial_\sigma w_j(\sigma, t) - K_j e^{A_j \sigma} \sum_{l \neq j} F_{lj} x_l(t), \quad \sigma \in [0, r_j] \tag{10}$$

$$w_j(r_j, t) = 0 \tag{11}$$

Remark 1. Substituting $\sigma = r_j$ into (7), the boundary condition (11) is guaranteed by the feedback law (6). In later analysis, it is evident that the boundary condition (11) plays an important role in the stabilization of the target cascade (9)–(11). However, in the sampled-data control, the continuous-time feedback (6) is replaced by the sampled-data feedback $u_j(t) = v_j(r_j, t) = K_j\left(e^{A_j r_j} x_j(t_k^j) + \int_0^{r_j} e^{A_j(r_j - \delta)} B_j v_j(\delta, t_k^j) d\delta\right)$, $t \in [t_k^j, t_{k+1}^j)$, $k \in \mathbb{Z}_0^+$ where t_k^j is the sampling instant of the jth subsystem, $\mathbb{Z}_0^+$ stands for the set of non-negative integrals. Substituting the above sampled-data control law into (7), the boundary condition (11) becomes non-homogeneous such that

$$w_j(r_j, t) = K_j\left(e^{A_j r_j}\left(x_j(t_k^j) - x_j(t)\right) + \int_0^{r_j} e^{A_j(r_j - \delta)} B_j \left(v_j(\delta, t_k^j) - v_j(\delta, t)\right) d\delta\right) \neq 0,$$
$$t \in [t_k^j, t_{k+1}^j), k \in \mathbb{Z}_0^+$$

Thus it is difficult to apply the PDE-based method to sampled-data control. ■

Theorem 1. *Consider the closed-loop system consisting of plant (2)–(4) and controller (6). Given tuning parameters $0 < \varepsilon < \alpha$, let parameters $\lambda_j > 0$, and $n^j \times n^j$ matrices $P_j > 0$, $m^j \times m^j$ matrices $U_j > 0$, $n^l \times n^l$ matrices $P_l > 0$ for $l = 1, \ldots, M$ and $l \neq j$, satisfy the LMIs:*

$$\Phi_j = \begin{bmatrix} \phi_j & P_j B_j & 0 & P_j \mathscr{F}_j \\ * & -U_j & 0 & 0 \\ * & * & -\lambda_j I_{n^j} & -\lambda_j \mathscr{F}_j \\ * & * & * & -\Pi_j \end{bmatrix} < 0 \tag{12}$$

$$\Psi_j = \begin{bmatrix} U_j & U_j K_j \\ K_j^T U_j^T & \frac{\lambda_j}{\delta_j} I_{m^j} \end{bmatrix} > 0 \tag{13}$$

where Φ_j is a symmetric matrix, I is a unit matrix with appropriate dimension, $\delta_j = r_j e^{2(1+2\alpha)r_j + 2|A_j|r_j}$ with $|A_j| = \sqrt{\lambda_{\max}\left(A_j^T A_j\right)}$, and

$$\phi_j = (A_j + B_j K_j)^T P_j + P_j (A_j + B_j K_j) + 2\alpha_j P_j,$$

$$\Pi_j = \mathrm{diag}_{l=1,\ldots,M}\left\{\frac{2\varepsilon}{M-1} P_l, l \neq j\right\},$$

$$\mathscr{F}_j = \mathrm{row}_{l=1,\ldots,M}\left\{F_{lj}, l \neq j\right\}.$$

Then the closed-loop large-scale system is exponentially stable with a decay rate $\rho = \alpha - \varepsilon$. ■

Proof. The PDE-based Lyapunov–Krasovskii functional (LKF) is selected as

$$V_j(t) = V_{P_j}(t) + V_{U_j}(t) \tag{14}$$

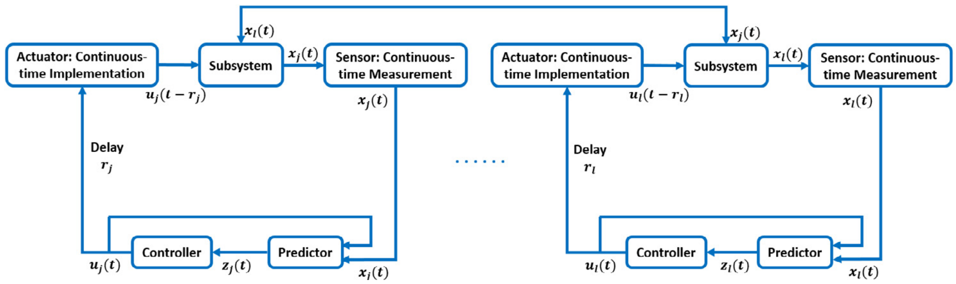

Fig. 1. Large-scale continuous-time systems under input delays.

where

$$V_{P_j}(t) = x_j^T(t)P_jx_j(t), \quad P_j > 0 \tag{15}$$

$$V_{U_j}(t) = \int_0^{r_j} e^{(1+2\alpha)\sigma} w_j^T(\sigma,t)U_jw_j(\sigma,t)d\sigma, \quad U_j > 0 \tag{16}$$

As suggested in Freirich and Fridman (2016) for decentralized control, if the Lyapunov candidate (14) along the solution of closed-loop system (9)–(11) satisfies

$$\dot V_j(t) + 2\alpha V_j(t) \le \frac{2\varepsilon}{M-1}\sum_{l\neq j} V_l(t) \tag{17}$$

then we have

$$\dot V(t) + 2(\alpha-\varepsilon)V(t) \le 0 \tag{18}$$

where $V(t) = \sum_{j=1}^M V_j(t)$, which implies the exponential stability of the closed-loop system.

Taking the time derivative of (15) along (9), we have

$$\begin{aligned}
&\dot V_{P_j}(t) + 2\alpha V_{P_j}(t)\\
&= x_j^T(t)\big(2P_j(A_j+B_jK_j)+2\alpha P_j\big)x_j(t)\\
&\quad + 2x_j^T(t)P_jB_jw_j(0,t) + 2x_j^T(t)P_j\sum_{l\neq j}F_{lj}x_l(t)
\end{aligned} \tag{19}$$

Taking the time derivative of (16) along (10), employing (11) and the integration by parts in σ, we have

$$\begin{aligned}
&\dot V_{U_j}(t) + 2\alpha V_{U_j}(t)\\
&= 2\int_0^{r_j} e^{(1+2\alpha)\sigma} w_j^T(\sigma,t)U_j\partial_\sigma w_j(\sigma,t)d\sigma\\
&\quad - 2\int_0^{r_j} e^{(1+2\alpha)\sigma} w_j^T(\sigma,t)U_jK_je^{A_j\sigma}d\sigma \sum_{l\neq j}F_{lj}x_l(t)\\
&\quad + 2\alpha\int_0^{r_j} e^{(1+2\alpha)\sigma} w_j^T(\sigma,t)U_jw_j(\sigma,t)d\sigma\\
&= -w_j^T(0,t)U_jw_j(0,t) - 2\xi_j^T(t)\sum_{l\neq j}F_{lj}x_l(t)\\
&\quad - \int_0^{r_j} e^{(1+2\alpha)\sigma} w_j^T(\sigma,t)U_jw_j(\sigma,t)d\sigma
\end{aligned} \tag{20}$$

where $\xi_j^T(t) = \int_0^{r_j} e^{(1+2\alpha)\sigma} w_j^T(\sigma,t)U_jK_je^{A_j\sigma}d\sigma$.

Utilizing Jensen's inequality, $\xi_j^T(t)$ on the right side of (20) satisfies

$$\begin{aligned}
\left|\xi_j^T(t)\right|^2 &= \left|\int_0^{r_j} e^{(1+2\alpha)\sigma} w_j^T(\sigma,t)U_jK_je^{A_j\sigma}d\sigma\right|^2\\
&\le r_j\int_0^{r_j}\left|e^{(1+2\alpha)\sigma} w_j^T(\sigma,t)U_jK_je^{A_j\sigma}\right|^2 d\sigma\\
&\le r_j\int_0^{r_j} e^{2(1+2\alpha)\sigma}\left|w_j^T(\sigma,t)U_jK_j\right|^2\left|e^{A_j\sigma}\right|^2 d\sigma\\
&\le \underbrace{r_je^{2(1+2\alpha)r_j+2|A_j|r_j}}_{\delta_j}\int_0^{r_j}\left|w_j^T(\sigma,t)U_jK_j\right|^2 d\sigma
\end{aligned} \tag{21}$$

Combining (19) with (20), applying S-procedure (see e.g. Section 3.2.3 of Fridman (2014)) with $\lambda_j > 0$, where we employ (21), we have

$$\begin{aligned}
&\dot V_j(t) + 2\alpha V_j(t) - \frac{2\varepsilon}{M-1}\sum_{l\neq j}V_l(t)\\
&\quad + \frac{1}{\lambda_j}\left(\delta_j\int_0^{r_j}\left|w_j^T(\sigma,t)U_jK_j\right|^2 d\sigma - \left|\xi_j^T(t)\right|^2\right)\\
&\le -\int_0^{r_j} w_j^T(\sigma,t)\left(U_j - \frac{1}{\lambda_j}\delta_jU_jK_jK_j^TU_j^T\right)w_j(\sigma,t)d\sigma\\
&\quad + \eta_j^T(t)\mathrm{diag}\left\{I, I, \frac{1}{\lambda_j}I, I\right\}\Phi_j\mathrm{diag}\left\{I, I, \frac{1}{\lambda_j}I, I\right\}\eta_j(t).
\end{aligned} \tag{22}$$

Here $\eta_j(t) = \mathrm{col}\{x_j(t), w_j(0,t), \xi_j(t), \mathrm{col}_{l=1,\ldots,M}\{x_l(t), l\neq j\}\}$, I is the unit matrix of appropriate dimension.

Applying Schur complement lemma, inequalities (17)–(18) are implied by LMI-condition (12)–(13). ■

Remark 2. Given any large delays, as long as the couplings among the large-scale systems are not solid, the PDE-based LMIs (12)–(13) are always feasible. Indeed, for $F_{lj} = 0$ in (1), which implies there is no interaction among subsystems, the LMIs (12)–(13) are reduced to

$$\begin{bmatrix}\phi_j & P_jB_j\\ * & -U_j\end{bmatrix} < 0 \tag{23}$$

Since $(A_j+B_jK_j)$ are assumed to be Hurwitz, for some $\alpha > 0$, there exist P_j such that $\phi_j < 0$. Then there exist U_j that satisfy (23). Fix next $\varepsilon \in (0,\alpha)$. Applying Schur complement to (13) and (12) with P_j, U_j subject to (23), respectively, we obtain

$$U_j - \tfrac{1}{\lambda_j}\delta_jU_jK_jK_j^TU_j^T > 0, \quad \text{and}$$

$$\begin{aligned}
&\begin{bmatrix}\phi_j & P_jB_j & 0\\ * & -U_j & 0\\ * & * & -\lambda_jI_{n^j}\end{bmatrix} + \begin{bmatrix}P_j\mathscr{F}_j\\ 0\\ -\lambda_j\mathscr{F}_j\end{bmatrix}\Pi_j^{-1}\begin{bmatrix}\mathscr{F}_j^TP_j^T & 0 & -\lambda_j\mathscr{F}_j^T\end{bmatrix}\\
&= \begin{bmatrix}\phi_j+P_j\mathscr{F}_j\Pi_j^{-1}\mathscr{F}_j^TP_j^T & P_jB_j & -\lambda_jP_j\mathscr{F}_j\Pi_j^{-1}\mathscr{F}_j^T\\ * & -U_j & 0\\ * & * & -\lambda_jI_{n^j}+\lambda_j^2\mathscr{F}_j\Pi_j^{-1}\mathscr{F}_j^T\end{bmatrix} < 0,\\
&\quad\Updownarrow\\
&\phi_j + P_jB_jU_j^{-1}B_j^TP_j^T + \tfrac{M-1}{2\varepsilon}P_j\sum_{l\neq j}\left(F_{lj}P_l^{-1}F_{lj}^T\right)P_j^T\\
&+ \lambda_j^2\tfrac{(M-1)^2}{4\varepsilon^2}P_j\sum_{l\neq j}\left(F_{lj}P_l^{-1}F_{lj}^T\right)\\
&\times\left(\lambda_jI_{n^j} - \tfrac{\lambda_j^2(M-1)}{2\varepsilon}\sum_{l\neq j}\left(F_{lj}P_l^{-1}F_{lj}^T\right)\right)^{-1}\\
&\times\sum_{l\neq j}\left(F_{lj}P_l^{-1}F_{lj}^T\right)P_j^T < 0
\end{aligned} \tag{24}$$

Let λ_j $(j = 1, \ldots, M)$ be large scalars. By selecting F_{lj} such that

$$F_{lj}F_{lj}^T \leq \frac{1}{\lambda_j}\lambda_{\max}^{-1}\left(P_l^{-1}\right)I \tag{25}$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of the matrix, we achieve the feasibility of LMIs (12)–(13). ■

2.1. Comparison between the decentralized and centralized analysis

In this section, treating the large-scale system as a global system, we apply a full-order LKF to stability analysis, and compare the conventionally centralized method with the decentralized one of Theorem 1.

Please note the delay r_j of each subsystem appears in the limit of integration of (16). In order to construct a global LKF, this should be avoided. Thus we apply the rescaled unity-interval notation to (5) so that (5) becomes $v_j(\sigma, t) = u_j(t + r_j(\sigma - 1))$, $\sigma \in [0, 1]$. Accordingly, the target closed-loop system (9)–(11) becomes

$$\dot{x}_j(t) = (A_j + B_jK_j)x_j(t) + B_jw_j(0, t) + \sum_{l\neq j} F_{lj}x_l(t), \tag{26}$$

$$r_j\partial_t w_j(\sigma, t) = \partial_\sigma w_j(\sigma, t) - r_jK_je^{A_jr_j\sigma}\sum_{l\neq j} F_{lj}x_l(t) \tag{27}$$

$$w_j(1, t) = 0, \quad \sigma \in [0, 1] \tag{28}$$

To analyze the large-scale systems as a global system, we integrate (26)–(28) for $j = 1, 2, \ldots, M$ into

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}w(0, t) \tag{29}$$

$$\bar{R}\partial_t w(\sigma, t) = \partial_\sigma w(\sigma, t) - \bar{K}\bar{E}(\sigma)\bar{F}x(t) \tag{30}$$

$$w(1, t) = 0 \tag{31}$$

where $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix}$, $w(\sigma, t) = \begin{bmatrix} w_1(\sigma,t) \\ w_2(\sigma,t) \\ \vdots \\ w_M(\sigma,t) \end{bmatrix}$, and $\bar{A} = \begin{bmatrix} A_1{+}B_1K_1 & F_{21} & \cdots & F_{M1} \\ F_{12} & A_2{+}B_2K_2 & \cdots & F_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ F_{1M} & F_{2M} & \cdots & A_M{+}B_MK_M \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_M \end{bmatrix}$, $\bar{K} = \begin{bmatrix} r_1K_1 & 0 & \cdots & 0 \\ 0 & r_2K_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_MK_M \end{bmatrix}$, $\bar{F} = \begin{bmatrix} 0 & F_{21} & \cdots & F_{M1} \\ F_{12} & 0 & \cdots & F_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ F_{1M} & F_{2M} & \cdots & 0 \end{bmatrix}$, $\bar{E}(\sigma) = \begin{bmatrix} e^{A_1r_1\sigma} & 0 & \cdots & 0 \\ 0 & e^{A_2r_2\sigma} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{A_Mr_M\sigma} \end{bmatrix}$, $\bar{R} = \begin{bmatrix} r_1I_{m_1} & & & \\ & r_2I_{m_2} & & \\ & & \ddots & \\ & & & r_MI_{m_M} \end{bmatrix}$

Proposition 1. *Consider the closed-loop system consisting of plant (2)–(4) and controller (6). Given tuning parameters $\rho > 0$ and $\mu > \max\{r_1, \ldots, r_M\}$, let a parameter $\lambda > 0$, $(n^1 + \cdots + n^M) \times (n^1 + \cdots + n^M)$ matrix $\bar{P} > 0$, $(m^1 + \cdots + m^M) \times (m^1 + \cdots + m^M)$ matrix $\bar{U} > 0$, satisfy the LMIs:*

$$\Phi = \begin{bmatrix} \bar{A}^T\bar{P} + \bar{P}\bar{A} + 2\rho\bar{P} & \bar{P}\bar{B} & -\lambda\bar{F}^T \\ * & -\bar{R}\bar{U} & 0 \\ * & * & -\lambda I \end{bmatrix} < 0 \tag{32}$$

$$\Psi = \begin{bmatrix} 2\rho\bar{R}\bar{U}\left(\mu I - \bar{R}\right) & \bar{R}\bar{U}\bar{K} \\ \bar{K}^T\bar{U}\bar{R} & \frac{\lambda}{\delta}I \end{bmatrix} > 0 \tag{33}$$

where Φ is a symmetric matrix, I is a unit matrix with appropriate dimension, $\delta = e^{4\rho\mu}\left|\mathrm{diag}\{e^{2|A_1|r_1}, \ldots, e^{2|A_M|r_M}\}\right|$. Then the closed-loop system is exponentially stable with a decay rate ρ. ■

Proof. The global LKF is selected as

$$V(t) = V_{\bar{P}}(t) + V_{\bar{U}}(t) \tag{34}$$

where

$$V_{\bar{P}}(t) = x^T(t)\bar{P}x(t), \quad \bar{P} > 0 \tag{35}$$

$$V_{\bar{U}}(t) = \int_0^1 e^{2\rho\mu\sigma}w^T(\sigma, t)\bar{R}\bar{U}\bar{R}w(\sigma, t)d\sigma, \quad \bar{U} > 0 \tag{36}$$

Note that the tuning parameter $\mu > 0$ is inserted into $V_{\bar{U}}$ of (36) in order to take into account the asymmetric structure of (27) with r_j multiplying $\partial_t w_j$. Taking the time-derivative of (35)–(36) along (29)–(31), and following a similar argument of the proof of Theorem 1, we conclude that the inequality $\dot{V}(t) + 2\rho V(t) \leq 0$ is implied by (32)–(33). ■

Comparing the LMIs (12)–(13) derived by the decentralized analysis proposed in the paper with the LMIs (32)–(33) derived by the conventional method using a full-order LKF, it is evident that (12)–(13) have essentially less decision variables and are of smaller order comparatively to (32)–(33). In the sampled-data case with asynchronous sampling, the decentralized analysis leads to essentially simpler results than the centralized one, where multiple integral terms should be inserted into LKF to take care of multiple samplings.

3. ODE-based sampled-data control under continuous-time measurements

In this section, we consider a more complicated case.

As revealed in Fig. 2, the sensor is able to continuously measure the plant state $x_j(t)$, whereas the continuously changing control signal $u_j(t)$ is sampled at the time instants ζ_k^j and sent through a controller-to-actuator network subject to a large delay r_j. As analyzed in Remark 1, when the control signals are sampled, the PDE-based method may not be applied trivially to NCSs. Instead, the ODE-based approach is employed.

The sampling series $\{\zeta_k^j\}$ satisfy

$$0 = \zeta_0^j < \zeta_1^j < \zeta_2^j < \cdots, \quad \lim_{k\to\infty} \zeta_k^j = \infty, \quad \zeta_{k+1}^j - \zeta_k^j \leq h_j \tag{37}$$

The actuator is a zero-order hold and is assumed to be event-driven (update its output once it receives new data). Therefore, the updating instants of the actuator satisfies

$$t_k^j = \zeta_k^j + r_j, \quad t_k^j < t_{k+1}^j, \quad k \in \mathbb{Z}_0^+ \tag{38}$$

As a consequence, the plant (1) is converted into

$$\dot{x}_j(t) = A_jx_j(t) + B_ju_j(\zeta_k^j) + \sum_{l\neq j} F_{lj}x_l(t), \quad t \in [t_k^j, t_{k+1}^j), k \in \mathbb{Z}_0^+ \tag{39}$$

Based on (1), the predictor state is introduced below

$$z_j(t) = e^{A_jr_j}x_j(t) + \int_{t-r_j}^t e^{A_j(t-\tau)}B_ju_j(\tau)d\tau \tag{40}$$

and the control input is designed as

$$u_j(t) = K_jz_j(t) = K_j\left(e^{A_jr_j}x_j(t) + \int_{t-r_j}^t e^{A_j(t-\tau)}B_ju_j(\tau)d\tau\right) \tag{41}$$

For stability analysis, taking the time-derivative of (40) along (39), the dynamics of $z_j(t)$ satisfies

$$\begin{aligned} \dot{z}_j(t) = {} & A_jz_j(t) + B_ju_j(t) + e^{A_jr_j}B_j\left(u_j(\zeta_k^j) - u_j(t - r_j)\right) \\ & + e^{A_jr_j}\sum_{l\neq j} F_{lj}x_l(t), \quad t \in [t_k^j, t_{k+1}^j), k \in \mathbb{Z}_0^+ \end{aligned} \tag{42}$$

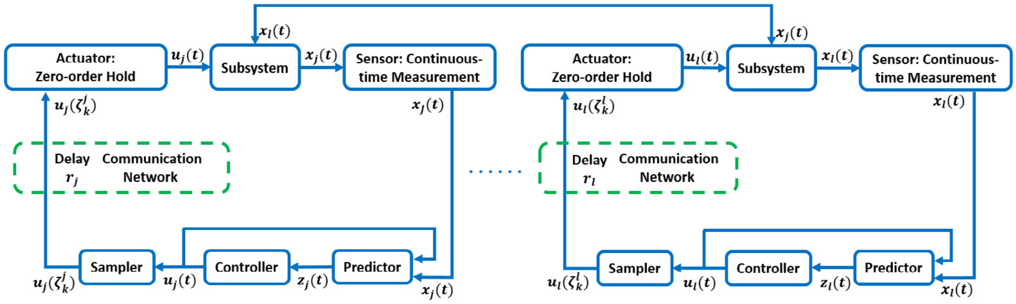

Fig. 2. Large-scale sampled-data NCSs with continuous-time measurement under controller-to-Actuator delays.

Combining (40) with (41), the inverse transformation is brought in as

$$
\begin{aligned}
x_l(t) &= e^{-A_l r_l} z_l(t) - \int_{t-r_l}^{t} e^{A_l(t-\tau-r_l)} B_l u_l(\tau) d\tau \\
&= e^{-A_l r_l} z_l(t) - \int_{t-r_l}^{t} e^{A_l(t-\tau-r_l)} B_l K_l z_l(\tau) d\tau \\
&= e^{-A_l r_l} z_l(t) - \xi_l(t)
\end{aligned} \tag{43}
$$

where

$$
\begin{aligned}
\xi_l(t) &= \int_{t-r_l}^{t} e^{A_l(t-\tau-r_l)} B_l K_l z_l(\tau) d\tau \\
&= \int_{0}^{r_l} e^{A_l(\theta-r_l)} B_l K_l z_l(t-\theta) d\theta
\end{aligned} \tag{44}
$$

Substituting (41) and (43) into (42), we get the dynamics of $z_j(t)$ for stability analysis as follows:

$$
\begin{aligned}
\dot{z}_j(t) &= (A_j + B_j K_j) z_j(t) + e^{A_j r_j} B_j K_j \big(z_j(t_k^j - r_j) - z_j(t - r_j)\big) \\
&\quad + e^{A_j r_j} \sum_{l \neq j} F_{lj} \big(e^{-A_l r_l} z_l(t) - \xi_l(t)\big) \\
&= (A_j + B_j K_j) z_j(t) - e^{A_j r_j} B_j K_j \upsilon_j(t) \\
&\quad + e^{A_j r_j} \sum_{l \neq j} F_{lj} \big(e^{-A_l r_l} z_l(t) - \xi_l(t)\big), \\
&\quad t \in [t_k^j, t_{k+1}^j),\ k \in \mathbb{Z}_0^+
\end{aligned} \tag{45}
$$

where $\upsilon_j(t) = z_j(t - r_j) - z_j(\zeta_k^j) = z_j(t - r_j) - z_j(t_k^j - r_j)$.

Theorem 2. *Consider the closed-loop system consisting of plant (39) and controller (41). Given tuning parameters $0 < \varepsilon < \alpha$, let $n^j \times n^j$ matrices $P_j, W_j, U_j > 0$ and $n^l \times n^l$ matrices $P_l, W_l > 0$ for $l = 1, \ldots, M$ and $l \neq j$, satisfy the LMI:*

$$
\begin{bmatrix} \Phi_j & \Psi_j \\ * & -H_j \end{bmatrix} < 0 \tag{46}
$$

where

$$
\Psi_j = \begin{bmatrix} (A_j + B_j K_j)^T H_j \\ -K_j^T B_j^T e^{A_j^T r_j} H_j \\ (\mathscr{F}_j^z)^T e^{A_j^T r_j} H_j \\ (\mathscr{F}_j^\xi)^T e^{A_j^T r_j} H_j \end{bmatrix}, \quad H_j = h_j^2 e^{2\alpha h_j} U_j \tag{47}
$$

and Φ_j is a symmetric matrix such that

$$
\Phi_j = \begin{bmatrix} \phi_j & -P_j e^{A_j r_j} B_j K_j & P_j e^{A_j r_j} \mathscr{F}_j^z & P_j e^{A_j r_j} \mathscr{F}_j^\xi \\ * & -\frac{\pi^2}{4} e^{-2\alpha r_j} U_j & 0 & 0 \\ * & * & -\Pi_j^z & 0 \\ * & * & * & -\Pi_j^\xi \end{bmatrix} \tag{48}
$$

with

$$
\begin{aligned}
\phi_j &= (A_j + B_j K_j)^T P_j + P_j (A_j + B_j K_j) + 2\alpha P_j + r_j \bar{W}_j \\
\bar{W}_j &= K_j^T B_j^T \left(\int_0^{r_j} e^{A_j^T(\theta - r_j)} W_j e^{A_j(\theta - r_j)} d\theta \right) B_j K_j \\
\Pi_j^z &= \frac{2\varepsilon}{M-1} \mathrm{diag}_{l=1,\ldots,M} \{P_l,\ l \neq j\}, \\
\Pi_j^\xi &= \frac{1}{M-1} \mathrm{diag}_{l=1,\ldots,M} \{e^{-2\alpha r_l} W_l,\ l \neq j\}, \\
\mathscr{F}_j^z &= \mathrm{row}_{l=1,\ldots,M} \{F_{lj} e^{-A_l r_l},\ l \neq j\}, \\
\mathscr{F}_j^\xi &= \mathrm{row}_{l=1,\ldots,M} \{-F_{lj},\ l \neq j\}.
\end{aligned}
$$

Then the closed-loop system is exponentially stable with a decay rate $\rho = \alpha - \varepsilon$. ■

Proof. The Lyapunov–Krasovskii functional is selected as

$$
V_j(t) = V_{P_j}(t) + V_{W_j}(t) + V_{U_j}(t) \tag{49}
$$

where

$$
V_{P_j}(t) = z_j^T(t) P_j z_j(t), \quad P_j > 0 \tag{50}
$$

$$
\begin{aligned}
V_{W_j}(t) &= r_j \int_0^{r_j} \int_{t-\theta}^{t} e^{2\alpha(s-t)} z_j^T(s) K_j^T B_j^T e^{A_j^T(\theta - r_j)} W_j \\
&\qquad \times e^{A_j(\theta - r_j)} B_j K_j z_j(s) ds d\theta, \quad W_j > 0
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
V_{U_j}(t) &= h_j^2 e^{2\alpha h_j} \int_{t_k^j - r_j}^{t} e^{2\alpha(s-t)} \dot{z}_j^T(s) U_j \dot{z}_j(s) ds \\
&\quad - \frac{\pi^2}{4} \int_{t_k^j - r_j}^{t - r_j} e^{2\alpha(s-t)} [z_j(s) - z_j(t_k^j - r_j)]^T U_j \\
&\quad \times [z_j(s) - z_j(t_k^j - r_j)] ds, \quad U_j > 0, \\
&\qquad t \in [t_k^j, t_{k+1}^j), \quad k \in \mathbb{Z}_0^+
\end{aligned} \tag{52}
$$

where $V_{U_j}(t) > 0$ and $V_{U_j}((t_k^j)^{-1}) \geq V_{U_j}(t_k^j)$ by Wirtinger's inequality in Liu and Fridman (2012), Selivanov and Fridman (2016a) and Section 7.4 of Fridman (2014). The term $V_{W_j}(t)$ is employed to compensate $\xi_l(t)$ in (45), whereas $V_{U_j}(t)$ is utilized to compensate $\upsilon_j(t)$ in (45).

If the Lyapunov candidate (49) along the solution of closed-loop system (45) satisfies

$$
\begin{aligned}
&\dot{V}_j(t) + 2\alpha V_j(t) + e^{-2\alpha r_j} \xi_j^T(t) W_j \xi_j(t) \\
&\leq \frac{2\varepsilon}{M-1} \sum_{l \neq j} V_l(t) + \frac{1}{M-1} \sum_{l \neq j} e^{-2\alpha r_l} \xi_l^T(t) W_l \xi_l(t)
\end{aligned} \tag{53}
$$

then we have

$$
\dot{V}(t) + 2(\alpha - \varepsilon) V(t) \leq 0 \tag{54}
$$

where $V(t) = \sum_{j=1}^{M} V_j(t)$, which implies the exponential stability of the closed-loop system.

Taking the time-derivative of (50), we have

$$\dot{V}_{P_j}(t) + 2\alpha V_{P_j}(t) = 2z_j^T(t)P_j\dot{z}_j(t) + 2\alpha z_j^T(t)P_j z_j(t) \tag{55}$$

By Jensen's inequality in Gu et al. (2003) and Sections 3.5.5 and 3.8 of Fridman (2014), it is not hard to calculate

$$\begin{aligned}
&\dot{V}_{W_j}(t) + 2\alpha V_{W_j}(t) \\
&= r_j z_j^T(t)K_j^T B_j^T \left(\int_0^{r_j} e^{A_j^T(\theta-r_j)} W_j e^{A_j(\theta-r_j)} d\theta\right) B_j K_j z_j(t) \\
&\quad - r_j \int_0^{r_j} e^{-2\alpha\theta} z_j^T(t-\theta) K_j^T B_j^T e^{A_j^T(\theta-r_j)} \\
&\qquad \times W_j e^{A_j(\theta-r_j)} B_j K_j z_j(t-\theta) d\theta \\
&\le r_j z_j^T(t)\bar{W}_j z_j(t) \\
&\quad - e^{-2\alpha r_j}\left(\int_0^{r_j} z_j^T(t-\theta)K_j^T B_j^T e^{A_j^T(\theta-r_j)} d\theta\right) \\
&\qquad \times W_j \left(\int_0^{r_j} e^{A_j(\theta-r_j)} B_j K_j z_j(t-\theta) d\theta\right) \\
&= r_j z_j^T(t)\bar{W}_j z_j(t) - e^{-2\alpha r_j}\xi_j^T(t) W_j \xi_j(t)
\end{aligned} \tag{56}$$

where $\bar{W}_j$ has been given underneath (48).

Taking the time-derivative of (52), we have

$$\begin{aligned}
&\dot{V}_{U_j}(t) + 2\alpha V_{U_j}(t) \\
&= h_j^2 e^{2\alpha h_j}\dot{z}_j^T(t)U_j\dot{z}_j(t) - \frac{\pi^2}{4}e^{-2\alpha r_j} v_j^T(t)U_j v_j(t)
\end{aligned} \tag{57}$$

Synthesizing (55)–(57) and substituting (45), we get

$$\begin{aligned}
&\dot{V}_j(t) + 2\alpha V_j(t) + e^{-2\alpha r_j}\xi_j^T(t)W_j\xi_j(t) \\
&\quad - \frac{2\varepsilon}{M-1}\sum_{l\neq j} V_l(t) - \frac{1}{M-1}\sum_{l\neq j} e^{-2\alpha r_l}\xi_l^T(t)W_l\xi_l(t)
\end{aligned} \tag{58}$$

$$\le \eta_j^T(t)\Phi_j\eta_j(t) + \eta_j^T(t)\Psi_j H_j^{-1}\Psi_j^T\eta_j(t) \tag{59}$$

where $\eta(t) = \mathrm{col}\{z_j(t), v_j(t), \mathrm{col}_{l=1,\dots,M}\{z_l(t), l \neq j\}, \mathrm{col}_{l=1,\dots,M}\{\xi_l(t), l \neq j\}\}$, and Φ_j, Ψ_j and H_j have been given in (47)–(48). By the Schur complement, inequalities (53)–(54) are implied by LMI-condition (46). ■

When $h_j = 0$ in (37), (52) and (47), we derive the following corollary for the ODE-based continuous-time control.

Corollary 1. *Consider the closed-loop system consisting of plant (1) and controller (41). Given tuning parameters $0 < \varepsilon < \alpha$, let $n^j \times n^j$ matrices $P_j, W_j > 0$ and $n^l \times n^l$ matrices $P_l, W_l > 0$ for $l = 1, \dots, M$ and $l \neq j$, satisfy the LMI:*

$$\Phi_j = \begin{bmatrix} \phi_j & P_j e^{A_j r_j}\mathscr{F}_j^z & P_j e^{A_j r_j}\mathscr{F}_j^\xi \\ * & -\Pi_j^z & 0 \\ * & * & -\Pi_j^\xi \end{bmatrix} < 0 \tag{60}$$

where all elements of Φ_j in (60) are defined exactly the same as those corresponding elements in (48). Then the closed-loop system is exponentially stable with a decay rate $\rho = \alpha - \varepsilon$. ■

Remark 3. Similar to Remark 2, we check the feasibility of the ODE-based LMI (60) for non-strong coupling. When $F_{lj} = 0$ in (1), which implies there is no interaction among subsystems, the LMI (60) is reduced to

$$\bar{\phi}_j = (A_j + B_j K_j)^T P_j + P_j(A_j + B_j K_j) + 2\alpha P_j < 0. \tag{61}$$

Given $\alpha > \varepsilon > 0$, applying Schur complement to (60) with P_j subject to (61), we obtain

$$\begin{aligned}
&\bar{\phi}_j + r_j K_j^T B_j^T \left(\int_0^{r_j} e^{A_j^T(\theta-r_j)} W_j e^{A_j(\theta-r_j)} d\theta\right) B_j K_j \\
&+ \tfrac{M-1}{2\varepsilon} P_j e^{A_j r_j} \sum_{l\neq j}\left(F_{lj} e^{-A_l r_l} P_l^{-1} e^{-A_l^T r_l} F_{lj}^T\right) e^{A_j^T r_j} P_j^T \\
&+ (M-1) P_j e^{A_j r_j} \sum_{l\neq j} e^{2\alpha r_l}\left(F_{lj} W_l^{-1} F_{lj}^T\right) e^{A_j^T r_j} P_j^T < 0
\end{aligned} \tag{62}$$

Set $W_j = w_j I$ $(j = 1, \dots, M)$, where $w_j > 0$ is a small scalar and I is a unit matrix. By selecting F_{lj} such that

$$F_{lj}F_{lj}^T \le w_l^2 I, \quad F_{lj}F_{lj}^T \le w_l \lambda_{\max}^{-1}\left(e^{-A_l r_l} P_l^{-1} e^{-A_l^T r_l}\right) I, \tag{63}$$

we achieve the feasibility of LMI (60). ■

4. ODE-based sampled-data control under discrete-time measurements

In this section, we consider the most complicated case of sampled-data feedback with sampled-data measurements. As shown in Fig. 3, let $\{s_k^j\}$ with $k \in \mathbb{Z}_0^+$ be sampling instants of the jth subsystem such that

$$0 = s_0^j < s_1^j < s_2^j < \cdots, \quad \lim_{k\to\infty} s_k^j = \infty, \quad s_{k+1}^j - s_k^j \le h_j \tag{64}$$

The state $x_j(t)$ is sampled by the sensor at sampling time s_k^j, and the sampled-data $x_j(s_k^j)$ is transmitted to the controller by the communication network subject to a known constant sensor-to-controller delay $r_j^0 > 0$. A control signal is calculated by the controller and transmitted to the actuator (which is a zero-order hold) through the communication network subject to a known constant controller-to-actuator delay $r_j^1 > 0$. The controller and actuator are assumed to be event-driven, which means they update their outputs as soon as they receive new data. Therefore the updating instants of the controller and actuator are respectively $\zeta_k^j = s_k^j + r_j^0$ and $t_k^j = \zeta_k^j + r_j^1 = s_k^j + r_j$, where $r_j = r_j^0 + r_j^1$.

There are two different cases about the communication delay: (1) small delay which is not greater than sampling interval (see Fig. 4, left), (2) large delay (see Fig. 4, right). In the first case, the delay is limited to be "small" and the system may be stabilized without predictor. Thus the predictor-based controller proposed in this section concentrates on the second case that the delay length is allowed to be large.

Under the sampled-data and zero-order hold, the plant (1) is transformed into

$$\dot{x}_j(t) = A_j x_j(t) + B_j u_j(s_k^j) + \sum_{l\neq j} F_{lj} x_l(t), \quad t \in [t_k^j, t_{k+1}^j), k \in \mathbb{Z}_0^+ \tag{65}$$

To construct a predictor-based controller for (65), define a piece-wise function such that

$$u_j(s) = u_j(s_k^j), \quad s \in [s_k^j, s_{k+1}^j), \quad k \in \mathbb{Z}_0^+ \tag{66}$$

and $u_j(s) \equiv 0$ for $s < 0$. By (66), the plant (65) recovers the form of (1) such that

$$\dot{x}_j(t) = A_j x_j(t) + B_j u_j(t - r_j) + \sum_{l\neq j} F_{lj} x_l(t), \quad t \in [t_k^j, t_{k+1}^j), k \in \mathbb{Z}_0^+ \tag{67}$$

According to (66)–(67), the change of variable is introduced as

$$z_j(t) = e^{A_j r_j} x_j(t) + \int_{t-r_j}^{t} e^{A_j(t-\theta)} B_j u_j(\theta) d\theta, \quad t \ge 0 \tag{68}$$

and $z_j(t) \equiv 0$ for $t < 0$.

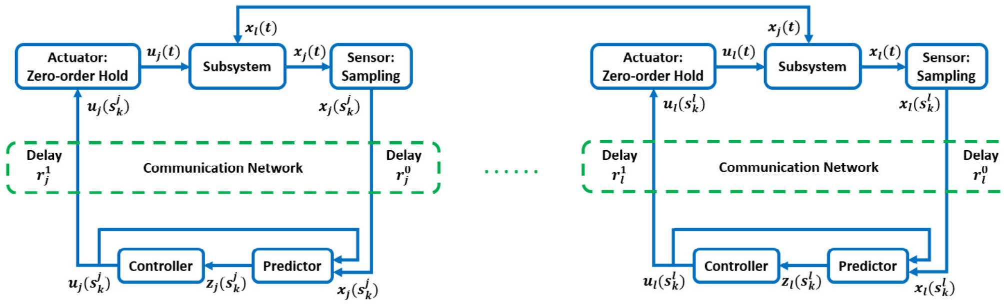

Fig. 3. Large-scale sampled-data NCSs under transmission delays.

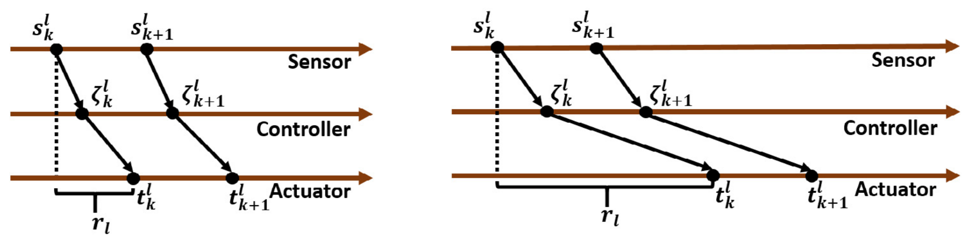

Fig. 4. Two different cases of transmission delay: (1) the delay is less than or equal to a sampling interval, (2) the delay is larger than sampling interval.

The control law is designed as

$$u_j(s_k^j) = K_j z_j(s_k^j) = K_j\Big(e^{A_j r_j} x_j(s_k^j) + \int_{s_k^j - r_j}^{s_k^j} e^{A_j(s_k^j-\theta)} B_j u_j(\theta) d\theta\Big), \quad k \in \mathbb{Z}_0^+ \tag{69}$$

Taking the time-derivative of (68) along (67), the dynamics of $z_j(t)$ for stability analysis is derived as

$$\dot{z}_j(t) = A_j z_j(t) + B_j u_j(t) + e^{A_j r_j} \sum_{l\neq j} F_{lj} x_l(t), \quad t \in [t_k^j, t_{k+1}^j),\ k \in \mathbb{Z}_0^+ \tag{70}$$

According to (66) and (69), we have

$$u_j(t) = u_j(s_k^j) = K_j z_j(s_k^j) = K_j z_j\big(t-(t-s_k^j)\big), \quad t \in [s_k^j, s_{k+1}^j), \quad k \in Z_0^+ \tag{71}$$

The inverse transformation of (68) is calculated as

$$x_l(t) = e^{-A_l r_l} z_l(t) - \int_{t-r_l}^{t} e^{A_l(t-\theta-r_l)} B_l u_l(\theta) d\theta = e^{-A_l r_l} z_l(t) - \xi_l(t) \tag{72}$$

where

$$\xi_l(t) = \int_{t-r_l}^{t} e^{A_l(t-\theta-r_l)} B_l u_l(\theta) d\theta = \int_{-r_l}^{0} e^{A_l s} B_l u_l(t-s-r_l) ds \tag{73}$$

Substituting (71) and (72) into (70), we get the closed-loop time-delay systems as follows:

$$\dot{z}_j(t) = A_j z_j(t) + B_j K_j z_j\big(t-\tau_j(t)\big) + e^{A_j r_j} \sum_{l\neq j} F_{lj}\left(e^{-A_l r_l} z_l(t) - \xi_l(t)\right), \quad t \geq 0. \tag{74}$$

where $\tau_j(t) = t - s_k^j$, $t \in [s_k^j, s_{k+1}^j)$, $k \in Z_0^+$ and $0 \leq \tau_j(t) \leq h_j$.

To compensate the distributed input term $\xi_l(t)$ on the right side of (74) which comes from (72)–(73), we use the following equality.

$$\begin{aligned} |\xi_l(t)|^2 &= \left| \int_{-r_l}^{0} e^{A_l s} B_l u_l(t-s-r_l) ds \right|^2 \\ &\leq r_l \int_{-r_l}^{0} \left| e^{A_l s} B_l u_l(t-s-r_l) \right|^2 ds \\ &\leq r_l \int_{-r_l}^{0} \left| e^{A_l s} B_l \right|^2 |u_l(t-s-r_l)|^2 ds \\ &\leq r_l \int_{-r_l}^{0} \left| e^{A_l s} B_l \right|^2 ds \sup_{\theta\in[-h_l-r_l,0]} |K_l z_l(t+\theta)|^2 \\ &= \delta_l \sup_{\theta\in[-h_l-r_l,0]} |z_l(t+\theta)|^2 \end{aligned} \tag{75}$$

where $\delta_l = r_l \int_{-r_l}^{0} \left|e^{A_l s} B_l\right|^2 ds\, |K_l|^2$. Thus we have

$$\delta_l \sup_{\theta\in[-h_l-r_l,0]} |z_l(t+\theta)|^2 - |\xi_l(t)|^2 \geq 0 \tag{76}$$

Theorem 3. *Consider the closed-loop system consisting of plant (65) and controller (69). Given positive tuning parameters ε_1, ε_2 and α such that $\varepsilon_1 + \varepsilon_2 < \alpha$, let a parameter $\lambda_l > 0$, $n^j \times n^j$ matrices $P_j, U_j > 0$, $n^j \times n^j$ matrices P_{2j}, P_{3j}, and $n^l \times n^l$ matrices $P_l > 0$ for $l = 1, \ldots, M$ and $l \neq j$, satisfy the LMIs:*

$$\Phi_j\big|_{\tau_j(t)\to 0} < 0, \quad \Phi_j\big|_{\tau_j(t)\to h_j} < 0, \quad P_l - \lambda_l \delta_l I_{n^l} > 0, \tag{77}$$

where $\Phi_j = \{\Phi_{ij}^j\}$ is the symmetric matrix composed of

$$\begin{aligned} &\Phi_{11}^j = (A_j + B_jK_j)^T P_{2j} + P_{2j}^T (A_j + B_jK_j) + 2\alpha_j P_j \\ &\Phi_{12}^j = (A_j + B_jK_j)^T P_{3j} - P_{2j}^T + P_j, \quad \Phi_{13}^j = -\tau_j(t) P_{2j}^T B_j K_j, \\ &\Phi_{14}^j = P_{2j}^T e^{A_j r_j} \mathrm{row}_{l=1,\ldots,M}\{F_{lj} e^{-A_l r_l}, l \neq j\}, \\ &\Phi_{15}^j = P_{2j}^T e^{A_j r_j} \mathrm{row}_{l=1,\ldots,M}\{-F_{lj}, l \neq j\}, \\ &\Phi_{22}^j = -P_{3j} - P_{3j}^T + (h_j - \tau_j(t)) U_j, \quad \Phi_{23}^j = -\tau_j(t) P_{3j}^T B_j K_j, \\ &\Phi_{24}^j = P_{3j}^T e^{A_j r_j} \mathrm{row}_{l=1,\ldots,M}\{F_{lj} e^{-A_l r_l}, l \neq j\}, \\ &\Phi_{25}^j = P_{3j}^T e^{A_j r_j} \mathrm{row}_{l=1,\ldots,M}\{-F_{lj}, l \neq j\}, \\ &\Phi_{33}^j = -e^{-2\alpha h_j} \tau_j(t) U_j, \\ &\Phi_{44}^j = -\tfrac{2\varepsilon_1}{M-1} \mathrm{diag}_{l=1,\ldots,M}\{P_l, l \neq j\} \\ &\Phi_{55}^j = -\tfrac{2\varepsilon_2}{M-1} \mathrm{diag}_{l=1,\ldots,M}\{\lambda_l I_l, l \neq j\} \end{aligned}$$

with $I_{n^l} \in \mathbb{R}^{n^l \times n^l}$ being an identity matrix.

Table 1
The results with different feedback schemes.

	Under the decay rate $\rho = \alpha - \varepsilon = 0.001$	
	Input delay r	Sampled interval h
Continuous-time Predictor-free ($u_j(t) = K_j x_j(t)$, Section 3.6.2 of Fridman (2014))	0.061	–
Sampled-data Predictor-free ($u_j(s_k^j) = K_j x_j(s_k^j)$, Section 7.4.2 of Fridman (2014))	0.06	0.001
Continuous-time Predictor-based (Backstepping-based PDE method, Theorem 1)	0.37	–
Continuous-time Predictor-based (Reduction-based ODE method, Corollary 1)	0.2	–
Sampled-data Predictor-based with Continuous-time Measurement (Theorem 2)	0.15	0.001

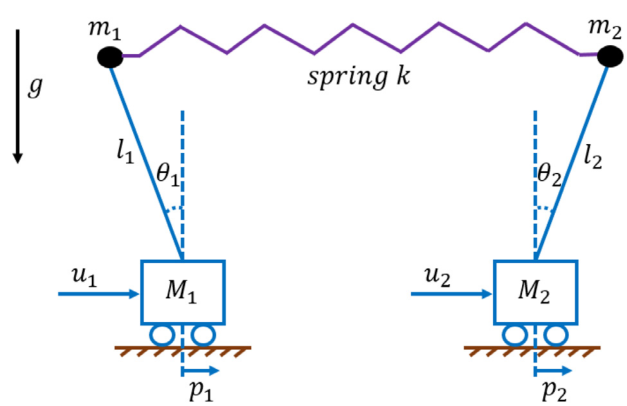

Fig. 5. Two coupled cart–pendulum systems.

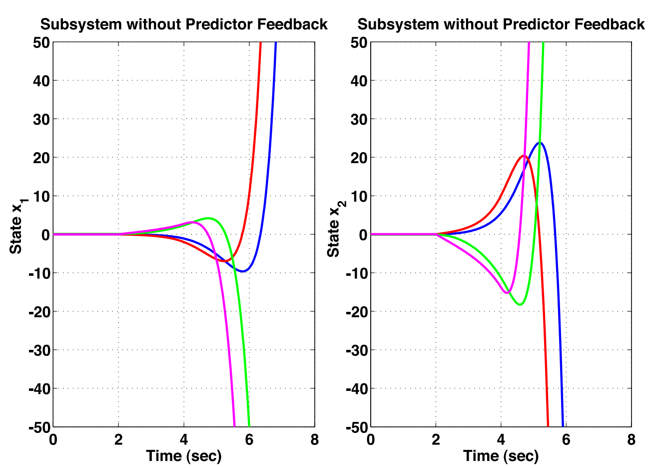

Fig. 6. Decentralized feedback under small delay $r_1 = r_2 = 0.2$ s without predictor.

Then the closed-loop system is exponentially stable with a decay rate ρ which is a unique positive solution of the equation $\rho = \alpha - \varepsilon_1 - \varepsilon_2 e^{2\rho(h+r)}$. ■

Proof. For stability analysis, the Lyapunov–Krasovskii functional is selected as $V_j(t) = V_{P_j}(t) + V_{U_j}(t)$, where

$$\begin{aligned} V_{P_j}(t) &= z_j^T(t) P_j z_j(t), \quad P_j > 0 \\ V_{U_j}(t) &= (h_j - \tau_j(t)) \int_{t-\tau_j(t)}^{t} e^{2\alpha(s-t)} \dot{z}_j^T(s) U_j \dot{z}_j(s) ds, \quad U_j > 0 \end{aligned} \tag{78}$$

The term $V_{U_j}(t)$ is used to compensate $\tau_j(t)$ in (74).

Taking the time-derivative of the first equation of (78), we have

$$\dot{V}_{P_j}(t) + 2\alpha V_{P_j}(t) = 2 z_j^T(t) P_j \dot{z}_j(t) + 2\alpha z_j^T(t) P_j z_j(t) \tag{79}$$

Table 2
Comparison between centralized and decentralized LMIs.

Theorem	No. of decision variables	No. of lines	No. of tuning parameters
Theorem 1	24	13×13, 2×2	2
Proposition 1	40	18×18, 4×4	2

By Jensen's inequality, the time-derivative of the second equation of (78) satisfies

$$\begin{aligned} \dot{V}_{U_j}(t) + 2\alpha V_{U_j}(t) &= -\int_{t-\tau_j(t)}^{t} e^{2\alpha(s-t)} \dot{z}_j^T(s) U_j \dot{z}_j(s) ds \\ &\quad + (h_j - \tau_j(t)) \dot{z}_j^T(t) U_j \dot{z}_j(t) \\ &\leq -e^{-2\alpha h_j} \tau_j(t) v_j^T(t) U_j v_j(t) \\ &\quad + (h_j - \tau_j(t)) \dot{z}_j^T(t) U_j \dot{z}_j(t) \end{aligned} \tag{80}$$

where $v_j(t) = \frac{1}{\tau_j(t)} \int_{t-\tau_j(t)}^{t} \dot{z}_j(s) ds$ with $\lim_{\tau_j(t)\to 0} v_j(t) = \dot{z}_j(t)$. By the descriptor representation of (74), we have

$$\begin{aligned} 0 &= 2\left[z_j^T(t) P_{2j}^T + \dot{z}_j^T(t) P_{3j}^T \right] \Big[(A_j + B_j K_j) z_j(t) - \tau_j(t) B_j K_j v_j(t) \\ &\quad + e^{A_j r_j} \sum_{l \neq j} F_{lj} \left(e^{-A_l r_l} z_l(t) - \xi_l(t) \right) - \dot{z}_j(t) \Big] \end{aligned} \tag{81}$$

Above all, synthesizing (79), (80), (81), (76), we get

$$\begin{aligned} &\dot{V}_j(t) + 2\alpha V_j(t) - \tfrac{2\varepsilon_1}{M-1} \sum_{l \neq j} V_l(t) \\ &\quad - \tfrac{2\varepsilon_2}{M-1} \sum_{l \neq j} \sup_{\theta \in [-h_l - r_l, 0]} V_l(t+\theta) \\ &\quad \tfrac{2\varepsilon_2}{M-1} \sum_{l \neq j} \lambda_l \left(\delta_l \sup_{\theta \in [-h_l - r_l, 0]} |z_l(t+\theta)|^2 - |\xi_l(t)|^2 \right) \\ &\leq \eta_j^T(t) \Phi_j \eta_j(t) \\ &\quad - \tfrac{2\varepsilon_2}{M-1} \sum_{l \neq j} \sup_{\theta \in [-h_l - r_l, 0]} z_l^T(t+\theta) \big(P_l - \lambda_l \delta_l I_{n^l} \big) \\ &\quad \times \sup_{\theta \in [-h_l - r_l, 0]} z_l(t+\theta) \end{aligned} \tag{82}$$

where $\eta_j(t) = \mathrm{col}\{z_j(t), \dot{z}_j(t), v_j(t), \mathrm{col}_{l=1,\ldots,M}\{z_l(t), l \neq j\}, \mathrm{col}_{l=1,\ldots,M}\{\xi_l(t), l \neq j\}\}$. ■

5. Example

In this section, we use an example of two coupled inverted pendulums on two carts (shown in Fig. 5) from Freirich and Fridman (2016), Heemels et al. (2013) and Dolk et al. (2017) under the decentralized control scheme.

The system matrices are $A_1 = A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2.9156 & 0 & -0.0005 & 0 \\ 0 & 0 & 0 & 1 \\ -1.6663 & 0 & 0.0002 & 0 \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 0 \\ -0.0042 \\ 0 \\ 0.0167 \end{bmatrix}$, $F_{21} = F_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0011 & 0 & 0.0005 & 0 \\ 0 & 0 & 0 & 0 \\ -0.0003 & 0 & -0.0002 & 0 \end{bmatrix}$. The control gains are selected as $K_1 = [\,11396 \;\; 7196.2 \;\; 573.96 \;\; 1199.0\,]$, $K_2 = [\,29241 \;\; 18135 \;\; 2875.3 \;\; 3693.9\,]$.

The simulation results under different feedback schemes are shown in Tables 1–2 and Figs. 6–7. Given the decay rates and

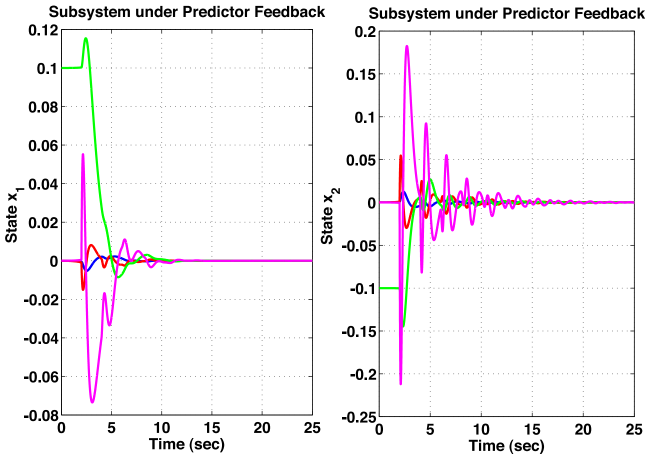

Fig. 7. Decentralized feedback under large delay $r_1 = r_2 = 2$ s with predictor.

sampled intervals, the predictor-based controller in this example always leads to a larger input delay than the predictor-free controller.

As revealed in Table 2, the LMIs of Theorem 1 (decentralized LKFs) have less decision variables and are of reduced-order comparatively to LMIs of Proposition 1 (a centralized LKF). The advantage should be more apparent when the number of subsystems M is large.

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