Quantized Control Under Round-Robin Communication Protocol

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***Abstract*—This paper analyzes the exponential stability of a discrete-time linear plant in feedback control over a communication network with N sensor nodes, dynamic quantization, large communication delays, variable sampling intervals and Round-Robin scheduling. The closed-loop system is modelled as a switched system with multiple ordered time-varying delays and bounded disturbances. We propose a time-triggered zooming algorithm implemented at the sensors that preserves exponential stability of the closed-loop system. A direct Lyapunov approach is presented for initialization of the zoom variable. The proposed framework can be applied to the plants with polytopic type uncertainties. The effectiveness of the method are illustrated on cart-pendulum and quadruple-tank processes.**

***Index Terms*—Networked control systems, dynamic quantization, Round-Robin protocol, switched time-delay systems, Lyapunov method.**

I. Introduction

THE rapidly developing wireless communication technology enables networked control systems (NCSs) with increased flexibility, ease of installation and reduced costs [9], [26]. In many such systems, the transmissions are constrained by bandwidth limitations and interference [3], [7], [33].

In such systems, one of the constraints associated with control over communication networks is that only a subset of sensors and actuators can transmit their data over the channel at each transmission instant. Therefore, protocols are needed to schedule which node is given access to the network at each time instant. There are three main classes of scheduling protocols: (i) periodic protocols, of which Round-Robin is a special case [7], [20], [21]; (ii) quadratic protocols, which include try-once-discard protocol [7], [20], [21], [30]; and (iii) stochastic protocols [2], [3], [18], [28].

Another communication constraint is that transmitted data should be quantized before they are sent from the sensor to the controller/actuator [11], [32], [34]. Quantization is implemented by a device that converts a real-valued signal into a piecewise constant one with a finite set of values. Quantized control has been paid considerable attention in recent years. When the system is affected by a static quantizer, a simple approach is to treat the quantization interval as uncertainty [6], [25], and to bound the uncertainty by using the sector bound approach [5]. Dynamic quantization was proposed in [1], where the quantizer incorporated an adjustable "zoom" variable. General types of dynamic quantizers were studied in [11], [12], [13].

By simultaneously considering dynamic quantization and scheduling protocols, a unified framework was provided in [20] for the analysis of NCSs via a hybrid system approach. However, delays were not included in the analysis. A linear matrix inequality (LMI)-based time-triggered zooming algorithm was presented in [17] via a time-delay system approach for NCSs with dynamic quantization and variable communication delays. As pointed out in [17], taking communication delays into consideration leads to additional challenges: (i) the closed-loop system and the resulting solution bounds should be formulated in terms of updating time instants at the actuators, while the zooming algorithm should be given in terms of sampling instants at the sensors; (ii) the solution bounds should include additional bounds on the first time interval of the delay length [15]. In [17], the zooming algorithm was proposed in terms of sampling instants at the sensors and a direct Lyapunov approach was presented for initialization of the zoom variable. However, scheduling protocols were not taken into account. This observation and the need for scheduling in wireless NCSs motivate us to develop a time-delay system approach for linear NCSs under scheduling and dynamic quantization.

In this paper, we consider the stability analysis of discrete-time NCSs with N sensor nodes. The system involves dynamic quantization, large communication delays, variable sampling intervals and Round-Robin scheduling protocol. The closed-loop quantized system is modelled as a switched system with multiple and ordered time-varying delays and bounded disturbances. In the presence of the Round-Robin protocol, a time-triggered zooming algorithm, which is implemented at the sensors is proposed and it is shown to lead to an exponentially stable closed-loop system. After each zooming-in instant, we suggest waiting for all the N latest transmitted measurements to arrive at the controller side and then sending them together to the actuator side. Following [15] we present a direct Lyapunov approach for initialization of the zoom variable. Polytopic type uncertainties in the plant model can be easily incorporated in the framework.

The remainder of the paper is organized as follows. Sec-

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tion II defines the model of the considered quantized discrete-time NCSs under Round-Robin scheduling. An input-to-state stability (ISS) condition is derived by a Lyapunov method for the switched closed-loop system model. Based on this ISS condition, Section III proposes an LMI-based zooming algorithm for the dynamic quantization. It is shown that it leads to exponential stability of the closed-loop system. Two illustrative examples are discussed in Section IV. The conclusions and future work are finally stated in Section V.

Notations: The superscript 'T' stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space with vector norm $|\cdot|$, $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices, and the notation $P > 0$, for $P \in \mathbb{R}^{n\times n}$ means that P is symmetric and positive definite. The symbol $*$ represents the symmetric term of a symmetric matrix. $\mathbb{Z}^+$ and $\mathbb{N}$ denote the set of non-negative and positive integers, respectively. $\lfloor x \rfloor$ denotes the largest integer k such that $k \leq x$, i.e., $\lfloor x \rfloor = \max\{k \in \mathbb{Z} : k \leq x\}$. $\lceil x \rceil$ denotes the smallest integer k such that $k \geq x$, i.e., $\lceil x \rceil = \min\{k \in \mathbb{Z} : k \geq x\}$.

II. NCS MODEL AND PROBLEM FORMULATION

In this section, we present the considered discrete-time NCS model and some preliminary results on the problem to be solved in the paper.

A. Quantized NCS under Round-Robin scheduling

The quantized NCS is depicted schematically in Figure 1. It consists of a linear discrete-time plant, N distributed sensors and quantizers, a controller node and an actuator node, which are all connected via communication networks. The discrete-time plant is given by

$$x(k+1) = Ax(k) + Bu(k), \quad k \in \mathbb{Z}^+, \tag{1}$$

where $x(k) \in \mathbb{R}^n$ denotes the state of the plant, and $u(k) \in \mathbb{R}^{n_u}$ the control input. The matrices A and B may be certain or uncertain. The initial condition is given by $x(0) = x_0$. It is assumed that x_0 may be unknown, but satisfies the bound $|x_0| < X_0$, where $X_0 > 0$ is known. The assumption on boundedness of the initial state is common, e.g., for interval observer design [23].

The measurement outputs of the plant are described by $y_i(k) = C_i x(k) \in \mathbb{R}^{n_i}$, $i = 1, \ldots, N$, $\sum_{i=1}^{N} n_i = n_y$. We denote $C = \left[\begin{array}{ccc} C_1^T & \cdots & C_N^T \end{array}\right]^T$, $y(k) = \left[\begin{array}{ccc} y_1^T(k) & \cdots & y_N^T(k) \end{array}\right]^T \in \mathbb{R}^{n_y}$. Following [6], we consider the quantization effect from the sensors to the controller.

Let $z_i(k) \in \mathbb{R}^{n_i}$, $i = 1, \ldots, N$, be the vectors to be quantized. The quantizers are piecewise constant functions $q_i(z_i(k))$: $\mathbb{R}^{n_i} \to \mathbb{D}_i$, where $\mathbb{D}_i$ is a finite subset of $\mathbb{R}^{n_i}$, $i = 1, \ldots, N$. It is assumed that there exist real numbers $M_i > \Delta_i > 0$, $i = 1, \ldots, N$, such that the following two conditions hold:

(a) if $|z_i(k)| \leq M_i$, then $|q_i(z_i(k)) - z_i(k)| \leq \Delta_i$;

(b) if $|z_i(k)| > M_i$, then $|q_i(z_i(k))| > M_i - \Delta_i$,

where Δ_i and M_i are the quantization interval bounds and ranges, respectively. Condition (a) gives a bound on the quantization interval when the quantizer does not saturate, and condition (b) provides a way to detect saturation.

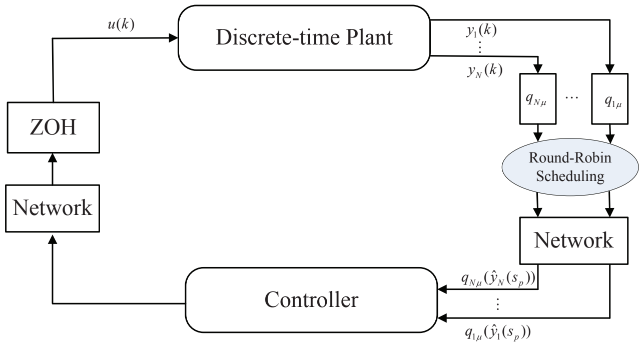

Fig. 1. Networked control systems with quantizers and Round-Robin scheduling

We consider quantized measurements as in [11]:

$$q_{i\mu}(z_i(k)) := \mu(k) q_i\Big(\frac{z_i(k)}{\mu(k)}\Big), \; i = 1, \ldots, N, \tag{2}$$

where $\mu(k) > 0$ is the zoom variable. The range of the quantizer $q_{i\mu}$, is $\mu(k) M_i$ and the quantization interval is $\mu(k)\Delta_i$, $i = 1, \ldots, N$. The zoom variable $\mu(k)$ changes dynamically in order to achieve exponential stability.

Let s_p represent the unbounded and monotonously increasing sequence of sampling instants, i.e.,

$$\begin{array}{l} 0 = s_0 < s_1 < \cdots < s_p < \cdots, \quad p \in \mathbb{Z}^+, \\ \lim\limits_{p\to\infty} s_p = \infty, \; s_{p+1} - s_p \leq \text{MATI}, \end{array} \tag{3}$$

where $\{s_p\}$ is a subsequence of $\mathbb{Z}^+$ and MATI denotes the maximum allowable transmission interval. Denote by $q_\mu(\hat{y}(s_p)) = [q_{1\mu}^T(\hat{y}_1(s_p)) \; \cdots \; q_{N\mu}^T(\hat{y}_N(s_p))]^T \in \mathbb{R}^{n_y}$ the output information submitted to the scheduling protocol. At each sampling instant s_p, one of the outputs $y_i(s_p) \in \mathbb{R}^{n_i}$ is quantized and transmitted over the network, that is, one of the $q_{i\mu}(\hat{y}_i(s_p))$ values is updated with the recent quantized output $q_{i\mu}(y_i(s_p))$. Let $i_p^* \in \mathcal{I} = \{1, \ldots, N\}$ denote the active output node at the sampling instant s_p, which will be chosen according to the scheduling protocol below.

Consider Round-Robin scheduling for the choice of the active quantized output node: $q_{i\mu}(y_i(k)) = q_{i\mu}(C_i x(k))$, $k \in \mathbb{Z}^+$, is transmitted only at the sampling instant $k = s_{N\ell+i-1}$, $\ell \in \mathbb{Z}^+$, $i = 1, \ldots, N$. After each transmission and reception, the values in $q_{i\mu}(y_i(k))$ are updated with the newly received values, while the values of $q_{j\mu}(y_j(k))$ for $j \neq i$ remain the same, as no additional information is received. This leads to the constrained data exchange expressed as

$$q_{i\mu}(\hat{y}_i(s_p)) = \left\{\begin{array}{ll} q_{i\mu}(y_i(s_p)) = q_{i\mu}(C_i x(s_p)), & p = N\ell + i - 1, \\ q_{i\mu}(\hat{y}_i(s_{p-1})), & p \neq N\ell + i - 1, \end{array}\right. \; \ell \in \mathbb{Z}^+. \tag{4}$$

It is assumed that data packet loss does not occur. Denote by t_p the updating time instant of the zero-order holder (ZOH). Suppose that the updating data at the instant t_p on the actuator side has experienced a variable transmission delay $\eta_p = t_p - s_p$. As in [19], the delays may be either smaller or larger than the sampling interval provided that the transmission

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order of data packets is maintained. Assume that the network-induced delay η_p and the time span between the updating and the current sampling instants are bounded:

$$t_{p+1} - 1 - t_p + \eta_p \leq \tau_M^N,\ 0 \leq \eta_m \leq \eta_p \leq \eta_M,\ p \in \mathbb{Z}^+, \tag{5}$$

where τ_M^N, η_m and η_M are known non-negative integers. Then we have

$$\begin{aligned} (t_{p+1}-1) - s_p &= s_{p+1} - s_p + \eta_{p+1} - 1 \\ &\leq \text{MATI} + \eta_M - 1 = \tau_M^N, \\ (t_{p+1}-1) - s_{p-N+j} &= s_{p+1} - s_{p-N+j} + \eta_{p+1} - 1 \\ &\leq (N-j+1)\,\text{MATI} + \eta_M - 1 \\ &= (N-j+1)\tau_M^N - (N-j)\eta_M \\ &\quad + N - j \triangleq \tau_M^j,\ j = 1,\ldots,N-1, \\ t_{p+1} - t_p &\leq \tau_M^N - \eta_m + 1. \end{aligned} \tag{6}$$

B. A switched system model

Next, we introduce a switched system model as the closed-loop system of NCS provided above. Suppose that the controller and the actuator are event-driven. The most recent output information on the controller side is denoted by $q_\mu(\hat{y}(s_p))$. Assume that there exists a matrix $K = [K_1\ \ldots\ K_N]$, $K_i \in \mathbb{R}^{m \times n_i}$ such that $A + BKC$ is Schur stable. Consider the static output feedback controller

$$u(k) = Kq_\mu(\hat{y}(s_p)),\ k \in [t_p, t_{p+1}-1],\ k \in \mathbb{Z}^+. \tag{7}$$

Due to (4), the controller (7) can be represented as

$$\begin{aligned} u(k) &= K_{i_p^*} q_{i_p^*\mu}(y_{i_p^*}(t_p - \eta_p)) \\ &\quad + \sum_{i=1, i\neq i_p^*}^{N} K_i q_{i\mu}(\hat{y}_i(t_{p-1} - \eta_{p-1})),\ k \in [t_p, t_{p+1}-1], \end{aligned} \tag{8}$$

where i_p^* is the index of the active node at s_p and η_p is the communication delay. The closed-loop system with Round-Robin scheduling is modeled as a switched system:

$$\begin{aligned} x(k+1) &= Ax(k) + \sum_{j=1}^{N} A_{\theta(i,j)} x(t_{p-N+j} - \eta_{p-N+j}) \\ &\quad + \sum_{j=1}^{N} B_{\theta(i,j)} \omega_{\theta(i,j)}(k),\ k \in [t_p, t_{p+1}-1],\ i = 1,\ldots,N, \end{aligned} \tag{9}$$

where $A_{\theta(i,j)} = BK_{\theta(i,j)}C_{\theta(i,j)}$, $B_{\theta(i,j)} = BK_{\theta(i,j)}$,

$$p = \begin{cases} N\ell + i - 1, & \text{for } i \in \mathcal{I}\backslash\{N\},\ \ell \in \mathbb{N} \\ N\ell - 1, & \text{for } i = N,\ \ell \in \mathbb{N} \end{cases}$$

$$\theta(i,j) = \begin{cases} i + j, & \text{if } i + j \leq N, \\ i + j - N, & \text{if } i + j > N,\ j = 1,\ldots,N, \end{cases}$$

and

$$\begin{aligned} \omega_{\theta(i,j)}(k) &= q_{\theta(i,j)\mu}(y_{\theta(i,j)}(s_{p-N+j})) - y_{\theta(i,j)}(s_{p-N+j}), \\ &\quad i \in \mathcal{I},\ j = 1,\ldots,N, \end{aligned}$$

denote the quantization intervals. If $|y_{\theta(i,j)}(s_{p-N+j})| \leq \mu(k)M_{\theta(i,j)}$, then $|\omega_{\theta(i,j)}(k)| \leq \mu(k)\Delta_{\theta(i,j)}$, $i \in \mathcal{I}$, $j = 1,\ldots,N$, for $k \in [t_p, t_{p+1}-1]$.

We represent $t_{p-N+j} - \eta_{p-N+j} = k - \tau_j(k)$, $j = 1,\ldots,N$, where

$$\begin{aligned} &\tau_\vartheta(k) < \tau_{\vartheta-1}(k),\ \vartheta = 2,\ldots,N, \\ &\tau_\vartheta(k) = k - t_{p-N+\vartheta} + \eta_{p-N+\vartheta}, \\ &\tau_{\vartheta-1}(k) = k - t_{p-N+\vartheta-1} + \eta_{p-N+\vartheta-1}, \\ &\tau_j(k) \in [\eta_m, \tau_M^j],\quad k \in [t_p, t_{p+1}-1],\ j = 1,\ldots,N. \end{aligned} \tag{10}$$

Therefore, (9) can be considered as a system with N time-varying interval delays, where $\tau_\vartheta(k) < \tau_{\vartheta-1}(k)$, $\vartheta = 2,\ldots,N$.

The objective of this paper is to find an LMI-based time-triggered zooming algorithm (i.e., to choose a suitable time-varying parameter $\mu(k)$) for exponential stability of the switched system (9). To do so, we first present a lemma for ISS of system (9) with static quantization (i.e., $\mu(k) \equiv \mu$). This lemma plays a key role in achieving the main results.

C. ISS under Round-Robin scheduling and static quantization

Definition 1: The switched system (9) is said to be ISS if there exist constants $b > 0$, $0 < \kappa < 1$ and $b' > 0$ such that, for initial condition $x_{t_{N-1}} \in \underbrace{\mathbb{R}^n \times \cdots \times \mathbb{R}^n}_{\tau_M^1 + 1 \text{ times}}$ and for disturbances ω_i, $i = 1,\ldots,N$, the solutions of the switched system (9) satisfy

$$\begin{aligned} |x(k)|^2 &\leq b\kappa^{2(k - t_{N-1})}\|x_{t_{N-1}}\|_c^2 \\ &\quad + b' \max\{|\omega(t_{N-1})|^2, \ldots, |\omega(k)|^2\},\ k \geq t_{N-1}, \end{aligned}$$

where $\|x_{t_{N-1}}\|_c = \sup_{t_{N-1} - \tau_M^1 \leq s \leq t_{N-1}} |x(s)|$ and $\omega = \text{col}\{\omega_1,\ldots,\omega_N\}$.

Consider first static quantizers with a constant zoom variable $\mu(k) \equiv \mu$. We apply the following discrete-time Lyapunov functional to (9) with time-varying delay from the maximum delay interval $[\eta_m, \tau_M^1]$ [4]:

$$\begin{aligned} V(x_k) &= x^T(k)Px(k) + \sum_{s=k-\eta_m}^{k-1} \lambda^{k-s-1} x^T(s) S_0 x(s) \\ &\quad + \eta_m \sum_{j=-\eta_m}^{-1} \sum_{s=k+j}^{k-1} \lambda^{k-s-1} \eta^T(s) R_0 \eta(s) \\ &\quad + \sum_{s=k-\tau_M^1}^{k-\eta_m-1} \lambda^{k-s-1} x^T(s) S_1 x(s) \\ &\quad + (\tau_M^1 - \eta_m) \sum_{j=-\tau_M^1}^{-\eta_m-1} \sum_{s=k+j}^{k-1} \lambda^{k-s-1} \eta^T(s) R_1 \eta(s), \\ &\quad \eta(k) = x(k+1) - x(k), \\ &\quad P > 0,\ S_i > 0,\ R_i > 0,\ i = 0,1,\ 0 < \lambda < 1, \end{aligned} \tag{11}$$

where $x_k(j) \triangleq x(k+j)$, $j = -\tau_M^1,\ldots,-1,0$, and $x(k) = x_0$, $k = -\tau_M^1,\ldots,-1,0$. Following [4], we find conditions such that

$$V(x_{k+1}) - \lambda V(x_k) - \sum_{i=1}^{N} b_i |\omega_i(k)|^2 \leq 0,\ k \geq t_{N-1}, \tag{12}$$

holds, where $0 < \lambda < 1$, $b_i > 0$, $i = 1,\ldots,N$. Then we arrive at the following conditions to guarantee (12) and thus, for ISS of the switched system (9).

Lemma 1: Given scalar $0<\lambda<1$, positive integers $0\leq\eta_m<\tau_M^N$, and $K_i,\ i=1,\ldots,N$, assume that there exist scalars $b_i>0,\ i=1,\ldots,N$, $n\times n$ matrices $P>0,\ S_i>0,\ R_i>0,\ i=0,1,\ G^i_{\ell,\vartheta}\ i=1,\ldots,N,\ \ell=1,\ldots,N,\ \vartheta=2,\ldots,N+1,\ \ell<\vartheta$, such that the following LMIs are feasible:

$$\Omega_i=\begin{bmatrix} R_1 & * \\ (G^i_{\ell,\vartheta})^T & R_1\end{bmatrix}\geq 0, \tag{13}$$

$$\begin{bmatrix} \Psi & * & * \\ PF_0^i & -P & * \\ H(F_0^i-F_1) & 0 & -H\end{bmatrix}<0, \tag{14}$$

where

$$\begin{aligned}
&F_0^i=[A\ 0_{n\times n}\ A_{\theta(i,N)}\ldots A_{\theta(i,1)}\ 0_{n\times n}\ B_{\theta(i,N)}\ldots B_{\theta(i,1)}],\\
&F_1=[I_n\ 0_{n\times((N+2)n+n_y)}],F_2=[0_{n\times n}\ I_n\ 0_{n\times((N+1)n+n_y)}],\\
&\ldots,\ F_{N+3}=[0_{n\times((N+2)n)}\ I_n\ \ 0_{n\times n_y}],\\
&\Sigma=\mathrm{diag}\{S_0-\lambda P,-\lambda^{\eta_m}(S_0-S_1),0_{(Nn)\times(Nn)},-\lambda^{\tau_M^1}S_1,\\
&\quad -b_{\theta(i,N)}I_{\theta(i,N)},\cdots,-b_{\theta(i,1)}I_{\theta(i,1)}\},\\
&\Psi=\Sigma-\lambda^{\eta_m}(F_1-F_2)^TR_0(F_1-F_2)\\
&\quad-\lambda^{\tau_M^1}\sum_{i=2}^{N+2}(F_i-F_{i+1})^TR_1(F_i-F_{i+1})\\
&\quad-2\lambda^{\tau_M^1}\sum_{j=2}^{N+1}(F_j-F_{j+1})^T\sum_{s=j+1}^{N+2}G^i_{j-1,s-1}(F_s-F_{s+1}),\\
&H=\eta_m^2R_0+(\tau_M^1-\eta_m)^2R_1,\ i=1,\ldots,N.
\end{aligned} \tag{15}$$

Let $\mu>0$ be a constant and $|\omega_i(k)|\leq\mu\Delta_i,\ i=1,\ldots,N$. Then the solutions of the switched system (9) with the initial conditions $x_{t_{N-1}}\in\underbrace{\mathbb{R}^n\times\cdots\times\mathbb{R}^n}_{\tau_M^1+1\ \text{times}}$ satisfy the following inequalities:

$$V(x_k)\leq\lambda^{k-t_{N-1}}V(x_{t_{N-1}})+\frac{\mu^2}{1-\lambda}\sum_{i=1}^{N}b_i\Delta_i^2,\ k\geq t_{N-1}. \tag{16}$$

Proof: See Appendix A. □

In order to derive a bound on $V(t_{N-1})$ in terms of x_0 in a simple way, we suggest waiting for all the N latest transmitted measurements $q_{1\mu}(y_1(s_0)),\ q_{2\mu}(y_2(s_1)),\ldots,\ q_{N\mu}(y_N(s_{N-1}))$ on the controller side and then sending them together to the actuator side. This is a reasonable approach which can be easily implemented. Then for $k=0,1,\ldots,t_{N-1}-1$, (1) is given by

$$x(k+1)=Ax(k),\ k=0,1,\ldots,t_{N-1}-1. \tag{17}$$

Remark 1: A common Lyapunov functional (11) has been applied to the switched system (9) to derive sufficient conditions for ISS. It should be pointed out that the multiple Lyapunov functional method and dwell time approach can be utilized to find a suitable switching signal to improve performance [8], [27].

III. Main Results: dynamic quantization of NCSs under Round-Robin scheduling

In the following, based on ISS of system (9), we present the main results on dynamic quantization of NCSs in the presence of Round-Robin scheduling. By defining the initial and level sets in Section III-A, in Section III-B we propose an LMI-based time-triggered zooming algorithm for exponential stability of the switched system (9). In Section III-C, we develop a novel Lyapunov-based method to initialize the zoom parameter. Under the Round-Robin protocol scheduling, digit "1" is transmitted in the protocol along with the measurements at the zooming-in sampling instants (otherwise, digit "0" is transmitted). Thus, on the controller side it is known whether the zoom variable μ of the received measurement is updated or not. Once the value of μ is updated, all N latest transmitted measurements are waited on the controller side and then sent together to the actuator side.

A. Initial and level sets

Given positive numbers σ and ρ, the initial and level sets are defined as

$$\begin{aligned}\mathcal{S}_\sigma=\{x_{t_{N-1}}\in\underbrace{\mathbb{R}^n\times\cdots\times\mathbb{R}^n}_{\tau_M^1+1\ \text{times}}:V(x_{t_{N-1}})<\sigma,\\ x^T(k)Px(k)<\sigma,\ k\in[t_{N-1}-\eta_M,t_{N-1}]\}\end{aligned} \tag{18}$$

and

$$\mathcal{X}_{k^*,\rho}=\{x_k\in\underbrace{\mathbb{R}^n\times\ldots\times\mathbb{R}^n}_{\tau_M^1+1\ \text{times}}:V(x_k)<\rho,k=k^*,k^*+1,\ldots\},$$

respectively. Given positive numbers μ, M_0, $\beta<1$ and $\nu<1$, the following lemma ensures that all solutions of (9) with $x_{t_{N-1}}\in\mathcal{S}_{\mu^2M_0^2}$ stay inside the region $\mathcal{X}_{t_{N-1},(1+\beta\nu^2)\mu^2M_0^2}$ for all $k\geq t_{N-1}$, and enter a smaller region $\mathcal{X}_{t_{N-1}+\mathrm{T},\nu^2\mu^2M_0^2}$ in a finite time T.

Lemma 2: Given $M_j>0,\ j=0,1,\ldots,N,\ \Delta_i>0,\ i=1,\ldots,N$, positive integers $0\leq\eta_m<\tau_M^N$ and tuning parameters $0<\lambda<1,\ 0<\nu<1$, assume that there exist scalars $0<\beta<1,\ b_i,\ i=1,\ldots,N$, $n\times n$ matrices $P>0$, $S_i>0,\ R_i>0,\ i=0,1,\ G^i_{\ell,j}\ i=1,\ldots,N,\ \ell=1,\ldots,N,\ j=2,\ldots,N+1,\ \ell<j$, such that LMIs (13)–(14) and

$$(1+\beta\nu^2)M_0^2C_i^TC_i<PM_i^2,\ i=1,\ldots,N, \tag{19}$$

$$\frac{1}{1-\lambda}\sum_{i=1}^{N}b_i\Delta_i^2<\beta\nu^2M_0^2 \tag{20}$$

hold. Let $\mu>0$ be a constant. Then the solutions of (9) that start in the region $\mathcal{S}_{\mu^2M_0^2}$

(i) satisfy $|C_ix(t_p-\eta_p)|=|y_i(t_p-\eta_p)|<\mu M_i,\ p\in\mathbb{Z}^+$, (implying $|\omega_i(k)|\leq\mu\Delta_i$ for all $i\in\mathcal{I}$ and $k=t_{N-1},t_{N-1}+1,\ldots$);

(ii) remain in the set $\mathcal{X}_{t_{N-1},(1+\beta\nu^2)\mu^2M_0^2}$;

(iii) enter a smaller set $\mathcal{X}_{t_{N-1}+\mathrm{T},\nu^2\mu^2M_0^2}$ in a finite time $\mathrm{T}=\lceil\tilde{\mathrm{T}}\rceil$, where $\tilde{\mathrm{T}}$ is the solution of

$$\lambda^{\tilde{\mathrm{T}}}=(1-\beta)\nu^2. \tag{21}$$

The proof of Lemma 2 follows from [17]. The second inequality in (18) allows us to guarantee the bounds on $y(s_p),\ s_p<t_{N-1}$ by verifying (19).

Remark 2: The functional $V(x_k)$ is a standard Lyapunov functional for delay-dependent analysis. The LMIs of Lemma 2 are feasible for small enough delay bound τ_M^N, large enough quantization ranges $M_1,\ldots,M_N$ and small enough

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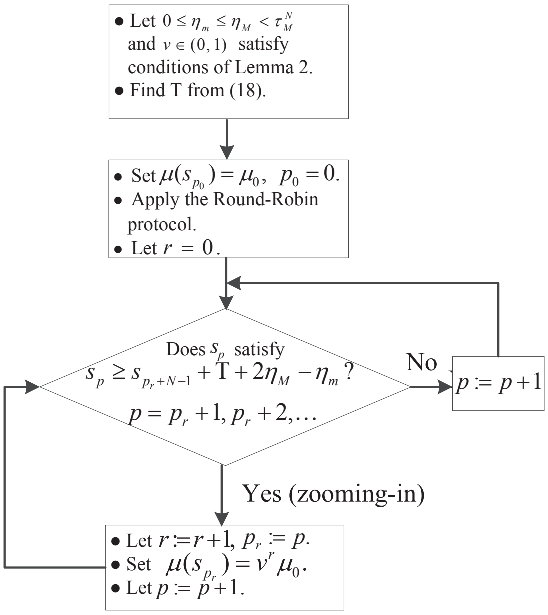

Fig. 2. The algorithm for dynamic quantization and Round-Robin scheduling

quantization intervals $\Delta_1, \ldots, \Delta_N$. Indeed, the LMIs (13) and (14) are feasible for $\tau_M^N = 0$ (i.e., in the absence of delay) since $A + BKC$ is Schur stable. Hence, (13) and (14) are feasible for small enough τ_M^N. The LMIs (19) and (20) are feasible for large enough quantization ranges and small enough quantization intervals. Moreover, the initial values of λ and ν can be set to be 1. It is noted that the conditions are sufficient only and always may be improved.

B. Dynamic quantization and zooming algorithm

In this section, we consider dynamic quantizers with the zoom variable μ. Zooming is performed on the sensor level. Therefore, in the closed-loop system $\mu = \mu(s_p)$ is constant on $[t_p, t_{p+1} - 1]$.

Given $\mu_0 > 0$, let $\mu = \mu_0$, $x_{t_{N-1}} \in \mathcal{S}_{\mu^2 M_0^2} = \mathcal{S}_{\mu_0^2 M_0^2}$. We will show how to choose μ_0 in Theorem 1 below. Assume that LMIs of Lemma 2 are feasible. In the sequel, we propose a zooming-in algorithm in Figure 2, where μ is decreased and thus, the resulting quantization interval is reduced to drive the state of (9) to the origin exponentially.

Definition 2: If there exist constants $b > 0$ and $0 < \kappa < 1$ such that

$$|x(k)|^2 \le b\kappa^{2(k-t_{N-1})}\mu_0^2 M_0^2, \quad \forall k \ge t_{N-1},\ k \in \mathbb{N},$$

for the solutions of the system (9) initialized with $x_{t_{N-1}} \in \mathcal{S}_{\mu_0^2 M_0^2}$, then the system (9) with $|\omega_i(k)| \le \mu\Delta_i$, $i = 1, \ldots, N$, is said to be exponentially stable with decay rate κ for some choice of the zoom variable $\mu > 0$.

Proposition 1: Assume that the LMIs of Lemma 2 are feasible. Given $\mu_0 > 0$, let $\mu = \mu_0$, $x_{t_{N-1}} \in \mathcal{S}_{\mu_0^2 M_0^2}$. Then under the algorithm in Figure 2, the system (9) is exponentially stable with a decay rate $\kappa = \nu^{\frac{1}{\bar\tau_M}}$, where

$$\bar\tau_M = \mathrm{T} + N\tau_M^N + 2\eta_M - N\eta_m - \eta_m + N. \qquad (22)$$

Proof: Set $r = 0$. Since

$$t_{p_1} - \eta_M = s_{p_1} + \eta_{p_1} - \eta_M \ge t_{N-1} + \mathrm{T} + \eta_{p_1} - \eta_m \ge t_{N-1} + \mathrm{T},$$

application of Lemma 2 with $\mu = \mu_0$ leads to

$$x^T(k)Px(k) \le V(x_k) < \nu^2\mu_0^2 M_0^2,\ \forall k \ge t_{p_1} - \eta_M,\ k \in \mathbb{N}.$$

Set $r = 1$. We wait for all N latest transmitted measurements to arrive into the reduced domain with $x^T(s_p)C_i^T C_i x(s_p) < \mu_0^2\nu^2 M_i^2$, $i = 1, \ldots, N$ for $p \ge p_1$, where $|\omega_i(k)| \le \mu_0\nu\Delta_i$, $k \ge t_{p_1}$, $k \in \mathbb{N}$. After sampling instant s_{p_1+N-1}, the resulting closed-loop system has initial condition

$$x_{t_{p_1+N-1}} \in \underbrace{\mathbb{R}^n \times \cdots \times \mathbb{R}^n}_{\tau_M^1 + 1 \text{ times}} : V(x_{t_{p_1+N-1}}) < \nu^2\mu_0^2 M_0^2. \qquad (23)$$

Then Lemma 2 is applied with $\mu = \mu_0\nu$, where t_{N-1} and η_{N-1} are changed by t_{p_1+N-1} and η_{p_1+N-1}, respectively. Thus the solutions of (9) initiated by (23) remain in a region $\mathcal{X}_{t_{p_1+N-1},(1+\beta\nu^2)\nu^2\mu_0^2 M_0^2}$ for all $k \ge t_{p_1+N-1}$, $k \in \mathbb{N}$. Since $s_{p_1+N-1} = t_{p_1+N-1} - \eta_{p_1+N-1} \ge t_{p_1+N-1} - \eta_M$, from (19) it follows that

$$x^T(s_p)C_i^T C_i x(s_p) < \frac{x^T(s_p)Px(s_p) \cdot \nu^2\mu_0^2 M_i^2}{(1+\beta\nu^2)\nu^2\mu_0^2 M_0^2} < \nu^2\mu_0^2 M_i^2,$$
$$i = 1, \ldots, N,\ \ \forall p \ge p_1 + N - 1,$$

and thus, $|\omega_i(k)| \le \nu\mu_0\Delta_i, i = 1, \ldots, N,\ k \ge t_{p_1+N-1} = s_{p_1+N-1} + \eta_{p_1+N-1}$. Therefore, for $k \ge t_{p_2} - \eta_M \ge t_{p_1+N-1} + \mathrm{T}$,

$$\begin{aligned} V(x_k) &\le \lambda^{k - t_{p_1+N-1}} V(x_{t_{p_1+N-1}}) + \frac{\nu^2\mu_0^2}{1-\lambda}\sum_{i=1}^N b_i\Delta_i^2 \\ &\le \lambda^{\mathrm{T}} V(x_{t_{p_1+N-1}}) + \frac{\nu^2\mu_0^2}{1-\lambda}\sum_{i=1}^N b_i\Delta_i^2 \\ &< (1-\beta)\nu^2 \cdot \nu^2\mu_0^2 M_0^2 + \beta\nu^2\mu_0^2 M_0^2 \cdot \nu^2 \\ &= \nu^4\mu_0^2 M_0^2. \end{aligned}$$

Similarly, for $r = 2, 3, \ldots$ we have $V(x_k) < \nu^{2r}\mu_0^2 M_0^2$ for all $k \in [t_{p_r} - \eta_M, t_{p_{r+1}} - \eta_M - 1]$. Noting that

$$\begin{aligned} &r\mathrm{T} + (r-1)(2\eta_M - \eta_m + N - 1) + t_{N-1} \le t_{p_r} - \eta_M \\ &\le k \le t_{p_{r+1}} - \eta_M - 1 < (r+1)\bar\tau_M - 1 + t_{N-1}, \end{aligned}$$

we obtain

$$\begin{aligned} V(x_k) &< \nu^{2r}\mu_0^2 M_0^2 < \nu^{2\left(\frac{k - t_{N-1}}{\bar\tau_M} - \frac{\bar\tau_M - 1}{\bar\tau_M}\right)}\mu_0^2 M_0^2 \\ &= \nu^{-\frac{2(\bar\tau_M - 1)}{\bar\tau_M}}\left(\nu^{\frac{1}{\bar\tau_M}}\right)^{2(k - t_{N-1})}\mu_0^2 M_0^2, \\ &\quad k \in [t_{p_r} - \eta_M, t_{p_{r+1}} - \eta_M - 1],\ r \in \mathbb{N}. \end{aligned}$$

Then the following holds for $k \ge t_{N-1}$

$$|x(k)|^2 \le \nu^{-\frac{2(\bar\tau_M - 1)}{\bar\tau_M}}[\lambda_{min}(P)]^{-1}\left(\nu^{\frac{1}{\bar\tau_M}}\right)^{2(k - t_{N-1})}\mu_0^2 M_0^2. \ \square$$

Remark 3: In the above analysis, it is assumed that data packet dropout does not occur. However, for small delays $\eta_p < s_{p+1} - s_p$, if the number of successive packet dropouts is upper bounded by $\bar d$, in the presence of Round-Robin scheduling we could accommodate for packet dropouts by modeling them as prolongations of the transmission interval and replace T by $\mathrm{T} + 2\bar d \cdot \mathrm{MATI}$ in the algorithm.

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C. Initialization of the zoom variable

The algorithm of the previous section is given in terms of the initial set $\mathcal{S}_{\mu_0^2 M_0^2}$ that involves the bound on $V(x_{t_{N-1}})$. In this section, we find the ball of initial conditions $x(0) = x_0$, starting from which the solutions of (9) and (17) remain in the initial set $\mathcal{S}_{\mu_0^2 M_0^2}$. From (5) and the bound

$$\begin{aligned} s_{N-2} &= s_{N-2} - s_{N-3} + s_{N-3} - \cdots + s_1 - s_0 \\ &\leq (N-2)(\tau_M^N - \eta_m + 1), \end{aligned}$$

it holds that

$$\begin{aligned} t_{N-1} &\leq s_{N-2} + \tau_M^N + 1 \\ &\leq (N-2)(\tau_M^N - \eta_m + 1) + \tau_M^N + 1 \triangleq \hat{\tau}_M. \end{aligned} \tag{24}$$

Then following [15], we derive a bound on $V(x_{t_{N-1}})$ in terms of x_0 in the next lemma:

Lemma 3: [15] Consider Lyapunov functional $V(x_k)$ given by (11) and denote $V_0(k) = x^T(k)Px(k)$. Under the constant initial condition $x(k) = x_0,\ k < 0$, if there exist $0 < \lambda < 1$ and $c > 1$ such that the following inequalities

$$V_0(k+1) - cV_0(k) \leq 0, \tag{25a}$$

$$V(x_{k+1}) - \lambda V(x_k) - (c-1)V_0(k) \leq 0, \tag{25b}$$

hold for $k = 0, 1, \ldots, t_{N-1} - 1$ along (17), then we have

$$\begin{aligned} &V_0(k) \leq \lambda_{\max}(c^{\hat{\tau}_M} P)|x_0|^2,\ k = 0, 1, \ldots, t_{N-1}, \\ &V(x_{t_{N-1}}) \leq \lambda_{\max}(c^{\hat{\tau}_M} P + \Omega)|x_0|^2, \end{aligned} \tag{26}$$

where $\hat{\tau}_M$ is given by (24) and

$$\Omega = \eta_m S_0 + \lambda^{\eta_m}(\tau_M^1 - \eta_m)S_1. \tag{27}$$

As a consequence, we achieve our main result:

Theorem 1: Given $M_j > 0,\ j = 0, 1, \ldots, N,\ \Delta_i > 0,\ i = 1, \ldots, N$, positive integers $0 \leq \eta_m \leq \eta_M < \tau_M^N$ and tuning parameters $0 < \lambda < 1,\ 0 < \nu < 1,\ c > 1$, assume that there exist scalars $0 < \beta < 1,\ b_i,\ i = 1, \ldots, N,\ n \times n$ matrices $P > 0,\ S_i > 0,\ R_i > 0,\ i = 0, 1,\ G^i_{\ell,\vartheta}\ i = 1, \ldots, N,\ \ell = 1, \ldots, N,\ \vartheta = 2, \ldots, N+1,\ \ell < \vartheta$, such that the LMIs (13)–(14), (19)–(20) and the following LMIs are feasible:

$$\begin{bmatrix} -cP & * \\ PA & -P \end{bmatrix} < 0, \tag{28}$$

$$\begin{bmatrix} \tilde{\Psi} & * & * \\ P\tilde{F}_0 & -P & * \\ H(\tilde{F}_0 - \tilde{F}_1) & 0 & -H \end{bmatrix} < 0, \tag{29}$$

where

$$\begin{aligned} &\tilde{F}_0 = [A\ 0_{n\times((N+2)n)}],\ \tilde{F}_1 = [I_n\ 0_{n\times((N+2)n)}], \\ &\tilde{F}_2 = [0_{n\times n}\ I_n\ \ 0_{n\times((N+1)n)}], \ldots, \tilde{F}_{N+3} = [0_{n\times((N+2)n)}\ I_n], \\ &\tilde{\Sigma} = \mathrm{diag}\{S_0 - \lambda P - (c-1)P, -\lambda^{\eta_m}(S_0 - S_1), \\ &\qquad 0_{(Nn)\times(Nn)}, -\lambda^{\tau_M^1} S_1\}, \\ &\tilde{\Psi} = \tilde{\Sigma} - \lambda^{\eta_m}(\tilde{F}_1 - \tilde{F}_2)^T R_0(\tilde{F}_1 - \tilde{F}_2) \\ &\quad -\lambda^{\tau_M^1} \sum_{i=2}^{N+2} (\tilde{F}_i - \tilde{F}_{i+1})^T R_1(\tilde{F}_i - \tilde{F}_{i+1}) \\ &\quad -2\lambda^{\tau_M^1} \sum_{j=2}^{N+1} (\tilde{F}_j - \tilde{F}_{j+1})^T \sum_{s=j+1}^{N+2} G^1_{j-1,s-1}(\tilde{F}_s - \tilde{F}_{s+1}), \end{aligned} \tag{30}$$

and the notation H is given by (15). If the initial condition satisfies the inequality $|x_0| < X_0$, where $X_0 > 0$ is known, then the zooming-in algorithm of Section III-B starting with $\mu(s_0) = \mu_0$ with μ_0 given by

$$\mu_0^2 = \frac{\lambda_{\max}(c^{\hat{\tau}_M} P + \Omega)}{M_0^2} X_0^2 \tag{31}$$

exponentially stabilizes system (9) and (17), where $\hat{\tau}_M$ and Ω are given by (24) and (27), respectively.

Proof: From [15], it follows that the matrix inequalities (13), (28) and (29) guarantee (25) along (17) for $k = 0, 1, \ldots, t_{N-1} - 1$. Therefore, if the initial condition satisfies the inequality $|x_0| < X_0$, then

$$\begin{aligned} \max\{V_0(k), V(x_{t_{N-1}})\} &\leq \lambda_{\max}(c^{\hat{\tau}_M} P + \Omega)X_0^2 \\ &= \mu_0^2 M_0^2,\ k = 0, 1, \ldots, t_{N-1}, \end{aligned}$$

meaning that $x_{t_{N-1}} \in \mathcal{S}_{\mu_0^2 M_0^2}$. The result then follows from Proposition 1. □

Remark 4: Note that given a bound $X_0 > 0$ on the state initial conditions and the values of the quantizer range $M_i > 0$ and interval $\Delta_i > 0,\ i = 1, \ldots, N$, the equation (31) defines the initial value of the zoom variable, starting from which the exponential stability is guaranteed by using zooming-in only. If the initial value of the zoom variable is given by μ_0, then the zooming-in algorithm of Section III-B starting with $\mu(s_0) = \mu_0$ exponentially stabilizes all the solutions of (9) and (17) starting from the initial ball

$$|x_0| < X_0, \quad X_0 = \frac{\mu_0 M_0}{\sqrt{\lambda_{max}(c^{\hat{\tau}_M} P + \Omega)}}, \tag{32}$$

where $\hat{\tau}_M$ and Ω are given by (24) and (27), respectively. In order to maximize the initial ball (32), the condition $c^{\hat{\tau}_M} P + \Omega - \gamma I < 0$ can be added to the conditions of Theorem 1, where $\gamma > 0$ is to be minimized.

Remark 5: The conditions of Theorem 1 possess $(N+1)$ of $2n \times 2n$, one of $(N+5)n \times (N+5)n$, N of $((N+5)n + n_y) \times ((N+5)n + n_y)$ LMIs, and have the number $\frac{N^2(N+1)+5}{2} n^2 + 2.5n + N + 1$ of decision variables. The huge numerical complexity is caused by the switched closed-loop system (9) composed of N subsystems.

Remark 6: The LMIs of Theorem 1 are affine in the system matrices. Therefore, in the case of system matrices from the uncertain time-varying polytope

$$\Theta = \sum_{j=1}^{M} g_j(k)\Theta_j,\ 0 \leq g_j(k) \leq 1,$$

$$\sum_{j=1}^{M} g_j(k) = 1,\ \Theta_j = \begin{bmatrix} A^{(j)} & B^{(j)} \end{bmatrix},$$

where $g_j(k),\ j = 1, \ldots, M$, are uncertain time-varying parameters and the system matrices $A^{(j)}$ and $B^{(j)},\ j = 1, \ldots, M$, are known with appropriate dimensions, one have to solve these LMIs simultaneously for all the M vertices Θ_j, applying the same decision matrices.

Remark 7: The time-delay system approach has been developed in [16] and [18] for NCSs with non-quantized measurements under try-once-discard protocol and under stochastic protocol, respectively. The proposed zooming algorithm in the present paper for Round-Robin protocol could be extended

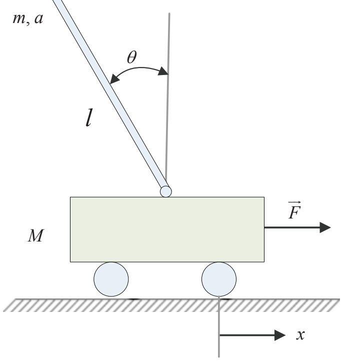

Fig. 3. Inverted pendulum system.

to the case of try-once-discard and stochastic protocols. In addition, to facilitate the static output-feedback controller design, it will be useful to eliminate the coupling between the Lyapunov matrices and system matrices. To this end, one may resort to the methods proposed in e.g., [24], and [35].

Remark 8: In a particular case of $N = 1$, the achieved conditions could be applied to the output tracking control that was studied in [14] and [31] for complex industrial processes. The discrete-time system theory for sampled-data control was applied in [14] and [31], whereas a time-delay approach is adopted in the present paper.

IV. Illustrative Examples

A. Example 1: inverted pendulum

The inverted pendulum system is widely used as a benchmark for testing control algorithm. The dynamics of the inverted pendulum on a cart shown in Figure 3 can be described in the following as in e.g., [36]:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{(a+ml^2)b}{a(M+m)+Mml^2} & -\frac{m^2gl^2}{a(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{mlb}{a(M+m)+Mml^2} & -\frac{mgl(M+m)}{a(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{a+ml^2}{a(M+m)+Mml^2} \\ 0 \\ \frac{ml}{a(M+m)+Mml^2} \end{bmatrix} u \tag{33}$$

with $M = 1.096\text{kg}$, $m = 0.109\text{kg}$, $l = 0.25\text{m}$, $g = 9.8\text{m/s}^2$, $a = 0.0034\text{kg} \cdot \text{m}^2$ and $b = 0.1\text{N/m/sec}$. In the model, x, θ, a and b represent cart position coordinate, pendulum angle from vertical, the friction of the cart and inertia of the pendulum, respectively.

By choosing a sampling time $\text{T}_s = 0.01s$, we obtain the following discrete-time system model:

$$x(k+1) = \begin{bmatrix} 1 & 0.01 & 0 & 0 \\ 0 & 0.9991 & 0.0063 & 0 \\ 0 & 0 & 1.0014 & 0.01 \\ 0 & -0.0024 & 0.2784 & 1.0014 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.0088 \\ 0.0001 \\ 0.0236 \end{bmatrix} u(k), \; k \in \mathbb{Z}^+. \tag{34}$$

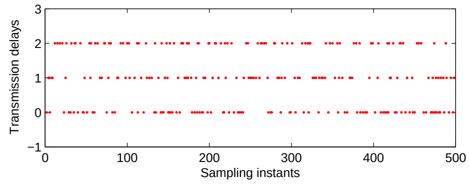

Fig. 4. Example 1: transmission delays

The pendulum can be stabilized by a state feedback $u(k) = Kx(k)$ with the gain $K = [K_1 \;\; K_2]$

$$K_1 = \begin{bmatrix} 0.9163 & 2.0169 \end{bmatrix}, \; K_2 = \begin{bmatrix} -27.4850 & -5.3437 \end{bmatrix},$$

which leads to the closed-loop system having eigenvalues $\{0.9419, 0.9865 + 0.0035i, \; 0.9865 - 0.0035i, \; 0.9813\}$. Suppose that the spatially distributed components of the state of the cart-pendulum system (34) are not accessible simultaneously.

Consider $N = 2$ and

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{35}$$

The quantizer is chosen as

$$q_\mu(y^i) = \begin{cases} 100\mu \,\text{sgn}(y^i), & \text{if } |y^i| > 100\mu, \\ \mu \left\lfloor \frac{y^i}{\mu} + 0.01 \right\rfloor, & \text{if } |y^i| \le 100\mu, \end{cases}$$

where y^i is the ith component of y, $i = 1, \dots, 4$. Therefore, we can take $M_1 = M_2 = 100$, $\Delta_1 = \Delta_2 = 0.01$. Choose $\mu_0 = 1$, $M_0 = 100$, $\nu = 0.9$, $\lambda = 0.984$, $c = 1.37$, $\tau_M^N = 4$, $\eta_m = 0$, $\eta_M = 2$. Then from (5), it follows that the network-induced delays η_p and the sampling intervals are bounded by $0 \le \eta_p \le 2$ and $1 \le s_{p+1} - s_p \le 3$, $p \in \mathbb{Z}^+$, respectively. It is observed that we allow network-induced delays larger than the sampling intervals.

The initial state is assumed to be $x_0 = [0.5 \; 0.3 \; -0.2 \; -0.9]^T$. In the simulation, the network-induced delays are generated randomly according to the aforementioned assumption, and shown in Figure 4. By Theorem 1 we find $\text{T} = \lceil -\frac{\ln(1-\beta)+2\ln\nu}{\ln\lambda} \rceil = 15$ from (21). Then the zooming-in algorithm of Section III-B and Round-Robin protocol with $\text{T} = 15$ and $\nu = 0.9$ exponentially stabilizes all the solutions of (9) and (17) starting from the initial ball $|x_0| < 1.2376$. Figure 5 shows the evolution of the zoom variable $\mu(k)$.

Moreover, it is found that the system is exponentially stable with a decay rate $\kappa = \nu^{\frac{1}{\bar{\tau}_M}} = 0.9964$, where (following (22)) $\bar{\tau}_M = \text{T} + 2\tau_M^N + 2\eta_M - 3\eta_m + 2 = 29$. The evolution of the control input and the state are depicted in Figure 6.

For the case of $N = 1$, i.e., the scheduling is not taken into account and $y(k) = [C_1^T \;\; C_2^T]^T x(k)$, we achieve a slightly better $\kappa = 0.9950$ for essentially larger initial ball $|x_0| < 11.1951$.

B. Example 2: quadruple-tank process

We also illustrate the efficiency of the given conditions on the example of the quadruple-tank process [10] described in

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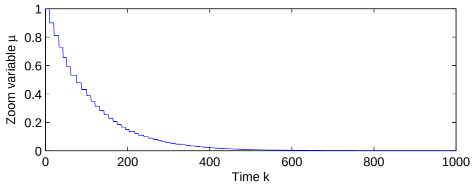
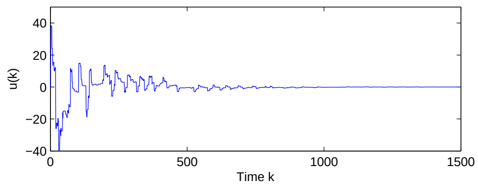
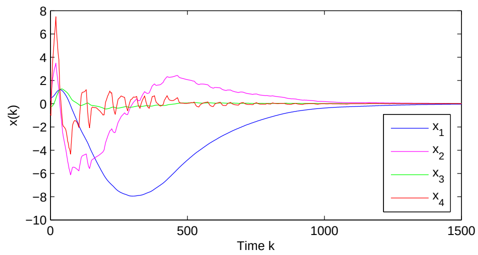


Fig. 5. Example 1: evolution of the zoom variable μ in the zooming-in algorithm



(a)



(b)

Fig. 6. Example 1: (a) evolution of the control input; (b) trajectory of the closed-loop system

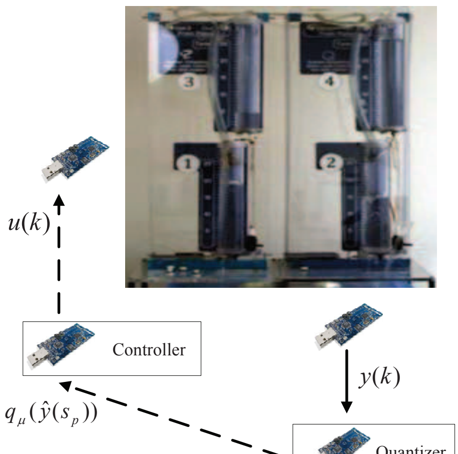


Fig. 7. Schematic diagram of the quadruple-tank process.

Figure 7. The linear discrete-time model obtained in [29] is given by:

$$x(k+1) = \begin{bmatrix} 0.975 & 0 & 0.042 & 0 \\ 0 & 0.977 & 0 & 0.044 \\ 0 & 0 & 0.958 & 0 \\ 0 & 0 & 0 & 0.956 \end{bmatrix} x(k) + \begin{bmatrix} 0.0515 & 0.0016 \\ 0.0019 & 0.0447 \\ 0 & 0.0737 \\ 0.0850 & 0 \end{bmatrix} u(k), \ k \in \mathbb{Z}^+. \tag{36}$$

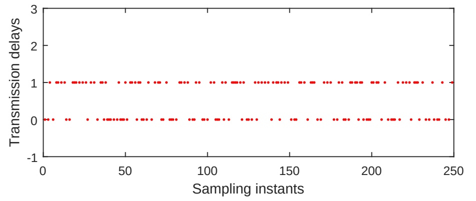


Fig. 8. Example 2: transmission delays

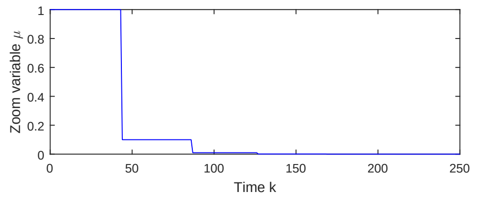


Fig. 9. Example 2: evolution of the zoom variable μ in the zooming-in algorithm

Here, the open-loop system is exponentially stable with a decay rate $\kappa = 0.9770$.

Consider $N = 2$ and choose the controller gain $K = [K_1 \quad K_2]$, where

$$K_1 = \begin{bmatrix} 0.0449 & -0.3007 \\ -0.3080 & 0.0469 \end{bmatrix}, K_2 = \begin{bmatrix} 0.1651 & -0.5644 \\ -0.6275 & 0.1681 \end{bmatrix}.$$

The measurement outputs are $y_i(k) = C_i x(k)$ with C_i, $i = 1, 2$, given by (35). Suppose that the components of the state of system (36) are not accessible simultaneously. The quantizer is chosen as

$$q_\mu(y^i) = \begin{cases} 150\mu \ \mathrm{sgn}(y^i), & \text{if } |y^i| > 150\mu, \\ \mu \left\lfloor \frac{y^i}{\mu} + 0.001 \right\rfloor, & \text{if } |y^i| \le 150\mu, \end{cases}$$

where y^i is the ith component of y, $i = 1, \ldots, 4$. Therefore, we can take $M_1 = M_2 = 150$, $\Delta_1 = \Delta_2 = 0.001$.

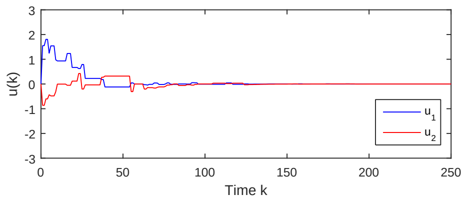
Choose $\tau_M^N = 2$, $\eta_m = 0$, $\eta_M = 1$. From (5), it follows that the network-induced delays η_p and the sampling intervals are bounded by $0 \le \eta_p \le 1$ and $1 \le s_{p+1} - s_p \le 2$, $p \in \mathbb{Z}^+$, respectively. The network-induced delays are depicted in Figure 8.

Then we find that given $\mu_0 = 1$, $M_0 = 100$, $\nu = 0.1$, $\lambda = 0.926$, $c = 1.10$, the zooming-in algorithm of Section III-B and Round-Robin protocol with $\mathrm{T} = 60$ exponentially stabilizes all the solutions of (9) and (17) starting from the initial ball $|x_0| < 23.7748$ with a decay rate $\kappa = 0.9667$. The decay rate for closed-loop system is improved compared to the one for the open-loop system. The evolution of the zoom variable $\mu(k)$ is presented in Figure 9. The evolution of the control input and the state with the initial state $x_0 = [5 \quad 2 \quad -2 \quad -4]^T$ are given in Figure 10.

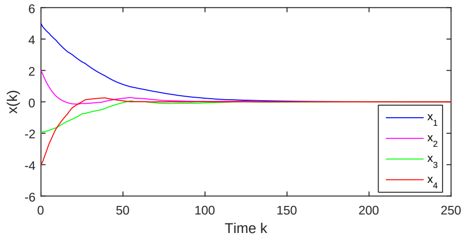
Moreover, for the case of $N = 1$, it is shown that the zooming-in algorithm with $\mathrm{T} = 60$ exponentially stabilizes all the solutions of the closed-loop system (9) and (17) ($N = 1$) starting from a larger initial ball $|x_0| < 91.4413$ with a slightly better decay rate $\kappa = 0.9647$.

V. Conclusions

This paper has investigated linear discrete-time NCSs that are subject to dynamic quantization, variable communication



(a)



(b)

Fig. 10. Example 2: (a) evolution of the control input; (b) trajectory of the closed-loop system

delays, variable sampling intervals and Round-Robin scheduling. An LMI-based time-triggered zooming algorithm, which includes proper initialization of the zoom parameter has been proposed for exponential stability of the switched closed-loop system. The interesting future research may include quantized input, stochastic communication delays and dynamic scheduling protocols.

APPENDIX A

Proof of Lemma 1: Given $i \in \mathcal{I}$, consider $k \in [t_p, t_{p+1} - 1]$, $k \in \mathbb{Z}_+$ and define $\xi(k) = \text{col}\{x(k), x(k-\eta_m), x(k-\tau_N(k)), \cdots, x(k-\tau_1(k)), x(k-\tau_M^1), \omega_1(k), \cdots, \omega_N(k)\}$. Applying Cauchy-Schwartz inequality, taking advantage of the ordered delays and using convex analysis [22], we have

$$V(x_{k+1}) - \lambda V(x_k) - \sum_{i=1}^{N} b_i|\omega_i(k)|^2 \leq \xi^T(k)[\Psi + (F_0^i)^T P F_0^i + (F_0^i - F_1)^T H (F_0^i - F_1)]\xi(k) \leq 0, \tag{37}$$

if $\Psi + (F_0^i)^T P F_0^i + (F_0^i - F_1)^T H (F_0^i - F_1) < 0$, i.e., by Schur complement, if (14) is feasible.

Since $|\omega_i(k)| \leq \mu\Delta_i$, $i = 1, \ldots, N$, the inequality (37) implies for $k \in [t_p, t_{p+1} - 1]$

$$\begin{aligned} V(x_k) &\leq \lambda V(x_{k-1}) + \mu^2 \sum_{i=1}^{N} b_i \Delta_i^2 \\ &\vdots \\ &\leq \lambda^{k-t_{N-1}} V(x_{t_{N-1}}) + \frac{\mu^2}{1-\lambda} \sum_{i=1}^{N} b_i \Delta_i^2, \end{aligned}$$

that completes the proof. □

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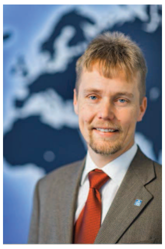

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