Automatica 48 (2012) 826–836

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Robust sampled-data control of a class of semilinear parabolic systems ☆

Emilia Fridman, Anatoly Blighovsky

School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel

ARTICLE INFO

Article history:
Received 15 October 2010
Received in revised form
17 April 2011
Accepted 20 September 2011
Available online 13 March 2012

Keywords:
Distributed parameter systems
Lyapunov method
Time-delays
Sampled-data control
LMIs

ABSTRACT

We develop sampled-data controllers for parabolic systems governed by uncertain semilinear diffusion equations with distributed control on a finite interval. Such systems are stabilizable by linear infinite-dimensional state-feedback controllers. For a realistic design, finite-dimensional realizations can be applied leading to local stability results. Here we suggest a sampled-data controller design, where the sampled-data (in time) measurements of the state are taken in a finite number of fixed sampling points in the spatial domain. It is assumed that the sampling intervals in time and in space are bounded. Our sampled-data static output feedback enters the equation through a finite number of shape functions (which are localized in the space) multiplied by the corresponding state measurements. It is piecewise-constant in time and it may possess an additional time-delay. The suggested controller can be implemented by a finite number of stationary sensors (providing discrete state measurements) and actuators and by zero-order hold devices. A direct Lyapunov method for the stability analysis of the resulting closed-loop system is developed, which is based on the application of Wirtinger's and Halanay's inequalities. Sufficient conditions for the exponential stabilization are derived in terms of Linear Matrix Inequalities (LMIs). By solving these LMIs, upper bounds on the sampling intervals that preserve the exponential stability and on the resulting decay rate can be found. The dual problem of observer design under sampled-data measurements is formulated, where the same LMIs can be used to verify the exponential stability of the error dynamics.

1. Introduction

We develop sampled-data controllers for parabolic systems governed by semilinear diffusion equations with distributed control. Such systems are stabilizable by linear infinite-dimensional state-feedback controllers. For a realistic design, finite-dimensional realizations (Balas, 1985; Candogan, Ozbay, & Ozaktas, 2008; Smagina & Sheintuch, 2006) can be applied. However, finite-dimensional control, which employs e.g. Galerkin truncation, leads to local stability results (Smagina & Sheintuch, 2006). In Hagen and Mezic (2003) the control input has been designed to enter the semilinear diffusion equation through a finite number of shape functions (e.g. step functions) and their respective amplitude values. Sufficient conditions have been derived for the global stabilization of the infinite-dimensional dynamics. For linear parabolic systems mobile collocated sensors and actuators (see Demetriou (2010) and references therein) or adaptive controllers (Krstic & Smyshlyaev, 2008; Smyshlyaev & Krstic, 2005) can be used. The latter methods are not easy to implement.

Sampled-data control of finite-dimensional systems have been studied extensively over the past decades (see e.g. Chen and Francis (1995), Naghshtabrizi, Hespanha, and Teel (2008), Fujioka (2009), Fridman (2010) and the references therein). Three main approaches have been used to control of sampled-data systems: the discrete-time, the time-delay and the impulsive system approaches. Unlike the other approaches, the discrete-time one does not take into account the inter-sampling behavior and seems not to be applicable to time-varying or nonlinear systems.

There are only a few references on sampled-data control of distributed parameter systems (Cheng, Radisavljevic, Chang, Lin, & Su, 2009; Logemann, Rebarber, & Townley, 2003, 2005). All these works use the discrete-time approach for linear time-invariant systems. Observability of parabolic systems under sampled-data measurements has been studied in Khapalov (1993). Recently a model-reduction-based approach to sampled-data control was introduced in Ghantasala and El-Farra (2010), Sun, Ghantasala, and El-Farra (2009), where a finite-dimensional controller was designed on the basis of a finite-dimensional system that captures the dominant (slow) dynamics of the infinite-dimensional system. The latter approach seems to be not applicable to systems with spatially-dependent diffusion coefficients and with uncertain

☆ This work was partially supported by Israel Science Foundation (grant No 754/10) and by Kamea Fund of Israel. The material in this paper was partially presented at the 18th IFAC World Congress, August 28–September 2, 2011, Milano, Italy. This paper was recommended for publication in revised form by Associate Editor Nicolas Petit under the direction of Editor Miroslav Krstic.

E-mail addresses: emilia@eng.tau.ac.il (E. Fridman), tolik_bl@yahoo.com (A. Blighovsky).

0005-1098/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2012.02.006

nonlinear terms. The existing sampled-data results are not applicable to the performance analysis of the closed-loop system, e.g. to the decay rate of the exponential convergence.

We suggest a sampled-data controller design for a one-dimensional semilinear diffusion equation, where the sampled-data in time measurements of the state are taken in a finite number of fixed sampling spatial points. It is assumed that the sampling intervals in time and in space may be variable, but bounded. The sampling instants (in time) may be uncertain. The diffusion coefficient and the nonlinearity may be unknown, but they satisfy some bounds. The sampled-data static output feedback controller is piecewise-constant in time. It can be implemented by a finite number of stationary sensors and actuators and by zero-order hold devices. Sufficient conditions for exponential stabilization are derived in terms of LMIs in the framework of time-delay approach to sampled-data systems. By solving these LMIs, *upper bounds on the sampling intervals that preserve the stability and on the resulting decay rate* can be found. Finally, the dual problem of observer design under sampled-data measurements is discussed.

We note that the LMI approach has been introduced in Fridman and Orlov (2009a), Fridman and Orlov (2009b) for some classes of distributed parameter systems, leading to simple finite-dimensional sufficient conditions for stability. The method in the present paper is based on the novel combination of Lyapunov–Krasovskii functionals with Wirtinger's and Halanay's inequalities. A numerical example illustrates the efficiency of the method. Some preliminary results will be presented in Fridman and Blighovsky (2011).

Notation. Throughout the paper $\mathbf{R}^n$ denotes the n dimensional Euclidean space with the norm $|\cdot|$, $\mathbf{R}^{n\times m}$ is the set of all $n\times m$ real matrices, and the notation $P>0$ with $P\in\mathbf{R}^{n\times n}$ means that P is symmetric and positive definite. The symmetric elements of the symmetric matrix will be denoted by $*$. Functions, continuous (continuously differentiable) in all arguments, are referred to as of class C (of class C^1). $L_2(0,l)$ is the Hilbert space of square integrable functions $z(\xi)$, $\xi\in[0,l]$ with the corresponding norm $\|z\|_{L_2}=\sqrt{\int_0^l z^2(\xi)d\xi}$. $\mathcal{H}^1(0,l)$ is the Sobolev space of absolutely continuous scalar functions $z:[0,l]\to R$ with $\frac{dz}{d\xi}\in L_2(0,l)$. $\mathcal{H}^2(0,l)$ is the Sobolev space of scalar functions $z:[0,l]\to R$ with absolutely continuous $\frac{dz}{d\xi}$ and with $\frac{d^2z}{d\xi^2}\in L_2(0,l)$.

2. Problem formulation and useful inequalities

Consider the following semilinear scalar diffusion equation

$$z_t(x,t)=\frac{\partial}{\partial x}[a(x)z_x(x,t)]+\phi(z(x,t),x,t)z(x,t) + u(x,t),\quad t\ge t_0,\ x\in[0,l],\ l>0, \tag{1}$$

with Dirichlet boundary conditions

$$z(0,t)=z(l,t)=0, \tag{2}$$

or with mixed boundary conditions

$$z_x(0,t)=\gamma z(0,t),\qquad z(l,t)=0,\quad \gamma\ge 0, \tag{3}$$

where subindexes denote the corresponding partial derivatives and γ may be unknown. In (1) $u(x,t)$ is the control input. The functions a and ϕ are of class C^1 and may be unknown. These functions satisfy the inequalities $a\ge a_0>0$, $\phi_m\le\phi\le\phi_M$, where a_0, ϕ_m and ϕ_M are known bounds.

It is well-known that the open-loop system (1) under the above boundary conditions may become unstable if ϕ_M is big enough (see Curtain and Zwart (1995) for $\phi\equiv\phi_M$). Moreover, a linear infinite-dimensional state feedback $u(x,t)=-Kz(x,t)$ with big enough $K>0$ exponentially stabilizes the system (see Proposition 1). In the present paper we develop a sampled-data controller design.

Consider (1) under the boundary conditions (2) or (3). Let the points $0=x_0<x_1<\cdots<x_N=l$ divide $[0,l]$ into N sampling intervals. We assume that N sensors are placed in the middle $\bar{x}_j=\frac{x_{j+1}+x_j}{2}$ $(j=0,\dots,N-1)$ of these intervals. Let $t_0<t_1<\cdots<t_k\dots$ with $\lim_{k\to\infty}t_k=\infty$ be sampling time instants. The sampling intervals in time and in space may be variable but bounded

$$0\le t_{k+1}-t_k\le h,\qquad x_{j+1}-x_j\le\Delta. \tag{4}$$

Sensors provide discrete measurements of the state:

$$y_{jk}=z(\bar{x}_j,t_k),\quad \bar{x}_j=\frac{x_{j+1}+x_j}{2},\quad j=0,\dots,N-1,\ t\in[t_k,t_{k+1}),\ k=0,1,2\dots \tag{5}$$

Our objective is to design for (1) an exponentially stabilizing (sampled-data in space and in time) controller

$$u(x,t)=-Kz(\bar{x}_j,t_k),\quad \bar{x}_j=\frac{x_{j+1}+x_j}{2},\quad x\in[x_j,x_{j+1}),\ j=0,\dots,N-1,\quad t\in[t_k,t_{k+1}),\ k=0,1,2\dots \tag{6}$$

with the gain $K>0$. The closed-loop system (1), (6) has the form:

$$\begin{aligned} z_t(x,t)=&\frac{\partial}{\partial x}[a(x)z_x(x,t)]+\phi(z(x,t),x,t)z(x,t)\\ &-Kz(\bar{x}_j,t_k),\quad t\in[t_k,t_{k+1}),\ k=0,1,2\dots\\ &x_j\le x<x_{j+1},\ j=0,\dots,N-1.\end{aligned} \tag{7}$$

By using the relation $z(\bar{x}_j,t_k)=z(x,t_k)-\int_{\bar{x}_j}^x z_\zeta(\zeta,t_k)d\zeta$, (7) can be represented as

$$\begin{aligned} z_t(x,t)=&\frac{\partial}{\partial x}[a(x)z_x(x,t)]+\phi(z(x,t),x,t)z(x,t)\\ &-K[z(x,t_k)-\int_{\bar{x}_j}^x z_\zeta(\zeta,t_k)d\zeta],\\ &x_j\le x<x_{j+1},\ j=0,\dots,N-1,\\ &t\in[t_k,t_{k+1}),\ k=0,1,2\dots\end{aligned} \tag{8}$$

We will start with the sampled-data in space and continuous in time controller

$$u(x,t)=-Kz(\bar{x}_j,t),\quad x_j\le x<x_{j+1},\ j=0,\dots,N-1. \tag{9}$$

Also a more general controller of the form

$$u(x,t)=-Kz(\bar{x}_j,t_k-\eta_k),\quad t\in[t_k,t_{k+1}),\ k=0,1,2\dots,\quad x_j\le x<x_{j+1},\ j=0,\dots,N-1,\ u(x,t)=0,\ t<t_0, \tag{10}$$

where $\eta_k\in[0,\eta_M]$ is an additional (control or measurement) delay, will be studied. Such a controller models e.g. network-based stabilization, where variable and uncertain sampling instants t_k may appear due to data packet dropouts, whereas η_k is network-induced delay (Gao, Chen, & Lam, 2008; Zhang, Branicky, & Phillips, 2001). Representing $t_k-\eta_k=t-\tau(t)$, where $\tau(t)=t-t_k+\eta_k$, we have $\tau(t)\in[0,\tau_M]$ with $\tau_M=h+\eta_M$. Finally, the dual problem of the observer design for semilinear diffusion equations under the sampled-data measurements is considered.

Remark 1. Our results will be applicable to convection–diffusion equation

$$z_t(x,t)=a_0z_{xx}(x,t)-\beta z_x(x,t)+\phi(z(x,t),x,t)z(x,t) + u(x,t),\quad t\ge t_0,\ x\in[0,l],\ l>0, \tag{11}$$

with constant and known $\beta \in R$, $a_0 > 0$ and unknown $\phi_m \le \phi \le \phi_M$ of class C^1 under the Dirichlet boundary conditions (2) or under the mixed boundary conditions

$$z_x(0,t) = \gamma_0 z(0,t), \qquad z(l,t) = 0, \quad \gamma_0 \ge \frac{\beta}{2a_0}, \tag{12}$$

where the measurements are given by (5). System (11) models many physical phenomena. Examples are numerous and among others include the problem of compressor rotating stall with air injection actuator (Hagen & Mezic, 2003), where $z(x,t)$ denotes the axial flow through the compressor.

Similar to Smyshlyaev and Krstic (2005), we change variables $\bar{z}(x,t) = e^{-\frac{\beta}{2a_0}x} z(x,t)$ in (11) and in the boundary conditions. This leads to

$$\begin{aligned}\bar{z}_t(x,t) &= a_0 \bar{z}_{xx}(x,t) + \phi_1(\bar{z}(x,t),x,t)\bar{z}(x,t) \\ &\quad + e^{-\frac{\beta}{2a_0}x} u(x,t), \quad t \ge t_0,\ x \in [0,l],\end{aligned} \tag{13}$$

where $\phi_1(\bar{z},x,t) = \phi(e^{\frac{\beta}{2a_0}x}\bar{z},x,t) - \frac{\beta^2}{4a_0}$ under the Dirichlet or under the mixed

$$\bar{z}_x(0,t) = \gamma \bar{z}(0,t), \qquad \bar{z}(l,t) = 0, \quad \gamma = \gamma_0 - \frac{\beta}{2a_0} \ge 0 \tag{14}$$

boundary conditions. In this case the control law (6) should be modified as follows:

$$\begin{aligned} &u(x,t) = -Ke^{-\frac{\beta}{2a_0}(\bar{x}_j - x)} z(\bar{x}_j,t_k) = -Ke^{\frac{\beta}{2a_0}x}\bar{z}(\bar{x}_j,t_k), \\ &x_j \le x < x_{j+1}, \qquad \bar{x}_j = \frac{x_{j+1}+x_j}{2}, \quad t \in [t_k,t_{k+1}). \end{aligned} \tag{15}$$

The closed-loop system (13), (15) has the form of (7), where z and ϕ should be replaced by $\bar{z}$ and ϕ_1 respectively and where $\phi_m - \frac{\beta^2}{4a_0} \le \phi_1 \le \phi_M - \frac{\beta^2}{4a_0}$. Thus, the stability conditions for (7) can be applied to the closed-loop system (13), (15). Similarly, the stability of (13) under the continuous in time $u(x,t) = -Ke^{\frac{\beta}{2a_0}x}\bar{z}(\bar{x}_j,t)$ (under the delayed $u(x,t) = -Ke^{\frac{\beta}{2a_0}x}\bar{z}(\bar{x}_j,t_k-\eta_k)$) controller is reduced to the stability of (1), (9) (of (1), (10)).

The following inequalities will be useful:

Lemma 1 (*Halanay, 1966 Halanay's Inequality*). *Let $0 < \delta_1 < 2\delta$ and let $V : [t_0 - h, \infty) \to [0,\infty)$ be an absolutely continuous function that satisfies*

$$\dot{V}(t) \le -2\delta V(t) + \delta_1 \sup_{-h\le\theta\le 0} V(t+\theta), \quad t \ge t_0. \tag{16}$$

Then

$$V(t) \le e^{-2\alpha(t-t_0)} \sup_{-h\le\theta\le 0} V(t_0+\theta), \quad t \ge t_0, \tag{17}$$

where $\alpha > 0$ is a unique positive solution of

$$\alpha = \delta - \frac{\delta_1 e^{2\alpha h}}{2}. \tag{18}$$

Lemma 2 (*Hardy, Littlewood, & Polya, 1988 Wirtinger's Inequality*). *Let $z \in \mathcal{H}^1(0,l)$ be a scalar function with $z(0) = 0$ or $z(l) = 0$. Then*

$$\int_0^l z^2(\xi)d\xi \le \frac{4l^2}{\pi^2}\int_0^l \left[\frac{dz}{d\xi}\right]^2 d\xi. \tag{19}$$

Moreover, if $z(0) = z(l) = 0$, then

$$\int_0^l z^2(\xi)d\xi \le \frac{l^2}{\pi^2}\int_0^l \left[\frac{dz}{d\xi}\right]^2 d\xi. \tag{20}$$

3. Well-posedness of the closed-loop system

We will establish the well-posedness of the closed-loop system under the Dirichlet boundary conditions (2). The well-posedness under the mixed conditions (3) can be proved similarly.

3.1. The continuous in time controller

We start with the well-posedness of the closed-loop system (1) under the continuous in time controller (9)

$$\begin{aligned} z_t(x,t) &= \frac{\partial}{\partial x}[a(x)z_x(x,t)] + \phi(z(x,t),x,t)z(x,t) \\ &\quad - Kz(x,t) + K\int_{\bar{x}_j}^x z_\zeta(\zeta,t)d\zeta, \\ &\quad x_j \le x < x_{j+1},\ \bar{x}_j = \frac{x_{j+1}+x_j}{2}, \\ &\quad j = 0,\dots,N-1,\ t \ge t_0,\ z(x,t_0) = z^{(0)}(x) \end{aligned} \tag{21}$$

and under the Dirichlet boundary conditions (2). Introduce the Hilbert space $H = L_2(0,l)$ with the norm $\|\cdot\|_{L_2}$ and with the scalar product $\langle\cdot,\cdot\rangle$. The boundary-value problem (21) can be rewritten as a differential equation

$$\dot{w}(t) = Aw(t) + F(t,w(t)), \quad t \ge t_0 \tag{22}$$

in H where the operator $A = \frac{\partial[a(x)\frac{\partial}{\partial x}]}{\partial x}$ has the dense domain

$$\mathcal{D}(A) = \{w \in \mathcal{H}^2(0,l) : w(0) = w(l) = 0\},$$

and the nonlinear term $F : R \times \mathcal{H}^1(0,l) \to L_2(0,l)$ is defined on functions $w(\cdot,t)$ according to

$$\begin{aligned} F(t,w(\cdot,t)) &= \phi(w(x,t),x,t)w(x,t) - Kw(x,t) \\ &\quad + K\int_{\bar{x}_j}^x w_\zeta(\zeta,t)d\zeta. \end{aligned}$$

It is well-known that A generates a strongly continuous exponentially stable semigroup T, which satisfies the inequality $\|T(t)\| \le \kappa e^{-\delta t}$, $(t \ge 0)$ with some constant $\kappa \ge 1$ and decay rate $\delta > 0$ (see, e.g., Curtain and Zwart (1995) for details). The domain $H_1 = \mathcal{D}(A) = A^{-1}H$ forms another Hilbert space with the graph inner product $\langle x,y\rangle_1 = \langle Ax,Ay\rangle$, $x,y \in H_1$. The domain $\mathcal{D}(A)$ is dense in H and the inequality $\|Aw\|_{L_2} \ge \mu\|w\|_{L_2}$ holds for all $w \in \mathcal{D}(A)$ and some constant $\mu > 0$. Operator $-A$ is positive, so that its square root $(-A)^{\frac{1}{2}}$ with

$$H_{\frac{1}{2}} = \mathcal{D}((-A)^{\frac{1}{2}}) = \{w \in \mathcal{H}^1(0,l) : w(0) = w(l) = 0\}$$

is well defined. Moreover, $H_{\frac{1}{2}}$ is a Hilbert space with the scalar product

$$\langle u,v\rangle_{\frac{1}{2}} = \langle (-A)^{\frac{1}{2}}u, (-A)^{\frac{1}{2}}v\rangle.$$

Denote by $H_{-\frac{1}{2}}$ the dual of $H_{\frac{1}{2}}$ with respect to the pivot space H. Then A has an extension to a bounded operator $A : H_{\frac{1}{2}} \to H_{-\frac{1}{2}}$. We have $H_1 \subset H_{\frac{1}{2}} \subset H$ with continuous embedding and the following inequality

$$\|(-A)^{\frac{1}{2}}w\|_{L_2} \ge \mu\|w\|_{L_2} \quad \text{for all } w \in H_{\frac{1}{2}} \tag{23}$$

holds. All relevant material on fractional operator degrees can be found, e.g., in Tucsnak and Weiss (2009).

A function $w : [t_0, T) \to H_{\frac{1}{2}}$ is called a strong solution of (22) if

$$w(t) - w(t_0) = \int_{t_0}^{t} [Aw(s) + F(s, w(s))]ds \tag{24}$$

holds for all $t \in [t_0, T)$. Here, the integral is computed in $H_{-\frac{1}{2}}$. Differentiating (24) we obtain (22).

Since the function ϕ of class C^1, the following Lipschitz condition

$$\|F(t_1, w_1) - F(t_2, w_2)\|_{L_2} \leq C[|t_1 - t_2| + \|(-A)^{\frac{1}{2}}(w_1 - w_2)\|_{L_2}] \tag{25}$$

with some constant $C > 0$ holds locally in $(t_i, w_i) \in R \times H_{\frac{1}{2}}$, $i = 1, 2$. Thus, Theorem 3.3.3 of Henry (1993) is applicable to (22), and by applying this theorem, a unique strong solution $w(t) \in H_{\frac{1}{2}}$ of (22), initialized with $z^{(0)} \in H_{\frac{1}{2}}$, exists locally. Since ϕ is bounded, there exists $C_1 > 0$ such that

$$\|F(t, w)\|_{L_2} \leq C_1\|(-A)^{\frac{1}{2}}w\|_{L_2}, \quad \forall w \in H_{\frac{1}{2}}.$$

Hence, the strong solution initialized with $z^{(0)} \in H_{\frac{1}{2}}$ exists for all $t \geq t_0$ (Henry, 1993).

3.2. The sampled in time and in space controller

Consider the boundary-value problem

$$\begin{aligned} z_t(x, t) = {} & \frac{\partial}{\partial x}[a(x)z_x(x, t)] + \phi(z(x, t), x, t)z(x, t) \\ & - Kz(\bar{x}_j, t_k - \eta_k), \quad x_j \leq x < x_{j+1}, \\ & j = 0, \ldots, N-1, \; t \in [t_k, t_{k+1}], \\ & (x, t_0) = z^{(0)}(x), \; z(x, t) = 0, \quad t < t_0 \end{aligned} \tag{26}$$

under the Dirichlet boundary conditions (2). We will use the step method for solution of time-delay systems (Kolmanovskii & Myshkis, 1999). For $t \in [t_0, t_1)$ the system has a form

$$\begin{aligned} z_t(x, t) = {} & \frac{\partial}{\partial x}[a(x)z_x(x, t)] + \phi(z(x, t), x, t)z(x, t) \\ & - Kz(\bar{x}_j, t_0 - \eta_0), \quad z(0, t) = z(l, t) = 0, \\ & z(x, t) = 0, \; t < t_0. \end{aligned} \tag{27}$$

By the above arguments, (27) has a unique strong solution $z(\cdot, t) \in H_{\frac{1}{2}}$, $t \in [t_0, t_1)$ for an arbitrary initial function $z^{(0)} \in H_{\frac{1}{2}}$. By considering next $t \in [t_k, t_{k+1})$, $k = 1, 2, \ldots$ we conclude that (26) has a unique strong solution for all $t \geq t_0$.

4. LMIs for the exponential stabilization

4.1. Stabilization via sampled in space controller

We will start with the stabilization via sampled-data in spatial variable controller which is continuous in time. In this case we assume that ϕ is upper bounded with $\phi \leq \phi_M < \infty$ (thus $\phi_m = -\infty$). Consider the closed-loop system (21) under the mixed boundary conditions (3). By using the Lyapunov function

$$V(t) = \int_0^l z^2(x, t)dx, \tag{28}$$

we will derive conditions that guarantee $\dot{V}(t) + 2\delta V(t) \leq 0$ along (21), (3). The latter inequality yields $V(t) \leq e^{-2\delta(t-t_0)}V(t_0)$ or

$$\int_0^l z^2(x, t)dx \leq e^{-2\delta(t-t_0)} \int_0^l z^2(x, t_0)dx \tag{29}$$

for the strong solutions of (21), (3) initialized with

$$z(\cdot, t_0) \in \mathcal{H}^1(0, l) : z_x(0, t_0) = \gamma z(0, t_0), \quad z(l, t_0) = 0. \tag{30}$$

If (29) holds, we will say that (21) under (3) is exponentially stable with the decay rate δ.

Differentiating V along (21) we find

$$\begin{aligned} \dot{V}(t) = {} & 2\int_0^l z(x, t)z_t(x, t)dx = 2\int_0^l z(x, t) \\ & \times \left[\frac{\partial}{\partial x}[a(x)z_x(x, t)] + \phi(z(x, t), x, t)z(x, t) \right. \\ & \left. - Kz(x, t)\right]dx + 2\sum_{j=0}^{N-1}\int_{x_j}^{x_{j+1}} Kz(x, t)[z(x, t) - z(\bar{x}_j, t)]dx. \end{aligned}$$

Integration by parts and substitution of the boundary conditions (3) lead to

$$\begin{aligned} & 2\int_0^l z(x, t)\frac{\partial}{\partial x}[a(x)z_x(x, t)]dx = 2a(x)z(x, t)z_x(x, t)\Big|_0^l \\ & -2\int_0^l a(x)z_x^2(x, t)dx \leq -2a_0\int_0^l z_x^2(x, t)dx. \end{aligned} \tag{31}$$

Therefore,

$$\begin{aligned} \dot{V}(t) \leq {} & -2a_0\int_0^l z_x^2(x, t)dx + 2\int_0^l (\phi_M - K)z^2(x, t)dx \\ & + 2\sum_{j=0}^{N-1}\int_{x_j}^{x_{j+1}} Kz(x, t)[z(x, t) - z(\bar{x}_j, t)]dx. \end{aligned} \tag{32}$$

By Young's inequality, for any scalar $\bar{R} > 0$ the following holds:

$$\begin{aligned} & -2K\sum_{j=0}^{N-1}\int_{x_j}^{x_{j+1}} [z(x, t)[z(x, t) - z(\bar{x}_j, t)]]dx \\ & \leq K\left[\bar{R}\int_0^l z^2(x, t)dx + \bar{R}^{-1}\sum_{j=0}^{N-1}\int_{\bar{x}_j}^{x_{j+1}} [z(x, t) \right. \\ & \left. - z(\bar{x}_j, t)]^2dx\right]. \end{aligned} \tag{33}$$

Then, application of Wirtinger's inequality (19) yields

$$\begin{aligned} & \int_{x_j}^{x_{j+1}} [z(x, t) - z(\bar{x}_j, t)]^2dx \\ & = \int_{x_j}^{\bar{x}_j} [z(x, t) - z(\bar{x}_j, t)]^2dx \\ & \quad + \int_{\bar{x}_j}^{x_{j+1}} [z(x, t) - z(\bar{x}_j, t)]^2dx \\ & \leq \frac{\Delta^2}{\pi^2}\int_{x_j}^{x_{j+1}} z_x^2(x, t)dx. \end{aligned} \tag{34}$$

Choosing next $\bar{R} = \frac{\Delta}{\pi}R$, we find from (32)–(34) that

$$\begin{aligned} \dot{V}(t) + 2\delta V(t) \leq {} & \left(R^{-1}K\frac{\Delta}{\pi} - 2a_0\right)\int_0^l z_x^2(x, t)dx \\ & + \left(RK\frac{\Delta}{\pi} + 2\delta + 2(\phi_M - K)\right) \\ & \times \int_0^l z^2(x, t)dx. \end{aligned} \tag{35}$$

By Wirtinger's inequality (19), $\dot{V}(t) + 2\delta V(t) \le 0$ if

$$R^{-1}K\frac{\Delta}{\pi} - 2a_0 \le 0,$$
$$RK\frac{\Delta}{\pi} + 2\delta + 2(\phi_M - K) + \frac{\pi^2}{bl^2}\left(R^{-1}K\frac{\Delta}{\pi} - 2a_0\right) \le 0, \tag{36}$$

where $b = 4$. Under the Dirichlet boundary conditions, application of (20) leads to the same conclusion with $b = 1$ in (36).

Note that inequalities (36) are feasible for small enough $\delta > 0$, $\Delta > 0$ iff $K > \phi_M - \frac{a_0\pi^2}{bl^2}$. We have proved

Proposition 1. (i) *Given $b = 4$, $K > \phi_M - \frac{a_0\pi^2}{bl^2}$, $R > 0$, let there exist $\Delta > 0$ and $\delta > 0$ such that the linear scalar inequalities (36) are feasible. Then the closed-loop system (1), (9) under the mixed boundary conditions (3) is exponentially stable with the decay rate δ (in the sense of (29)).*

(ii) *If the conditions of (i) hold with $b = 1$, then the closed-loop system (1), (9) under the Dirichlet boundary conditions (2) is exponentially stable with the decay rate δ.*

(iii) *The state-feedback controller $u = -Kz(x, t)$ exponentially stabilizes (1) with the decay rate $\delta > 0$ if $K \ge \phi_M - \frac{a_0\pi^2}{bl^2} + \delta$, where $b = 1$ corresponds to (2) and $b = 4$ to (3).*

Remark 2. The condition (36) of Proposition 1 cannot be improved for the diffusion equation

$$z_t(x, t) = z_{xx}(x, t), \tag{37}$$

where $x \in [0, \pi]$ under the mixed boundary conditions $z_x(0, t) = z(\pi, t) = 0$. The feasibility of (36) with $K = 0$, $a = 1$ guarantees the exponential decay rate $\delta = 0.25$ of the system. This is the exact decay rate since -0.25 is the rightmost eigenvalue of the operator $A = \frac{\partial^2}{\partial \xi^2}$ with the domain (Tucsnak & Weiss, 2009)

$$\mathcal{D}(A) = \{w \in \mathcal{H}^2(0, l) : w_x(0) = w(\pi) = 0\}.$$

The same conclusion is true for the Dirichlet boundary conditions with $\delta = 1$.

4.2. Stabilization via the time-delayed sampled-data controller

The time-delayed controller (10) will be designed for the diffusion Eq. (1) under the boundary conditions (2) or (3). Therefore, we will analyze the exponential stability of the closed-loop system (26), which can be represented as

$$\begin{aligned} z_t(x, t) &= \frac{\partial}{\partial x}[a(x)z_x(x, t)] + \phi(z(x, t), x, t)z(x, t) \\ &\quad - K\left[z(x, t - \tau(t)) - \int_{\bar{x}_j}^{x} z_\zeta(\zeta, t - \tau(t))d\zeta\right], \\ &\quad x_j \le x < x_{j+1},\ j = 0, \ldots, N-1,\ t \ge 0, \\ &\quad \tau(t) \in [0, \tau_M],\ z(x, t) = 0,\ t < t_0. \end{aligned} \tag{38}$$

In Fridman and Orlov (2009a) for $a \equiv a_0$ a Lyapunov functional of the form

$$\begin{aligned} V(t) &= (p_1 - a_0p_3)\int_0^l z^2(x, t)dx + a_0p_3\int_0^l z_x^2(x, t)dx \\ &\quad + \int_0^l \left[\tau_M r\int_{-\tau_M}^0\int_{t+\theta}^t e^{2\delta(s-t)}z_s^2(x, s)dsd\theta\right. \\ &\quad \left. + g\int_{t-\tau_M}^t e^{2\delta(s-t)}z^2(x, s)ds\right]dx \end{aligned} \tag{39}$$

with $l = \pi$ and some constants $p_3 > 0$, $p_1 > 0$, $r \ge 0$ and $g \ge 0$ was introduced for the exponential stability (with the decay rate $\delta > 0$) of the Dirichlet boundary value problem for the heat equation with time-delay (38), where the last integral term is deleted.

The main difficulty in the Lyapunov-based analysis of (38) is the "compensation" of the term $K\int_{\bar{x}_j}^{x} z_\zeta(\zeta, t - \tau(t))d\zeta$ with $\dot{\tau} = 1$ for $t \ne t_k$. An extension of the existing constructions of Lyapunov–Krasovskii functionals (such as the r-dependent term in (39) that "compensates" the term $Kz(x, t - \tau(t))$) seems not to be applicable. The method that we develop in this paper is based on the combination of the Lyapunov–Krasovskii functional for (38) with Halanay's inequality (17).

Remark 3. Numerical examples show that for $a \equiv a_0$ the term $-a_0p_3\int_0^l z^2(x, t)dx$ of V is useful if $K - \phi_M + a_0$ is comparatively small. In this case the above term allows to enlarge the upper bound on the delay which preserves the stability of the delayed diffusion equation. For greater values of K this term does not change the result.

We modify V as follows:

$$\begin{aligned} V(t) &= p_1\int_0^l z^2(x, t)dx + p_3\int_0^l a(x)z_x^2(x, t)dx \\ &\quad + \int_0^l \left[\tau_M r\int_{-\tau_M}^0\int_{t+\theta}^t e^{2\delta(s-t)}z_s^2(x, s)dsd\theta\right. \\ &\quad \left. + g\int_{t-\tau_M}^t e^{2\delta(s-t)}z^2(x, s)ds\right]dx + qz^2(0, t) \end{aligned} \tag{40}$$

where $p_3 > 0$, $p_1 > 0$, $r \ge 0$, $g \ge 0$. For the Dirichlet boundary conditions we choose $q = 0$, whereas for the mixed conditions (3) we consider $q = a(0)p_3\gamma$. Note that the resulting exponential decay rate for (38) will be less than δ.

Theorem 1. (i) *Consider the Dirichlet boundary value problem (26), (2) and let $b = 1$. Given positive scalars Δ, δ, $K > \phi_M - \frac{a_0\pi^2}{bl^2}$, τ_M, R and δ_1 such that $2\delta > \delta_1$, let there exist positive scalars p_1, p_2, p_3, r and g satisfying the following LMIs*

$$\delta p_3 \le p_2, \qquad \Delta KR^{-1}(p_2 + p_3) \le \pi\delta_1 a_0 p_3 \tag{41}$$

and

$$\bar{\Phi}^{\tau_M}_{|\phi=\phi_m} \le 0, \qquad \bar{\Phi}^{\tau_M}_{|\phi=\phi_M} \le 0, \tag{42}$$

where

$$\bar{\Phi}^{\tau_M} \triangleq \begin{bmatrix} \Phi_{11}^{\tau_M} - \lambda & \Phi_{12}^{\tau_M} & 0 & \Phi_{14}^{\tau_M} \\ * & \Phi_{22}^{\tau_M} & 0 & -Kp_3 \\ * & * & \Phi_{33}^{\tau_M} & re^{-2\delta\tau_M} \\ * & * & * & \Phi_{44}^{\tau_M} \end{bmatrix}, \tag{43}$$

$$\Phi_{11}^{\tau_M} = 2\delta p_1 + g + 2p_2\left(\phi + \frac{\Delta}{2\pi}KR\right) - re^{-2\delta\tau_M},$$

$$\Phi_{12}^{\tau_M} = p_1 - p_2 + p_3\phi, \qquad \Phi_{22}^{\tau_M} = r\tau_M^2 - 2p_3 + \frac{\Delta}{\pi}KRp_3,$$

$$\lambda = \frac{2a_0\pi^2}{bl^2}(p_2 - \delta p_3), \qquad \Phi_{33}^{\tau_M} = -(r + g)e^{-2\delta\tau_M},$$

$$\Phi_{14}^{\tau_M} = re^{-2\delta\tau_M} - Kp_2, \qquad \Phi_{44}^{\tau_M} = -2re^{-2\delta\tau_M} - \delta_1 p_1.$$

Then a unique strong solution to the Dirichlet boundary value problem (26), (2) initialized with

$$z(\cdot, t_0) \in H_{\frac{1}{2}}, \qquad z(x, t) \equiv 0, \quad t < t_0 \tag{44}$$

satisfies the inequality

$$p_1\int_0^l z^2(x, t)dx + p_3\int_0^l a(x)z_x^2(x, t)dx$$

$$\leq e^{-2\alpha(t-t_0)}[p_1 \int_0^l z^2(x,t_0)dx$$
$$+ p_3 \int_0^l a(x) z_x^2(x,t_0)dx + qz^2(0,t_0)], \quad t \geq t_0, \tag{45}$$

where $q = 0$ and where $\alpha > 0$ is a unique positive solution of (18).

(ii) *If the above conditions hold with $b = 4$, then a unique strong solution to the mixed boundary value problem (26), (3) initialized with*

$$z(\cdot, t_0) \in \mathcal{H}^1(0,l) : z_x(0,t_0) = \gamma z(0,t_0), \quad z(l,t_0) = 0,$$
$$z(x,t) \equiv 0, \quad t < t_0$$

satisfies the inequality (45), where $q = a(0)p_3\gamma$ and where $\alpha > 0$ is a unique positive solution of (18).

Proof 1. We start with (ii). Differentiating V we find

$$\begin{aligned}
\dot{V}(t) + 2\delta V(t) = {} & 2p_1 \int_0^l z(x,t) z_t(x,t)dx \\
& + 2p_3 \int_0^l a(x) z_x(x,t) z_{xt}(x,t)dx \\
& - \tau_M r \int_0^l \int_{t-\tau_M}^t e^{2\delta(s-t)} z_s^2(x,s)dsdx \\
& + \int_0^l [\tau_M^2 r z_t^2(x,t) + gz^2(x,t) \\
& - ge^{-2\delta\tau_M} z^2(x, t-\tau_M)]dx \\
& + 2a(0)p_3\gamma z(0,t) z_t(0,t) \\
& + 2\delta p_1 \int_0^l z^2(x,t)dx \\
& + 2\delta p_3 \int_0^l a(x) z_x^2(x,t)dx \\
& + 2\delta a(0) p_3 \gamma z^2(0,t).
\end{aligned} \tag{46}$$

By Jensen's inequality (Gu, Kharitonov, & Chen, 2003) we have

$$\begin{aligned}
& - \tau_M r \int_0^l \int_{t-\tau_M}^t e^{2\delta(s-t)} z_s^2(x,s)dsdx \\
& = -\tau_M r \int_0^l \int_{t-\tau_M}^{t-\tau(t)} e^{2\delta(s-t)} z_s^2(x,s)dsdx \\
& \quad - \tau_M r \int_0^l \int_{t-\tau(t)}^t e^{2\delta(s-t)} z_s^2(x,s)dsdx \\
& \leq -r \int_0^l e^{-2\delta\tau_M} \left[\int_{t-\tau_M}^{t-\tau(t)} z_s(x,s)ds\right]^2 dx \\
& \quad - r \int_0^l e^{-2\delta\tau_M} \left[\int_{t-\tau(t)}^t z_s(x,s)ds\right]^2 dx \\
& = -re^{-2\delta\tau_M} \int_0^l [z(x,t-\tau(t)) - z(x,t-\tau_M)]^2 dx \\
& \quad - re^{-2\delta\tau_M} \int_0^l [z(x,t) - z(x,t) - \tau(t)]^2 dx.
\end{aligned} \tag{47}$$

We apply further the descriptor method (Fridman, 2001; Fridman & Orlov, 2009a) to (38), where the left-hand side of

$$\begin{aligned}
& 2\int_0^l [p_2 z(x,t) + p_3 z_t(x,t)] \left[-z_t(x,t) + \frac{\partial}{\partial x}[a(x) z_x(x,t)] \right. \\
& \left. + \ \phi(z(x,t),x,t) z(x,t) - Kz(x,t-\tau(t))\right] dx \\
& + 2\sum_{j=0}^{N-1} \int_{x_j}^{x_{j+1}} [p_2 z(x,t) \\
& + p_3 z_t(x,t)] K \int_{\bar{x}_j}^x z_\zeta(\zeta, t-\tau(t)) d\zeta dx = 0
\end{aligned} \tag{48}$$

with some free scalar $p_2 > 0$ is added to $\dot{V}(t) + 2\delta V(t)$.

Integration by parts and substitution of the boundary conditions (3) lead to

$$\begin{aligned}
& 2p_3 \int_0^l z_t(x,t) \frac{\partial}{\partial x}[a(x) z_x(x,t)]dx \\
& = 2a(x) p_3 z_t(x,t) z_x(x,t)\Big|_0^l - 2p_3 \int_0^l a(x) z_{xt}(x,t) z_x(x,t)dx \\
& = -2a(0) p_3 \gamma z(0,t) z_t(0,t) - 2p_3 \int_0^l a(x) z_x(x,t) z_{xt}(x,t)dx, \\
& 2p_2 \int_0^l z(x,t) \frac{\partial}{\partial x}[a(x) z_x(x,t)]dx = -2a(0) p_2 \gamma z^2(0,t) \\
& \quad - 2p_2 \int_0^l a(x) z_x^2(x,t)dx.
\end{aligned} \tag{49}$$

Therefore, by adding the left-hand side of (48) to $\dot{V}(t) + 2\delta V(t)$ and by taking into account (46)–(48) (note that the term $2p_3 \int_0^l a(x) z_x(x,t) z_{xt}(x,t)dx$ in (46) is canceled by the corresponding term in (48)), we obtain

$$\begin{aligned}
\dot{V}(t) + 2\delta V(t) \leq {} & \int_0^l [\tau_M^2 r z_t^2(x,t) \\
& - re^{-2\delta\tau_M}[z(x,t-\tau(t)) - z(x,t-\tau_M)]^2]dx \\
& - re^{-2\delta\tau_M}[z(x,t) - z(x,t-\tau(t))]^2 dx \\
& + 2p_1 \int_0^l z(x,t) z_t(x,t)dx \\
& - 2p_2 \int_0^l a(x) {z_x}^2(x,t)dx \\
& + 2\delta p_3 \int_0^l a(x) z_x^2(x,t)dx \\
& + 2\delta p_1 \int_0^l z^2(x,t)dx \\
& + \int_0^l [gz^2(x,t) - ge^{-2\delta\tau_M} z^2(x,t-\tau_M)]dx \\
& + 2\int_0^l [p_2 z(x,t) + p_3 z_t(x,t)][-z_t(x,t) \\
& + \phi(z(x,t),x,t) z(x,t) - Kz(x,t-\tau(t))]dx \\
& + 2\sum_{j=0}^{N-1} \int_{x_j}^{x_{j+1}} [p_2 z(x,t) \\
& + p_3 z_t(x,t)] K \int_{\bar{x}_j}^x z_\zeta(\zeta, t-\tau) d\zeta dx \\
& + W_0,
\end{aligned} \tag{50}$$

where $W_0 = 2a(0)(\delta p_3 - p_2)\gamma z^2(0,t)$. The feasibility of the first inequality (41) implies $W_0 \leq 0$.

By Young's inequality, for any scalar $\bar{R} > 0$ we have

$$\begin{aligned}
& 2Kp_2 \sum_{j=0}^{N-1} \int_{x_j}^{x_{j+1}} z(x,t) \int_{x_j}^x z_\zeta(\zeta, t-\tau(t)) d\zeta dx \\
& \leq K\bar{R} p_2 \int_0^l z^2(x,t)dx
\end{aligned}$$

$$+K\bar{R}^{-1}p_2\sum_{j=0}^{N-1}\int_{x_j}^{x_{j+1}}\left[\int_{\bar{x}_j}^{x} z_\zeta(\zeta,t-\tau(t))d\zeta\right]^2 dx. \quad (51)$$

Wirtinger's inequality (19) yields (cf. (34))

$$\int_{x_j}^{x_{j+1}}\left[\int_{\bar{x}_j}^{x} z_\zeta(\zeta,t-\tau(t))d\zeta\right]^2 dx \le \frac{\Delta^2}{\pi^2}\int_{x_j}^{x_{j+1}} z_x^2(x,t-\tau(t))dx.$$

Choosing next $\bar{R}=\frac{\Delta}{\pi}R$, we find

$$\begin{aligned}2Kp_2\sum_{j=0}^{N-1}&\int_{\bar{x}_j}^{x_{j+1}} z(x,t)\int_{x_j}^{x} z_\zeta(\zeta,t-\tau(t))d\zeta dx\\ &\le \frac{\Delta}{\pi}KRp_2\int_0^l z^2(x,t)+\frac{\Delta}{\pi}KR^{-1}p_2\int_0^l z_x^2(x,t-\tau(t))dx,\\ 2Kp_3\sum_{j=0}^{N-1}&\int_{x_j}^{x_{j+1}}\int_{x_j}^{x} z_t(x,t)z_\zeta(\zeta,t-\tau(t))d\zeta dx\\ &\le \frac{\Delta}{\pi}KRp_3\int_0^l z_t^2(x,t)dx+\frac{\Delta}{\pi}KR^{-1}p_3\int_0^l z_x^2(x,t-\tau(t))dx. \quad (52)\end{aligned}$$

Set $\eta = col\{z(x,t), z_t(x,t), z(x,t-\tau_M), z(x,t-\tau(t))\}$. Then (50)–(52) implies

$$\begin{aligned}\dot{V}+2\delta V &\le \int_0^l \eta^T\bar{\Phi}^{\tau_M}_{|\lambda=\delta_1=0}\eta dx-2a_0(p_2-\delta p_3)\int_0^l z_x^2(x,t)dx\\ &\quad+\frac{\Delta}{\pi}KR^{-1}(p_3+p_2)\int_0^l z_x^2(x,t-\tau(t))dx\\ &\le \int_0^l \eta^T\bar{\Phi}^{\tau_M}_{|\delta_1=0}\eta dx\\ &\quad+\frac{\Delta}{\pi}KR^{-1}(p_3+p_2)\int_0^l z_x^2(x,t-\tau(t))dx, \quad (53)\end{aligned}$$

where $b=4$. The latter inequality follows from Wirtinger's inequality (19).

In order to apply further Halanay's inequality (17) we note that

$$\begin{aligned}&\dot{V}(t)+2\delta V(t)-\delta_1\sup_{\theta\in[-\tau_M,0]}V(t+\theta)\\ &\le \dot{V}(t)+2\delta V(t)-\delta_1 V(t-\tau(t))\\ &\le \dot{V}(t)+2\delta V(t)-\delta_1\int_0^\pi[p_1z^2(x,t-\tau(t))\\ &\quad+a(x)p_3z_x^2(x,t-\tau(t))]dx. \quad (54)\end{aligned}$$

Since $\bar{\Phi}^{\tau_M}$ defined by (43) is affine in $\phi\in[\phi_m,\phi_M]$, the feasibility of (42) implies the feasibility of $\bar{\Phi}^{\tau_M}\le 0$ for all $\phi\in[\phi_m,\phi_M]$. Therefore, (53), (41) and (42) yield

$$\begin{aligned}&\dot{V}(t)+2\delta V(t)-\delta_1\sup_{\theta\in[-\tau_M,0]}V(t+\theta)\\ &\le \int_0^l \eta^T\bar{\Phi}^{\tau_M}\eta dx+\frac{\Delta}{\pi}KR^{-1}(p_2+p_3)\\ &\quad\times\int_0^l z_x^2(x,t-\tau(t))dx-\delta_1 a_0p_3\\ &\quad\times\int_0^l z_x^2(x,t-\tau(t))dx\le 0. \quad (55)\end{aligned}$$

Application of Halanay's inequality, where $V(t_0)=\sup_{\theta\in[-\tau_M,0]}V(t_0+\theta)$ (due to (44)), completes the proof of (ii). Under the Dirichlet boundary conditions, the result of (i) is derived by using the above arguments, where in (55) Wirtinger's inequality (20) is applied. □

4.3. Sampled-data in time and in space controller

Lyapunov functionals (39) and (40) lead to sufficient conditions for any time-varying delays $\tau(t)\in[0,\tau_M]$ without taking into account the sawtooth evolution of the sampled-data induced delay $\tau(t)=t-t_k-\eta_k,\ t\in[t_k,t_{k+1})$. In the finite-dimensional case, in Fridman (2010) a novel construction of Lyapunov functional has been introduced for the sawtooth delays $\tau(t)=t-t_k,\ t\in[t_k,t_{k+1})$, which essentially improves the results. We extend the construction of Fridman (2010) to the diffusion equation. For the exponential stability analysis of the closed-loop system (8) we consider the following Lyapunov functional

$$\begin{aligned}V(t)&=p_1\int_0^l z^2(x,t)dx+\int_0^l[a(x)p_3z_x^2(x,t)\\ &\quad+r(t_{k+1}-t)\int_{t_k}^t e^{2\delta(s-t)}z_s^2(x,s)ds]dx+qz^2(0,t),\\ &\quad t\in[t_k,t_{k+1}),\ p_3>0,\ p_1>0,\ r>0, \quad (56)\end{aligned}$$

where $q=0$ corresponds to the Dirichlet and $q=a(0)p_3\gamma$ corresponds to the mixed boundary conditions. It is continuous in time since

$$\begin{aligned}V(t_k)&=\int_0^l[p_1z^2(x,t_k)+a(x)p_3z_x^2(x,t_k)]dx+qz^2(0,t_k)\\ &=V(t_k^-).\end{aligned}$$

Theorem 2. (i) *Consider the Dirichlet boundary value problem (26), (2) and let $b=1$. Given positive scalars $\Delta,\delta,K>\phi_M-\frac{a_0\pi^2}{bl^2}$, h, R and δ_1 such that $2\delta>\delta_1$, let there exist scalars $p_i>0$, $r>0$ and $y_i (i=1,2,3)$, satisfying (41) and the following LMIs*

$$\Phi^i{}_{|\phi=\phi_m}\le 0,\qquad \Phi^i{}_{|\phi=\phi_M}\le 0,\qquad i=0,1 \quad (57)$$

where

$$\Phi^0\triangleq\begin{bmatrix}\Phi_{11}-\lambda & \Phi_{12} & \Phi_{13}\\ * & hr+\Phi_{22} & \Phi_{23}\\ * & * & \Phi_{33}\end{bmatrix},$$

$$\Phi^1\triangleq\begin{bmatrix}\Phi_{11}-\lambda & \Phi_{12} & \Phi_{13} & hy_1\\ * & \Phi_{22} & \Phi_{23} & hy_2\\ * & * & \Phi_{33} & hy_3\\ * & * & * & -hre^{-2\delta h}\end{bmatrix},$$

$$\begin{aligned}&\Phi_{11}=2\delta p_1+2p_2\left(\phi+\frac{\Delta}{2\pi}KR\right)-2y_1,\\ &\Phi_{12}=p_1-p_2+p_3\phi-y_2,\qquad \Phi_{13}=y_1-y_3-Kp_2,\\ &\Phi_{22}=-2p_3+\frac{\Delta}{\pi}KRp_3,\qquad \Phi_{23}=y_2-Kp_3,\\ &\lambda=\frac{2a_0\pi^2}{bl^2}(p_2-\delta p_3),\qquad \Phi_{33}=2y_3-\delta_1p_1. \quad (58)\end{aligned}$$

Then a unique strong solution to the Dirichlet boundary value problem (26), (2) initialized with $z(\cdot,t_0)\in H_{\frac{1}{2}}$ satisfies the inequality (45) where $q=0$ and where $\alpha>0$ is a unique positive solution of (18).

(ii) *If the conditions of (i) hold with $b=4$, then a unique strong solution to the mixed boundary value problem (26), (3) initialized with (30) satisfies the inequality (45), where $q=a(0)p_3\gamma$ and where $\alpha>0$ is a unique positive solution of (18).*

Proof 2. For simplicity, we prove the result under the Dirichlet boundary conditions (2). Differentiating V, where $t \in [t_k, t_{k+1})$, we find

$$\begin{aligned}\dot V(t) + 2\delta V(t) = {}& 2p_1 \int_0^l z(x,t) z_t(x,t)dx \\ &+ 2p_3 \int_0^l a(x) z_x(x,t) z_{xt}(x,t)dx \\ &- r\int_0^l \int_{t_k}^t e^{2\delta(s-t)} z_s^2(x,s)dsdx \\ &+ \int_0^l [r(t_{k+1}-t) z_t^2(x,t) + 2\delta p_1 z^2(x,t) \\ &+ 2\delta a p_3 z_x^2(x,t)]dx.\end{aligned}$$

Denote $v_1(x,t) \overset{\Delta}{=} \frac{1}{t-t_k}\int_{t_k}^t z_s(x,s)ds$, where by $v_{1|t=t_k}$ we understand the following: $\lim_{t\to t_k^+} v_1 = z_t(x,t_k)$. By Jensen's inequality (Gu et al., 2003) we have

$$\begin{aligned}&-r\int_0^l \int_{t_k}^t e^{2\delta(s-t)} z_s^2(x,s)dsdx \\ &\le -r\frac{1}{t-t_k}\int_0^l e^{-2\delta h}\left[\int_{t_k}^t z_s(x,s)ds\right]^2 dx \\ &= -re^{-2\delta h}(t-t_k)\int_0^l v_1^2(x,t)dx. \qquad (59)\end{aligned}$$

We apply further the descriptor method to (8), where the left-hand side of (48) with $t-\tau(t) = t_k$ with some free scalar $p_2 > 0$ is added to $\dot V(t) + 2\delta V(t)$. Taking into account (31), we obtain

$$\begin{aligned}\dot V(t) + 2\delta V(t) \le {}& -re^{-2\delta h}(t-t_k)\int_0^l v_1^2(x,t)dx \\ &+ r\int_0^l (t_{k+1}-t) z_t^2(x,t)dx \\ &+ 2\delta p_3 \int_0^l a(x) z_x^2(x,t)dx \\ &+ 2\int_0^l [\delta p_1 z^2(x,t) + p_1 z(x,t) z_t(x,t) \\ &- a(x) p_2 {z_x}^2(x,t)]dx + 2\int_0^l [p_2 z(x,t) \\ &+ p_3 z_t(x,t)][-z_t(x,t) + \phi(z(x,t),x,t) z(x,t) \\ &- Kz(x,t-\tau(t))]dx + 2\sum_{j=0}^{N-1}\int_{x_j}^{x_{j+1}} [p_2 z(x,t) \\ &+ p_3 z_t(x,t)]K\int_{\bar x_j}^x z_\zeta(\zeta,t_k)d\zeta dx. \qquad (60)\end{aligned}$$

Young's and Wirtinger's inequalities (19) yield

$$\begin{aligned}&2K\sum_{j=0}^{N-1}\int_{\bar x_j}^{x_{j+1}}\int_{x_j}^x [p_2 z(x,t) + p_3 z_t(x,t)] z_\zeta(\zeta,t_k)d\zeta dx \\ &\le \frac{\Delta}{\pi}K\int_0^l [R(p_2 z^2(x,t) + p_3 z_t^2(x,t)) \\ &\quad + R^{-1}(p_2+p_3) z_x^2(x,t_k)]dx \qquad (61)\end{aligned}$$

for some $R > 0$ (cf. Eq. (1), (52)). Extending the free-weighting matrices method of He, Wu, She, and Liu (2004) to the infinite-dimensional case, we further insert free scalars y_1, y_2, y_3 by adding to $\dot V(t) + 2\delta V(t)$ the left-hand side of the following expression

$$\begin{aligned}&2\int_0^l [y_1 z(x,t) + y_2 z_t(x,t) + y_3 z(x,t_k)][-z(x,t) \\ &+ z(x,t_k) + (t-t_k) v_1(x,t)]dx = 0. \qquad (62)\end{aligned}$$

Set $\eta = col\{z(x,t), z_t(x,t), z(x,t_k), v_1\}$. Then (60)–(62) and Wirtinger's inequality (20) imply

$$\begin{aligned}\dot V(t) + 2\delta V(t) \le {}& -\int_0^l [2a_0(p_2 - \delta p_3) z_x^2(x,t) \\ &- \frac{\Delta}{\pi}KR^{-1}(p_2+p_3) z_x^2(x,t_k)]dx \\ &+ \int_0^l \eta^T \bar\Phi^s_{|\lambda=\delta_1=0}\eta dx \\ \le {}& \int_0^l \eta^T \bar\Phi^s_{|\delta_1=0}\eta dx + \frac{\Delta}{\pi}KR^{-1}(p_2+p_3) \\ &\times \int_0^l z_x^2(x,t)dx \qquad (63)\end{aligned}$$

where $b = 1$ and

$$\bar\Phi^s \overset{\Delta}{=} \begin{bmatrix} \Phi_{11}-\lambda & \Phi_{12} & \Phi_{13} & (t-t_k)y_1 \\ * & (t_{k+1}-t)r + \Phi_{22} & \Phi_{23} & (t-t_k)y_2 \\ * & * & \Phi_{33} & (t-t_k)y_3 \\ * & * & * & -(t-t_k)re^{-2\delta h}\end{bmatrix}.$$

We note that

$$\begin{aligned}&\dot V(t) + 2\delta V(t) - \delta_1 \sup_{\theta\in[-h,0]} V(t+\theta) \\ &\le \dot V(t) + 2\delta V(t) - \delta_1 V(t_k) \le \dot V(t) + 2\delta V(t) \\ &\quad - \delta_1\int_0^l [p_1 z^2(x,t_k) + a(x) p_3 z_x^2(x,t_k)]dx \\ &\le \int_0^l \eta^T\bar\Phi_s\eta dx + \left[\frac{\Delta}{\pi}KR^{-1}(p_2+p_3) - \delta_1 a_0 p_3\right] \\ &\quad \times \int_0^l z_x^2(x,t_k)dx, \qquad (64)\end{aligned}$$

if (41) is feasible and if $\bar\Phi^s \le 0$ for all $t \in [t_k, t_{k+1})$.

We will prove next that the four LMIs (57) yield $\bar\Phi^s \le 0$. Matrices Φ^0 and Φ^1 given by (58) are affine in ϕ. Therefore, $\Phi^j \le 0$ for all $\phi \in [\phi_m, \phi_M]$ if LMIs (57) are satisfied. For $t - t_k \to 0$ and $t - t_k \to h$ the matrix inequality $\bar\Phi_s \le 0$ leads to $\Phi^0 \le 0$ and $\Phi^1 \le 0$ with notations given in (58). Denote by $\eta_0 = col\{z(x,t), z_t(x,t), z(x,t_k)\}$. Then $\Phi^0 \le 0$ and $\Phi^1 \le 0$ imply for $t \in [t_k, t_{k+1})$

$$\frac{t_{k+1}-t}{t_{k+1}-t_k}\eta_0^T\Phi^0\eta_0 + \frac{t-t_k}{t_{k+1}-t_k}\eta^T\Phi^1\eta = \eta^T\Phi_h\eta \le 0, \quad \forall \eta \ne 0,$$

where

$$\Phi_h \overset{\Delta}{=} \begin{bmatrix} \Phi_{11}-\lambda & \Phi_{12} & \Phi_{13} & h\frac{t-t_k}{t_{k+1}-t_k}y_1 \\ * & h\frac{t_{k+1}-t}{t_{k+1}-t_k}r + \Phi_{22} & \Phi_{23} & h\frac{t-t_k}{t_{k+1}-t_k}y_2 \\ * & * & \Phi_{33} & h\frac{t-t_k}{t_{k+1}-t_k}y_3 \\ * & * & * & -h\frac{t-t_k}{t_{k+1}-t_k}re^{-2\delta h}\end{bmatrix} \le 0.$$

Since $\frac{h}{t_{k+1}-t_k} \ge 1$, the feasibility of $\Phi_h \le 0$ (by Schur complements) implies $\bar\Phi^s \le 0$. Therefore, inequalities (41), (64), $\bar\Phi^s \le 0$ and the

Halanay's inequality (17) yield $V(t) \leq e^{-2\alpha(t-t_0)}V(t_0)$ for $\alpha > 0$ satisfying (18), which completes the proof. □

4.4. Example

Consider the controlled diffusion equation

$$z_t(x,t) = \frac{\partial}{\partial x}[a(x)z_x(x,t)] + \phi(z(x,t))z(x,t) - \beta z_x(x,t) + u(x,t), \quad x \in [0,\pi],\ t \geq 0,\ a \geq 1 \quad (65)$$

under the Dirichlet (2) or under the mixed (12) (with $\gamma = \beta/2$) boundary conditions. We consider either $\beta = 0$, where a is assumed to be of class C^1 (and may be unknown), or $\beta = 1$ with $a \equiv 1$. The unknown function ϕ is assumed to be of class C^1 with $0 \leq \phi \leq 1.8$. The sampled-data control law is chosen as (see Remark 1)

$$\begin{aligned} &u(x,t) = -3e^{-\frac{\beta}{2}(\bar{x}_j - x)} z(\bar{x}_j, t_k - \eta_k), \\ &x_j \leq x < x_{j+1},\ \bar{x}_j = \frac{x_{j+1}+x_j}{2},\ t \in [t_k, t_{k+1}), \\ &x_{j+1} - x_j \leq \Delta,\ t_{k+1} - t_k \leq h,\ 0 \leq \eta_k \leq \eta_M. \end{aligned} \quad (66)$$

According to Proposition 1 and Remark 1, the state-feedback $u(x,t) = -3z(x,t)$ exponentially stabilizes (65), (12) and (65), (2) with the decay rates $1.45 + 0.25\,\beta^2$ and $2.2 + 0.25\,\beta^2$ respectively. Thus, for small enough sampling intervals and delay, the sampled-data controller stabilizes the system.

For the continuous in time controller $u(x,t) = -3e^{-\frac{\beta}{2}(\bar{x}_j - x)} z(\bar{x}_j, t)$ we apply Proposition 1, where for simplicity we choose $R = 1$. We find that the closed-loop system, where $\beta = 0$ or $\beta = 1$, under the Dirichlet boundary conditions remains exponentially stable till $\Delta \leq 2.09$. Therefore, the above controller exponentially stabilizes the system if the spatial domain is divided into two sub-domains with $x_{j+1} - x_j \leq 2.09$. Moreover, if we choose $x_1 = \frac{\pi}{2}$ in the middle of $[0,\pi]$, which corresponds to two sensors placed in $\bar{x}_0 = \frac{\pi}{4}$ and $\bar{x}_1 = \frac{3\pi}{4}$, then the above controller exponentially stabilizes the system with the decay rate 0.7.

For $\beta = 0$, $\phi \equiv 1.8$ and the continuous in space controller $u(x,t) = -3z(x,t_k)$, by using LMI Toolbox of Matlab we verify LMI conditions of Theorem 2 under the Dirichlet boundary conditions. Note that Matlab verifies the feasibility of the strict inequalities. We find that the closed-loop system preserve the exponential stability for $t_{k+1} - t_k \leq h = 0.66$. The corresponding bound for the time-varying delay which follows from Theorem 1 is essentially smaller: $t_{k+1} - t_k + \eta_k \leq \tau_M = 0.38$.

We consider further the controller (66), $0 \leq \phi \leq 1.8$ and apply Theorems 1 and 2 to the closed-loop system, where we choose $R = 1$. Theorem 2 is applied in the case of $\eta_k \equiv 0$. For $\beta = 0$ and $\beta = 1$, Tables 1 and 2 show the maximum values of Δ as functions of $\tau_M = h + \eta_M$ (that result from Theorem 1) and of h (that result from Theorem 2), which preserve the exponential stability of the system. The corresponding values of δ are also given, whereas the values of $\delta_1 < 2\delta$ are chosen to be close to 2δ, which leads to a small decay rate α but enlarges the sampling intervals. The values before the brackets correspond to $\beta = 0$, whereas the values in brackets correspond to $\beta = 1$. If these values coincide, only one number is given.

It is seen from the Tables 1 and 2 that in the sampled-data case with $t_{k+1} - t_k \leq 0.1$ and $\beta = 1$ under the Dirichlet boundary conditions the resulting $\Delta = 1.05 > \pi/3$, which leads to three sub-domains with the three sensors in the middle. In all the other cases (the delayed control under the mixed/Dirichlet boundary conditions and the sampled-data control under the Dirichlet boundary conditions with $\beta = 0$) four sensors corresponding to four sub-domains should be used. Considering further $t_{k+1} - t_k + \eta_k \leq 0.1$ and $\beta = 0$, $x_{j+1} - x_j = \pi/4, j = 0, \ldots, 3$ under the Dirichlet boundary conditions, we find that the conditions of Theorem 1 are feasible with $\delta = 1.2$, $\delta_1 = 2 \cdot 0.77 \cdot \delta$, which guarantees the exponential stability of the closed-loop system with the decay rate $\alpha = 0.252$.

Table 1
Dirichlet b.c. with $\beta = 0$ ($\beta = 1$).

δ	1000	1(1.2)	0.73	0.23
τ_M	0	0.1	0.2	0.3
$\Delta_{\vert\tau_M}$	2.09	0.97(1)	0.54(0.57)	0.1(0.14)
h	0	0.1	0.2	0.3
$\Delta_{\vert h}$	2.09	1(1.05)	0.65(0.69)	0.3(0.33)

Table 2
Mixed b.c with $\beta = 0$ ($\beta = 1$).

δ	1000	1	0.6	0.23
τ_M	0	0.1	0.2	0.3
$\Delta_{\vert\tau_M}$	2.08(2.09)	0.94(0.99)	0.46(0.50)	0(0)
h	0	0.1	0.2	0.3
$\Delta_{\vert h}$	2.08(2.09)	0.98(1.03)	0.56(0.62)	0.28(0.3)

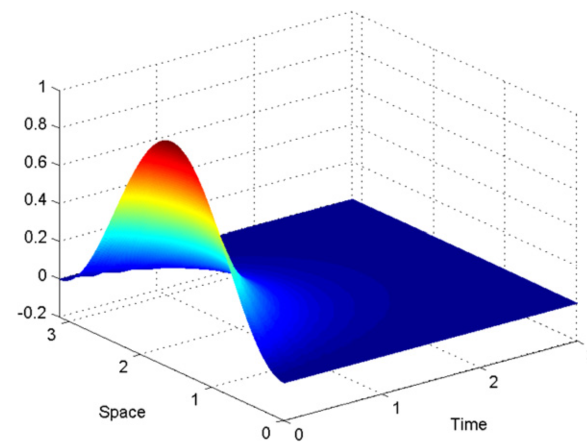

Fig. 1. Solution under the Dirichlet b.c. with $\Delta = \pi/2$, $h = 0.2$, $\phi(z) = 1.8\cos^2 z$ and $\beta = 0$.

We proceed further with the numerical simulations of the solutions to the closed-loop system under the Dirichlet boundary conditions with $a \equiv 1$ and $\beta = 0$, where we choose $z(x,0) = \sin^2 x$ and either $\phi(z) = 1.8\cos^2 z$ or $\phi \equiv 1.8$. We use a finite difference method. For the continuous in space controller $u(x,t) = -3z(x,t_k)$ and $\phi \equiv 1.8$, our numerical simulations confirm the predicted upper bound on $t_{k+1} - t_k \leq 0.66$ which preserves the stability. Thus, for $t_{k+1} - t_k > 0.68$ the system becomes unstable. Hence, the conditions of Theorem 2 for the sampled-data in time controller are not conservative.

Simulations of solutions under the sampled in spatial variable controller $u(x,t) = -3z(\bar{x}_j, t)$ with $x_{j+1} - x_j = \pi/2,\ j = 0, 1$, where the space domain is divided into two sub-domains, show that the closed-loop system is exponentially stable. This confirms the predicted by Proposition 1 behavior. Moreover, for $x_{j+1} - x_j = \pi/2,\ j = 0, 1$ the sampled-data in time and in space controller $u(x,t) = -3z(\bar{x}_j, t_k)$ preserves the stability for $t_{k+1} - t_k \leq 0.55$ (see Fig. 1, where $t_{k+1} - t_k = 0.2$, $\phi(z) = 1.8\cos^2 z$). The latter illustrates the conservatism of the presented method, where for $t_{k+1} - t_k \leq 0.2$ the corresponding value of the maximum Δ is 0.65, which results in five sub-domains.

4.5. The dual sampled-data observation problem

Consider the semilinear diffusion equation

$$z_t(x,t) = \frac{\partial}{\partial x}[a(x)z_x(x,t)] + f(z(x,t),x,t) + u(x,t),$$

$$t \geq t_0,\ x \in [0, l],\ l > 0,\ a > a_0 > 0 \tag{67}$$

under the Dirichlet boundary conditions (2), where u is the control input, a and f are *known functions* of class C^1 and $\phi_m \leq f_z \leq \phi_M$. The discrete measurements are given by (5) with the *known sampling instants* t_k. We suggest a nonlinear observer of the form

$$\begin{aligned} \hat{z}_t(x, t) &= \frac{\partial}{\partial x}[a(x)\hat{z}_x(x, t)] + f(\hat{z}(x, t), x, t) \\ &\quad + u(x, t) - K[y_{jk} - \hat{z}(\bar{x}_j, t_k)], \end{aligned} \tag{68}$$
$$t \in [t_k, t_{k+1}),\ k = 0, 1, 2 \ldots,$$
$$x_j \leq x < x_{j+1},\ j = 0, \ldots, N-1$$

under the Dirichlet boundary conditions, where $K > 0$ is the injection gain and where $\hat{z}(x, t_0) = 0$.

Then the estimation error $\hat{e} = z - \hat{z}$ satisfies the Dirichlet boundary value problem for the equation

$$\begin{aligned} \hat{e}_t(x, t) &= \frac{\partial}{\partial x}[a(x)\hat{e}_x(x, t)] + \phi\hat{e}(x, t) - K\hat{e}(\bar{x}_j, t_k), \\ &\quad t \in [t_k, t_{k+1}),\ x_j \leq x < x_{j+1}, \end{aligned} \tag{69}$$

where $\phi = \int_0^1 f_z(\hat{z} + \theta\hat{e}, x, t)d\theta$ with $\phi_m \leq \phi \leq \phi_M$. Hence, Theorem 2 gives sufficient conditions for the exponential stability of (69) under the Dirichlet boundary conditions. The dual observation problem under the mixed boundary conditions can be formulated and solved similarly.

Remark 4. The infinite-dimensional observer-based control of (1) under the sampled-data measurements have no advantages over the static output-feedback control in the following sense: the observer convergence should be faster than the system convergence, which may increase the number of sensors. Moreover, the observer design exploits the knowledge of the system and of the sampling (in time) instants and, thus, is not applicable to uncertain systems under uncertain sampling time instants/time-delays. However, the observer-based control may have a practical advantage being less sensitive to the measurement noise.

5. Conclusions

We have developed a sampled-data (in time and in space) controller design for a 1-D uncertain semilinear diffusion equation under the homogenous Dirichlet or under the mixed boundary conditions. Sufficient conditions for the exponential stabilization are derived in terms of LMIs. By solving these LMIs, upper bounds are found on the sampling intervals that preserve the exponential stability, as well as the resulting decay rate. A numerical example illustrates the efficiency of the method and its conservatism. Thus, the results are close to analytical ones if the controller is sampled in time only (and it is continuous in space) and are almost not conservative if the controller is sampled in space only. The conservatism of the method for the sampled-data in temporal and spatial variables controller may stem from the application of Halanay's inequality.

The presented method gives a *general framework for robust control of parabolic systems*: being formulated in terms of LMIs, our conditions can be further applied to systems with saturated actuators, to input-to-state stabilization. It gives tools for network-based control, where data packet dropouts (resulting in variable in time sampling) and network-induced delays are taken into account. Extension of the method to various classes of parabolic systems, as well as its improvement may be topics for the future research.

References

Balas, M. J. (1985). Distributed parameter control of nonlinear flexible structures with linear finite-dimensional controllers. *Journal of Mathematical Analysis and Applications*, *108*, 528–545.

Candogan, U.O., Ozbay, H., & Ozaktas, H.M. (2008). Controller implementation for a class of spatially-varying distributed parameter systems. In *Proceedings of the 17th IFAC world congress*. Seoul, Korea.

Chen, T., & Francis, B. (1995). Optimal sampled-data control systems. In *Communications and control engineering series*. London; New York: Springer.

Cheng, M. B., Radisavljevic, V., Chang, C. C., Lin, C. F., & Su, W. C. (2009). A sampled-data singularly perturbed boundary control for a diffusion conduction system with noncollocated observation. *IEEE Transactions of Automatic Control*, *54*(6), 1305–1310.

Curtain, R., & Zwart, H. (1995). *An introduction to infinite-dimensional linear systems theory*. New York: Springer-Verlag.

Demetriou, M. A. (2010). Guidance of mobile actuator-plus-sensor networks for improved control and estimation of distributed parameter systems. *IEEE Transactions on Automatic Control*, *55*, 1570–1584.

Fridman, E. (2010). A refined input delay approach to sampled-data control. *Automatica*, *46*, 421–427.

Fridman, E. (2001). New Lyapunov–Krasovskii functionals for stability of linear retarded and neutral type systems. *Systems & Control Letters*, *43*, 309–319.

Fridman, E., & Blighovsky, A. (2011)Sampled-data control and observation of semilinear convection–diffusion equations. In *Proceedings of the 18th IFAC world congress*. Milano, Italy.

Fridman, E., & Orlov, Y. (2009a). Exponential stability of linear distributed parameter systems with time-varying delays. *Automatica*, *45*(2), 194–201.

Fridman, E., & Orlov, Y. (2009b). An LMI approach to H_∞ boundary control of semilinear parabolic and hyperbolic systems. *Automatica*, *45*(9), 2060–2066.

Fujioka, H. (2009). A discrete-time approach to stability analysis of systems with aperiodic sample-and-hold devices. *IEEE Transactions on Automatic Control*, *54*(10), 2440–2445.

Gao, H., Chen, T., & Lam, J. (2008). A new system approach to network-based control. *Automatica*, *44*, 39–52.

Ghantasala, S., & El-Farra, N.H. (2010). Fault-tolerant control of sampled-data nonlinear distributed parameter systems. In *Proceedings of American control conference*. Baltimore, USA.

Gu, K, Kharitonov, V, & Chen, J. (2003). *Stability of time-delay systems*. Boston: Birkhauser.

Hagen, G., & Mezic, I. (2003). Spillover stabilization in finite-dimensional control and observer design for dissipative evolution equations. *SIAM Journal on Control and Optimization*, *42*(2), 746–768.

Halanay, A. (1966). *Differential equations: stability, oscillations, time lags*. New York: Academic Press.

Hardy, G. H., Littlewood, J. E., & Polya, G. (1988). *Inequalities*. Cambridge: Cambridge University Press.

Henry, D. (1993). *Geometric theory of semilinear parabolic equations*. New York: Springer-Verlag.

He, Y., Wu, M., She, J., & Liu, G. (2004). Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays. *Systems & Control Letters*, *51*, 57–65.

Khapalov, A. Y. (1993). Continuous observability for parabolic system under observations of discrete type. *IEEE Transactions on Automatic Control*, *38*(9), 1388–1391.

Kolmanovskii, V., & Myshkis, A. (1999). *Applied theory of functional differential equations*. Kluwer.

Krstic, M., & Smyshlyaev, A. (2008). Adaptive boundary control for unstable parabolic PDEs-part I: Lyapunov design. *IEEE Transactions on Automatic Control*, *53*, 1575–1591.

Logemann, H., Rebarber, R., & Townley, S. (2003). Stability of infinite-dimensional sampled-data systems. *Transactions of the American Mathematical Society*, *35*, 301–328.

Logemann, H., Rebarber, R., & Townley, S. (2005). Generalized sampled-data stabilization of well-posed linear infinite-dimensional systems. *SIAM Journal on Control and Optimization*, *44*(4), 1345–1369.

Naghshtabrizi, P., Hespanha, J., & Teel, A. (2008). Exponential stability of impulsive systems with application to uncertain sampled-data systems. *Systems & Control Letters*, *57*, 378–385.

Smagina, E., & Sheintuch, M. (2006). Using Lyapunov's direct method for wave suppression in reactive systems. *Systems & Control Letters*, *55*(7), 566–572.

Smyshlyaev, A., & Krstic, M. (2005). On control design for PDEs with space-dependent diffusivity or time-dependent reactivity. *Automatica*, *41*, 1601–1608.

Sun, Y., Ghantasala, S., & El-Farra, N.H. (2009). Networked control of spatially distributed processes with sensor-controller communication constraints. In *Proceedings of American control conference*. St. Louis, USA.

Tucsnak, M., & Weiss, G. (2009). *Observation and control for operator semigroups*. Birkhauser.

Zhang, W., Branicky, M., & Phillips, S. (2001). Stability of networked control systems. *IEEE Control System Magazine*, *21*, 84–99.

Emilia Fridman received the M.Sc. degree from Kuibyshev State University, USSR, in 1981 and the Ph.D. degree from Voronezg State University, USSR, in 1986, all in Mathematics. From 1986 to 1992 she was an Assistant and Associate Professor in the Department of Mathematics at Kuibyshev Institute of Railway Engineers, USSR. Since 1993 she has been at Tel Aviv University, where she is currently Professor of Electrical Engineering Systems. She has held visiting positions at the Weierstrass Institute for Applied Analysis and Stochastics in Berlin (Germany), INRIA in Rocquencourt (France), Ecole Centrale de Lille (France), Valenciennes University (France), Leicester University (UK), Kent University (UK), CINVESTAV (Mexico), Zhejiang University (China), St. Petersburg IPM (Russia).

Her research interests include time-delay systems, distributed parameter systems, robust control, singular perturbations, nonlinear control and asymptotic methods. She has published about 90 articles in international scientific journals. Currently she serves as Associate Editor in Automatica, SIAM Journal on Control and Optimization and in IMA Journal of Mathematical Control and Information.

Anatoly Blighovsky received his M.Sc. degree from the School of Electrical Engineering of Tel Aviv University, Israel in 2011. His research interests include stability and robust control of infinite-dimensional systems.