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Robust predictive extended state observer for a class of nonlinear systems with time-varying input delay

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ABSTRACT
This paper deals with asymptotic stabilisation of a class of nonlinear input-delayed systems via dynamic output feedback in the presence of disturbances. The proposed strategy has the structure of an observer-based control law, in which the observer estimates and predicts both the plant state and the external disturbance. A nominal delay value is assumed to be known and stability conditions in terms of linear matrix inequalities are derived for fast-varying delay uncertainties. Asymptotic stability is achieved if the disturbance or the time delay is constant. The controller design problem is also addressed and a numerical example with an unstable system is provided to illustrate the usefulness of the proposed strategy.

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Time-varying delay; nonlinear systems; output feedback; robust control; linear matrix inequality

1. Introduction

Time delays are ubiquitous in many engineering applications such as chemical or biological processes, oil or gas factories, and networked control. Large delays often lead to closed-loop instability if they are not taken into account and limit the achievable performance of conventional controllers (Fridman, 2014a).

When dealing with stabilisation of input-delayed systems, predictive feedback plays a key role. For the linear SISO case, the stabilisation problem in the presence of input/output delays can be solved by the so-called Smith Predictor (Smith, 1957), which is actually a predictive model-based controller formulated in the frequency domain. In the time domain, similar control strategies have been proposed (Artstein, 1982; Manitius & Olbrot, 1979), even for nonlinear time-varying systems (Bekiaris-Liberis & Krstic, 2012), all of them requiring a state predictor. However, in many cases, little attention is devoted to the predictor implementation, often assuming the stability of the open-loop process. This has been a matter of concern for some researchers (Engelborghs, Dambrine, & Roose, 2001; Mondié & Michiels, 2003; Zhong, 2004), as the discretisation of the integral terms involved may lead to instability of the closed-loop. For nonlinear systems, the implementation may be even more challenging as it requires the on-line integration of nonlinear functions. See the recent monograph (Karafyllis & Krstic, 2017).

In order to avoid integral terms in the control law, the idea of a predictor in observer form has been receiving increasing attention. It was first introduced in Besançon, Georges, and Benayache (2007) for systems with small input delays and extended in Najafi, Hosseinnia, Sheikholeslam, and Karimadini (2013) and Najafi, Sheikholeslam, Wang, and Hosseinnia (2014) to larger delays by using the cascade observer structure initiated in Germani, Manes, and Pepe (2002). The idea is to use a chain of observers so that each of them predicts the state over a fraction of the delay. This is known in the literature as the sequential predictors technique. Recently, this methodology has been exploited by some researchers (Ahmed-Ali, Cherrier, & Lamnabhi-Lagarrigue, 2012; Léchappé, Moulay, & Plestan, 2016; Mazenc & Malisoff, 2016; Sanz, Garcia, Fridman, & Albertos, 2018). However, in the context of sequential predictors, nonlinearities have not been addressed in any of the aforementioned works.

Disturbance rejection is also a central issue in process control, especially challenging for time-delay systems. Several works devoted to improving disturbance rejection of state predictors have been reported recently in the literature. The inverse optimality of a filtered state predictor with respect to a functional involving the disturbance was shown in Krstic (2008). A filtered prediction was also considered in Sanz, García, and Albertos (2018) with a frequency-domain approach. Additional delayed feedback was considered in Léchappé, Moulay, Plestan, Glumineau, and Chriette (2015) to reject constant disturbances. A modified prediction based on a disturbance observer was proposed in Sanz, García, and Albertos (2016), leading to rejection of polynomial-in-time disturbances and also better attenuation of sufficiently smooth signals. Similar results are also reported in Furtat, Fridman, and Fradkov (2018). For unknown sinusoidal disturbances, cancellation by means of adaptive control schemes has been also achieved in Basturk and Krstic (2015), Basturk (2017).

The observer structure of the sequential predictor approach makes it suitable to combine with a disturbance observer. This

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key idea was recently used in Sanz, García, Fridman, and Albertos (2017), where nonlinearities were introduced. The previous work is extended in different directions in this paper. First, an additional nonlinear term is used in the control law in order to counteract the nonlinearity. Second, a chain of observers of arbitrary length is considered here, while only a one-element chain was considered in Sanz et al. (2017) for simplicity. Third, a generator model of the disturbance is considered to achieve rejection of time-varying disturbances. These modifications lead to a substantially more complicated closed-loop stability analysis. A systematic design procedure is given to compute the observer gains of all elements in the chain, as well as the feedback gain for the controller. Stability is then guaranteed in spite of the nonlinearity and the time-varying delay.

The rest of the paper is structured as follows. The problem formulation and preliminaries are given in Section 2. The proposed strategy is developed in Section 3, while the closed-loop stability and the controller design are tackled in Section 4. The main results are illustrated through a numerical example in Section 5.

2. Preliminaries

The present work deals with the class of input-delayed systems defined by

$$\dot{x}(t) = Ax(t) + B[u(t-\tau(t)) + w(t) + g(t,x)], \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where A,B,C are known matrices of appropriate dimensions, $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^q$ is the measured output and $u \in \mathbb{R}$ is the control input, $w : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is an unknown external disturbance and $g : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n$ is a known nonlinearity. The time-varying delay is assumed to have the form

$$\tau(t) = h + \eta(t), \quad 0 \leq \eta(t) \leq \bar{\eta}, \tag{3}$$

where the nominal value of the delay, $h \geq 0$, is known. The unknown time-varying function $\eta(t)$ is supposed to be piecewise-continuous and non-negative. In addition, the following assumptions are made:

Assumption 2.1: The pair (A, B) is stabilisable and the pair (A, C) is detectable.

Assumption 2.2: The nonlinearity has the structure $g(t, x) = g^t(t)g^x(x)$. There exist known constants $c_1, c_2 > 0$ such that $|g^t(t)| \leq c_1$, $\forall t \geq 0$ and

$$|g^t(t_1) - g^t(t_2)| \leq c_2|t_1 - t_2|, \quad \forall t_1, t_2 \geq 0.$$

Furthermore, $g^x(0) = 0$ and there exists a known vector $m \in \mathbb{R}^n$ such that

$$|g^x(x_1) - g^x(x_2)| \leq |l^T(x_1 - x_2)|,$$

for all x_1, x_2 in some region $\mathcal{D}$, containing the origin.

Assumption 2.3: The disturbance signal can be modelled by the exogenous system

$$\dot{\xi}(t) = G\xi(t), \tag{4}$$

$$w(t) = H\xi(t), \tag{5}$$

where $G \in \mathbb{R}^{r\times r}$, $H \in \mathbb{R}^{1\times r}$ are known and form a completely observable pair and $\xi \in \mathbb{R}^r$ is a generator vector with unknown initial condition $\xi(0)$.

The first assumption is necessary for the stabilisation of (1)–(2) via dynamic output feedback. Assumption 2.2 basically implies Lipschitz continuity of the nonlinearity with respect to both arguments, since it is assumed to be the product of two Lipschitz functions. Assumption 2.3 allows to represent a variety of signals such as sinusoidal or polynomial disturbances. Let us state the following auxiliary lemma, which will be used in the stability proof.

Lemma 2.1: *Under Assumption* (2.2)*, the following holds:*

$$|g(t_1, x_1) - g(t_2, x_2)| \leq c_1|l^T(x_1 - x_2)| + c_2|t_1 - t_2||l^T x_2|,$$

for all $t_1, t_2 \geq 0$ *and any* $x_1, x_2 \in \mathcal{D}$.

Proof: Computing the norm and adding and subtracting $g^t(t_1)g^x(x_2)$ leads to

$$\begin{aligned}|g(t_1, x_1) - g(t_2, x_2)| &= |g^t(t_1)g^x(x_1) - g^t(t_2)g^x(x_2)| \\ &= |g^t(t_1)g^x(x_1) - g^t(t_1)g^x(x_2) \\ &\quad + g^t(t_1)g^x(x_2) - g^t(t_2)g^x(x_2)| \\ &\leq |g^t(t_1)||g^x(x_1) - g^x(x_2)| \\ &\quad + |g^x(x_2)||g^t(t_1) - g^t(t_2)|,\end{aligned}$$

and thus the result follows by employing the bounds stated in Assumption 2.2. ■

The goal is to find an observer-based output-feedback control law such that the system (1)–(2) is robustly stabilised for all time-varying delays described by (3) when either the disturbance or the delay are constant. It should be remarked that asymptotic stability in the presence of both time-varying disturbances and delays is not pursued in this work, for which a delay estimation strategy would be necessary.

3. Proposed strategy

Let us define $z(t) = [x^{\mathrm{T}}(t), \xi(t)]^{\mathrm{T}} \in \mathbb{R}^{n_z}$ as an extended state with $n_z = n + r$, containing both the system state and the disturbance. Then the dynamics (1)–(2) can be rewritten as

$$\dot{z}(t) = A_z z(t) + B_z[u(t-\tau(t)) + g(t,x)], \tag{6}$$

$$y(t) = C_z z(t), \tag{7}$$

where

$$A_z = \begin{bmatrix} A & BH \\ 0 & G \end{bmatrix}, \quad B_z = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \text{and} \quad C_z = \begin{bmatrix} C & 0 \end{bmatrix}.$$

Now, a predictive observer is adopted to obtain a future estimation of the augmented state h units of time ahead. Following the ideas in Germani et al. (2002), Besançon et al. (2007), and Najafi et al. (2013), the proposed observer has a chain structure, such that each of the observer states, denoted by $\bar{z}_j(t) = [\bar{x}_j^{\mathrm{T}}(t), \bar{\xi}_j(t)]^{\mathrm{T}}$, estimates a prediction of the augmented state over a fraction of the total delay, $z(t+h_j)$, with $h_j = \frac{j}{m}h$. The proposed observer is given by

$$\dot{\bar{z}}_1(t) = A_z\bar{z}_1(t) + B_z[u(t-\bar{h}_1) + g(t+h_1, \bar{x}_1(t))] + L_1\left(y(t) - C_z\bar{z}_1\left(t - \frac{h}{m}\right)\right), \quad (8)$$

$$\dot{\bar{z}}_j(t) = A_z\bar{z}_j(t) + B_z[u(t-\bar{h}_j) + g(t+h_j, \bar{x}_j(t))] + L_j\left(z_{j-1}(t) - \bar{z}_j\left(t - \frac{h}{m}\right)\right), \quad (9)$$

for $j = 2, \ldots, m$, being $L_1 \in \mathbb{R}^{n_z \times q}$, $L_j \in \mathbb{R}^{n_z \times n_z}$ and $\bar{h}_j = h - h_j = (1 - \frac{j}{m})h$. Note that the particular case $m = 1$ is feasible and then the observer is simply given by (8). Let us define the prediction error $\tilde{z}_j(t) = [\tilde{x}_j^{\mathrm{T}}(t), \tilde{\xi}_j(t)]^{\mathrm{T}}$, by

$$\tilde{z}_j(t) = z(t) - \bar{z}_j(t - h_j), \quad (10)$$

so that $\tilde{z}_j(t) \to 0$ implies $\bar{z}(t) \to z(t+h_j)$, as discussed above. Differentiating (10), using (6)–(9) and the Newton–Leibniz formula to rewrite $u(t-\tau(t)) = u(t-h) - \mathcal{I}(\dot{u})$ with $\mathcal{I}(\phi) \triangleq \int_{t-\tau(t)}^{t-h} \phi(s)\,\mathrm{d}s$, the error dynamics satisfies[1]

$$\dot{\tilde{z}}_1(t) = A_z\tilde{z}_1(t) - L_1C_z\tilde{z}_1\left(t - \frac{h}{m}\right) - B_z\mathcal{I}(\dot{u}) + B_z\delta g_1, \quad (11)$$

$$\dot{\tilde{z}}_j(t) = A_z\tilde{z}_j(t) - L_j\tilde{z}_j\left(t - \frac{h}{m}\right) + L_j\tilde{z}_{j-1}(t - \frac{h}{m}) - B_z\mathcal{I}(\dot{u}) + B_z\delta g_j, \quad (12)$$

where

$$\delta g_j = g(t, x(t)) - g(t, \bar{x}_j(t - h_j)). \quad (13)$$

Now, the proposed control law

$$u(t) = -K\bar{x}_m(t) - g(t+h, \bar{x}_m(t)) - H\bar{\xi}_m(t), \quad (14)$$

with $K \in \mathbb{R}^{1\times n}$, is composed of three terms, the first two providing internal stability and the third one mitigating the effect of the disturbance. Note that (14) is a slight departure from the control law proposed in Sanz et al. (2017), in which the nonlinear term was neglected.

Delaying (14) by h units of time and using (10) with $j = m$, one can prove that the following holds:

$$u(t-h) = -Kx(t) - w(t) - g(t, \bar{x}_m(t-h)) + F\tilde{z}_m(t), \quad (15)$$

where $F \triangleq [K, H]$. Using again the Newton–Leibniz equation into (1) and plugging (15) into the resulting expression , yields

$$\dot{x}(t) = (A - BK)x(t) + BF\tilde{z}_m(t) - B\mathcal{I}(\dot{u}) + B\delta g_m. \quad (16)$$

4. Closed-loop analysis

Let us define $\mu(t) = [x^{\mathrm{T}}(t), \tilde{z}_1^{\mathrm{T}}(t), \ldots, \tilde{z}_m^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^N$ as an augmented state, where $N = n + m \cdot n_z$, and whose dynamics can be obtained from (11)–(12) and (16) as

$$\dot{\mu}(t) = A_0\mu(t) + A_1\mu\left(t - \frac{h}{m}\right) - \Gamma_0\mathcal{I}(\dot{u}) + \Gamma_1\delta g, \quad (17)$$

where $\delta g = [\delta g_1, \ldots, \delta g_m]^{\mathrm{T}}$ and

$$A_0 = \begin{bmatrix} A - BK & 0 & \ldots & 0 & BF \\ 0 & A_z & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ 0 & & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 0 & A_z \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ 0 & -L_1C_z & \ddots & \ddots & 0 \\ \vdots & L_2 & -L_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & L_m & -L_m \end{bmatrix},$$

$$\Gamma_0 = \begin{bmatrix} B \\ B_z \\ \vdots \\ \vdots \\ B_z \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 0 & 0 & \ldots & 0 & B \\ 0 & B_z & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \ldots & 0 & B_z \end{bmatrix}.$$

In order to derive stability conditions for (17), the term $\mathcal{I}(\dot{u})$ should be rewritten as a function of the augmented state μ. Similarly, the bounds on the uncertain terms should be also expressed in that form. This is done in the following propositions.

Proposition 4.1: *The integral $\mathcal{I}(\dot{u})$ can be expressed as*

$$\mathcal{I}(\dot{u}) = \Phi_0 \int_{t-\eta(t)}^{t} \dot{\mu}(s)\,\mathrm{d}s - \Delta g - \varphi(t), \quad (18)$$

where $\Phi_0 = [-K, 0, \ldots, 0, F]$ and

$$\Delta g = g\left(t, \bar{x}_m(t-h)\right) - g\left(t - \eta(t), \bar{x}_m(t - \tau(t))\right),$$

$$\varphi(t) = \int_{t-\eta(t)}^{t} \dot{w}(\theta)\,\mathrm{d}\theta.$$

Proof: Introducing the change of variables $s = \theta - h$, the integral term can be written as $\mathcal{I}(\dot{u}) = \int_{t-\tau(t)}^{t-h} \dot{u}(s)\,\mathrm{d}s = \int_{t-\eta(t)}^{t} \dot{u}(\theta - h)\,\mathrm{d}\theta$. Differentiating (15) and plugging it into the integral expression just derived yields the desired result. Note that the term $\Delta g = \int_{t-\eta(t)}^{t} \frac{\mathrm{d}g}{\mathrm{d}\theta}(\theta, \bar{x}(\theta - h))\,\mathrm{d}\theta$ has been expanded for convenience. ■

Proposition 4.2: *The following inequalities hold:*

$$\mathcal{S}_1 \triangleq \mu^{\mathrm{T}}(t)M_1\mu(t) - |\delta g|^2 \geq 0,$$

$$\mathcal{S}_2 \triangleq \begin{bmatrix}\mu^{\mathrm{T}}(t) & \int_{t-\eta(t)}^{t}\dot{\mu}^{\mathrm{T}}(s)\,\mathrm{d}s\end{bmatrix}\begin{bmatrix}M_3 & -M_3\\ (*) & M_2+M_3\end{bmatrix}\begin{bmatrix}\mu(t)\\ \int_{t-\eta(t)}^{t}\dot{\mu}(s)\,\mathrm{d}s\end{bmatrix} - |\Delta g|^2 \geq 0,$$

where $M_1 = \mathrm{diag}\{0_n, \bar{M}_1, \ldots, \bar{M}_1\}$ *and*

$$\Phi_1{}^{\mathrm{T}} = [I_n, 0_{n\times 1}], \quad \Phi_2{}^{\mathrm{T}} = [I_n, 0_{n\times n_z}, \ldots, 0_{n\times n_z}, -\Phi_1{}^{\mathrm{T}}],$$
$$\bar{M}_1 = c_1^2\Phi_1 ll^{\mathrm{T}}\Phi_1^{\mathrm{T}}, \quad M_2 = 2c_1^2\Phi_2 ll^{\mathrm{T}}\Phi_2^{\mathrm{T}},$$
$$M_3 = 2c_2^2\eta^2\Phi_2 ll^{\mathrm{T}}\Phi_2^{\mathrm{T}}.$$

Proof: See Appendix. ■

Plugging (18) into (17), the closed-loop dynamics can be rewritten as

$$\dot{\mu}(t) = A_0\mu(t) + A_1\mu(t-h) - A_2\int_{t-\eta(t)}^{t}\dot{\mu}(s)\,\mathrm{d}s + \Gamma_0\Delta g + \Gamma_1\delta g + \Gamma_0\varphi(t), \tag{19}$$

where

$$A_2 = \begin{bmatrix} -BK & 0 & \ldots & 0 & BF\\ -B_zK & 0 & \ldots & 0 & B_zF\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ -B_zK & 0 & \ldots & 0 & B_zF\\ -B_zK & 0 & \ldots & 0 & B_zF\end{bmatrix}.$$

One can see that (19) is driven by $\varphi(t)$, which arises as a consequence of the unknown time-varying delay. Asymptotic stability to zero will only be possible if $\bar{\eta} = 0$ or $\lim_{t\to\infty}\dot{w}(t) = 0$. Otherwise, we look at a conveniently defined L_2-gain performance. For a given $\gamma > 0$, let us introduce the performance index

$$J = \int_0^{\infty} y^{\mathrm{T}}(t)y(t) - \gamma^2\bar{\eta}\psi(t)\,\mathrm{d}t, \tag{20}$$

where $\psi(t) = \int_{t-\eta(t)}^{t}\dot{w}(s)^2\,\mathrm{d}s > 0$ for all $0 \neq \dot{w} \in L_2[0,\infty)$. Next, an auxiliary result is given in Lemma 4.1, followed by a sufficient criterion for the closed-loop stability in Theorem 4.1. The design problem is then solved in Theorem 4.2.

Lemma 4.1: *Let us denote* $\mu_t(\theta) = \mu(t+\theta)$, $\theta \in [-h-\bar{\eta}, 0]$ *and* $\|\mu_t\|_W = \max_{[-h-\bar{\eta},0]}|\mu_t| + \|\mu_t\|_{L_2[-h-\bar{\eta},0]}$. *If there is a Lyapunov–Krasovskii functional satisfying*

$$\beta_1|\mu(t)|^2 \leq V(t,\mu_t,\dot{\mu}_t) \leq \beta_2\|\mu_t\|_W^2,$$

with $\beta_1, \beta_2 > 0$*, such that, along the solutions of* (19)*, the inequality*

$$\dot{V}(t,\mu_t,\dot{\mu}_t) + 2\alpha V(t,\mu_t,\dot{\mu}_t) + y^{\mathrm{T}}(t)y(t) - \gamma^2\bar{\eta}\psi(t) \leq 0 \tag{21}$$

holds locally, then (19) *is internally exponentially stable with decay rate* α *and achieves performance* $J < 0$ *for all* $\psi(t) \neq 0$ *and zero initial conditions. Furthermore, if* $\bar{\eta} = 0$*, then* $\mu(t)$ *converges to zero.*

Proof: Setting $\psi(t) = 0$ in (21) leads to $\dot{V} + 2\alpha V \leq 0$ and thus $|\mu(t)|^2 \leq \beta_1^{-1}V(t) \leq \beta_1^{-1}V(0)\mathrm{e}^{-2\alpha t} \leq \beta_2\beta_1^{-1}\mathrm{e}^{-2\alpha t}\|\mu_0\|_W^2$, where the second inequality follows by the comparison principle. This proves the internal α-exponential stability. On the other hand, integration from 0 to ∞ leads to $J < 0$, provided that $\mu_0 = 0$ implies $V(0,\mu_0,\dot{\mu}_0) = 0$. Finally, setting $\bar{\eta} = 0$ in (21) also leads to $\dot{V} + 2\alpha V \leq 0$ and thus by the same arguments as above, the convergence of $\mu(t)$ to zero follows. For additional details see Fridman (2014b) and the references therein. ■

Theorem 4.1: *Given scalars* $\gamma > 0$ *and* $\bar{h}, \bar{\eta} \geq 0$*, let there exist positive definite matrices* $P, Q, R, S \in \mathbb{R}^N$*, full matrices* $P_2, P_3 \in \mathbb{R}^N$ *and positive scalars* λ_1, λ_2 *such that*

$$\begin{bmatrix} (1,1) & (1,2) & P_2^{\mathrm{T}}A_1 + R\mathrm{e}^{-2\alpha\bar{h}} & -P_2^{\mathrm{T}}A_2 - \lambda_2M_3 & P_2^{\mathrm{T}}\Gamma_0 & P_2^{\mathrm{T}}\Gamma_1 & P_2^{\mathrm{T}}\Gamma_0\\ (*) & (2,2) & P_3^{\mathrm{T}}A_1 & -P_3^{\mathrm{T}}A_2 & P_3^{\mathrm{T}}\Gamma_0 & P_3^{\mathrm{T}}\Gamma_1 & P_3^{\mathrm{T}}\Gamma_0\\ (*) & (*) & -(S+R)\mathrm{e}^{-2\alpha\bar{h}} & 0 & 0 & 0 & 0\\ (*) & (*) & (*) & (3,3) & 0 & 0 & 0\\ (*) & (*) & (*) & (*) & -\lambda_2 & 0 & 0\\ (*) & (*) & (*) & (*) & (*) & -\lambda_1I_m & 0\\ (*) & (*) & (*) & (*) & (*) & (*) & -\gamma^2 \end{bmatrix} < 0, \tag{22}$$

where

$$(1,1) = A_0{}^{\mathrm{T}}P_2 + P_2{}^{\mathrm{T}}A_0 + S - R + \bar{\eta}^2Q + \lambda_1M_1 + \lambda_2M_3,$$
$$(1,2) = P - P_2{}^{\mathrm{T}} + A_0{}^{\mathrm{T}}P_3,$$
$$(2,2) = -P_3 - P_3{}^{\mathrm{T}} + \bar{h}^2R,$$
$$(3,3) = -U\mathrm{e}^{-2\alpha\bar{\eta}} + \lambda_2(M_2 + M_3).$$

Then the closed-loop composed of the plant (1)–(2)*, the observer* (8)–(9) *and the control law* (14) *is internally exponentially stable and achieves* $J < 0$ *for all* $\psi(t) \neq 0$*, with zero initial conditions and for any* $0 \leq h \leq \bar{h}$*. Furthermore, if* $\bar{\eta} = 0$*, then* $\mu(t)$ *converges to zero with decay rate* α*.*

Proof: Let us consider a Lyapunov–Krasovskii functional of the form (see e.g. Section 3.7 of Fridman, 2014a)

$$V(t) = \mu(t)^{\mathrm{T}}P\mu(t) + V_h(t) + V_\eta(t), \tag{23}$$

where

$$V_h(t) = \int_{t-h}^{t}\mathrm{e}^{-2\alpha(s-t)}\mu^{\mathrm{T}}(s)S\mu(s)\,\mathrm{d}s + h\int_{-h}^{0}\int_{t+\theta}^{t}\mathrm{e}^{2\alpha(s-t)}\dot{\mu}(s)^{\mathrm{T}}R\dot{\mu}(s)\,\mathrm{d}s, \tag{24}$$

$$V_\eta(t) = \bar{\eta}\int_{-\bar{\eta}}^{0}\int_{t+\theta}^{t}\mathrm{e}^{2\alpha(s-t)}\dot{\mu}(s)^{\mathrm{T}}Q\dot{\mu}(s). \tag{25}$$

Differentiating (23) and using the descriptor method (Fridman, 2001), one has that

$$\begin{aligned}\dot V(t) &= 2\mu^{\mathrm T}(t)P\dot\mu(t) + \mu^{\mathrm T}(t)S\mu(t)\\ &\quad - \mathrm e^{-2\alpha h}\mu^{\mathrm T}(t-h)S\mu(t-h) + h^2\dot\mu^{\mathrm T}(t)R\dot\mu(t)\\ &\quad - \mathrm e^{-2\alpha h}h\int_{t-h}^{t}\dot\mu^{\mathrm T}(s)R\dot\mu(t)\,\mathrm ds + \bar\eta^2\dot\mu^{\mathrm T}(t)Q\dot\mu(t)\\ &\quad - \mathrm e^{-2\alpha\bar\eta}\bar\eta\left(\underbrace{\int_{t-\bar\eta}^{t-\eta(t)}\dot\mu^{\mathrm T}(s)Q\dot\mu(t)\,\mathrm ds}_{\text{neglected}}\right.\\ &\quad \left. + \int_{t-\eta(t)}^{t}\dot\mu^{\mathrm T}(s)Q\dot\mu(t)\,\mathrm ds\right)\\ &\quad + 2[\mu^{\mathrm T}(t){P_2}^{\mathrm T} + \dot\mu^{\mathrm T}(t){P_3}^{\mathrm T}][\text{RHS of (19)} - \dot\mu(t)],\end{aligned}\tag{26}$$

where the last term in (26) can be added as it is identically zero. Now, by Jensen's inequality, it follows that

$$-\bar\eta\int_{t-\eta(t)}^{t}\dot\mu^{\mathrm T}(s)Q\dot\mu(t)\,\mathrm ds \le \int_{t-\eta(t)}^{t}\dot\mu^{\mathrm T}(s)\,\mathrm dsQ\int_{t-\eta(t)}^{t}\dot\mu(t)\,\mathrm ds,\tag{27}$$

$$\begin{aligned}-h\int_{t-h}^{t}\dot\mu^{\mathrm T}(s)R\mu(t)\,\mathrm ds &\le \int_{t-h}^{t}\dot\mu^{\mathrm T}(s)\,\mathrm dsR\int_{t-h}^{t}\dot\mu(t)\,\mathrm ds\\ &= -[\mu(t)-\mu(t-h)]R[\mu(t)\\ &\quad - \mu(t-h)].\end{aligned}\tag{28}$$

Then, using (26)–(28) and Jensen's inequality to bound $-\bar\eta\psi(t) \le -\varphi(t)^2$, one can write

$$\dot V(t) - 2\alpha V(t) + y^{\mathrm T}(t)y(t) - \gamma^2\bar\eta\psi(t) \le q^{\mathrm T}(t)\Xi q(t),\tag{29}$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & P_2^{\mathrm T}A_1 + R\mathrm e^{-2\alpha h} & -P_2^{\mathrm T}A_2 & P_2^{\mathrm T}\Gamma_0 & P_2^{\mathrm T}\Gamma_1 & P_2^{\mathrm T}\Gamma_0\\ (*) & \Xi_{22} & P_3^{\mathrm T}A_1 & -P_3^{\mathrm T}A_2 & P_3^{\mathrm T}\Gamma_0 & P_3^{\mathrm T}\Gamma_1 & P_3^{\mathrm T}\Gamma_0\\ (*) & (*) & -(S+R)\mathrm e^{-2\alpha h} & 0 & 0 & 0 & 0\\ (*) & (*) & (*) & -U\mathrm e^{-2\alpha\bar\eta} & 0 & 0 & 0\\ (*) & (*) & (*) & (*) & 0 & 0 & 0\\ (*) & (*) & (*) & (*) & (*) & 0 & 0\\ (*) & (*) & (*) & (*) & (*) & (*) & -\gamma^2 \end{bmatrix},\tag{30}$$

$$\Xi_{11} = {A_0}^{\mathrm T}P_2 + {P_2}^{\mathrm T}A_0 + S - R + \bar\eta^2Q,$$
$$\Xi_{12} = P - {P_2}^{\mathrm T} + {A_0}^{\mathrm T}P_3,$$
$$\Xi_{22} = -P_3 - {P_3}^{\mathrm T} + h^2R,$$

and

$$q(t) = \mathrm{col}\left\{\mu(t), \dot\mu(t), \mu(t-h), \int_{t-\eta(t)}^{t}\mu(s)\,\mathrm ds, \Delta g, \delta g, \varphi(t)\right\}.$$

To deal with the uncertain terms δg and Δg, the $\mathcal S$-procedure is invoked. Given the quadratic forms $\mathcal S_1, \mathcal S_2 \ge 0$ in Proposition 4.2, it is verified that $q^{\mathrm T}(t)\Xi q(t) \le 0$ if there exist scalars $\lambda_1, \lambda_2 > 0$ such that

$$q^{\mathrm T}(t)\Xi q(t) + \lambda_1\mathcal S_1 + \lambda_2\mathcal S_2 \le 0.\tag{31}$$

Rearranging (31) into a matrix form leads to (22). Since the LMI is convex in h, its feasibility for $\bar h$ implies its feasibility for any $0 \le h \le \bar h$. Finally, if (22) holds then so does (29). Furthermore, since $V(t, \mu_t, \dot\mu_t)$ in (23) clearly satisfies the lower and upper bounds in Lemma 4.1, then the theorem follows. ■

Theorem 4.2: *Given scalars $\gamma, \epsilon > 0$ and $\bar h, \bar\eta \ge 0$, let there exist positive definite matrices $W \in \mathbb R^n$, $P, Q, R, S \in \mathbb R^N$, full matrices $X \in \mathbb R^{1\times n}$, $P_{20} \in \mathbb R^{n\times n}$, $P_{21}, \ldots, P_{2m} \in \mathbb R^{n_z\times n_z}$, $Y_{21} \in \mathbb R^{n_z\times 1}$, $Y_{22}, \ldots, Y_{2m} \in \mathbb R^{n_z\times n_z}$ and positive scalars λ_1, λ_2 such that*

$$WA^{\mathrm T} + AW - X^{\mathrm T}B^{\mathrm T} - BX + 2\alpha W < 0,\tag{32}$$

$$\begin{bmatrix} (1,1) & (1,2) & \mathcal Y + R\mathrm e^{-2\alpha\bar h} & -P_2^{\mathrm T}A_2 - \lambda_2M_3 & P_2^{\mathrm T}\Gamma_0 & P_2^{\mathrm T}\Gamma_1 & P_2^{\mathrm T}\Gamma_0\\ (*) & (2,2) & \epsilon\mathcal Y & -\epsilon P_2^{\mathrm T}A_2 & \epsilon P_2^{\mathrm T}\Gamma_0 & \epsilon P_2^{\mathrm T}\Gamma_1 & \epsilon P_2^{\mathrm T}\Gamma_0\\ (*) & (*) & -(S+R)\mathrm e^{-2\alpha\bar h} & 0 & 0 & 0 & 0\\ (*) & (*) & (*) & (3,3) & 0 & 0 & 0\\ (*) & (*) & (*) & (*) & -\lambda_2 & 0 & 0\\ (*) & (*) & (*) & (*) & (*) & -\lambda_1I_m & 0\\ (*) & (*) & (*) & (*) & (*) & (*) & -\gamma^2 \end{bmatrix} < 0,\tag{33}$$

where

$$(1,1) = {A_0}^{\mathrm T}P_2 + {P_2}^{\mathrm T}A_0 + S - R + \bar\eta^2Q + \lambda_1M_1 + \lambda_2M_3,$$
$$(1,2) = P - {P_2}^{\mathrm T} + \epsilon{A_0}^{\mathrm T}P_2,$$
$$(2,2) = -\epsilon P_2 - \epsilon{P_2}^{\mathrm T} + \bar h^2R,$$
$$(3,3) = -U\mathrm e^{-2\alpha\bar\eta} + \lambda_2(M_2 + M_3),$$

$$\mathcal Y = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0\\ 0 & -Y_1C_z & \ddots & \ddots & 0\\ \vdots & Y_2 & -Y_2 & \ddots & \vdots\\ 0 & \ddots & \ddots & \ddots & 0\\ 0 & 0 & \ldots & Y_m & -Y_m \end{bmatrix},$$

and $P_2 = \mathrm{diag}\{P_{20}, \dots, P_{2m}\}$. Then the closed-loop composed of the plant (1)–(2), the observer (8)–(9) with $L_j = (P_{2j}^{\mathrm{T}})^{-1} Y_j$ and the control law (14) with $K = XW^{-1}$ is internally asymptotically stable and achieves $J < 0$ for all $\psi(t) \neq 0$, with zero initial conditions any for any $0 \leq h \leq \bar{h}$. Furthermore, if $\bar{\eta} = 0$, then the closed-loop converges exponentially to zero with decay rate α.

Proof: Given the complexity of linearising (22) to obtain both K and L simultaneously, the matrix K is simply computed to guarantee that the plant (1) under the controller $u(t) = -Kx(t) - g(t, x(t))$ is α-exponentially stable, which is guaranteed by (32). Now, let us consider as in Suplin, Fridman, and Shaked (2007) and Shustin and Fridman (2007) the simplifications $P_3 = \epsilon P_2$, with $\epsilon > 0$ a scalar tuning parameter and $P_2 = \mathrm{diag}\{P_{20}, \dots, P_{2m}\}$. Defining $Y_j = P_{2j}^{\mathrm{T}} L_j$, for $j = 1, 2, \dots, m$, $\mathcal{Y} = P_2^{\mathrm{T}} A_1$, and after some straightforward manipulations, the LMI (22) is transformed into (33), which completes the proof. ∎

5. Simulations

The proposed strategy is illustrated in this section with three examples. The first one is an academic example to validate the theoretical results stated in Theorem 4.2. The others are focused on physical systems to illustrate the usefulness of this approach. First, a servo motor with a nonlinear friction model is considered. Second, a simplified model of the longitudinal dynamics of an aircraft is also studied.

5.1 Example 1

Let us consider the following system:

$$\dot{x}_1(t) = x_2(t), \tag{34}$$

$$\dot{x}_2(t) = x_1(t) + x_1^2(t) \sin t + u(t - \tau(t)) + w(t), \tag{35}$$

where $\tau(t) = 0.2 + 0.05 \sin^2 t$ and $y(t) = x_1(t)$. The time-delay function matches (3) with $h = 0.2$ and $\bar{\eta} = 0.05$. The system matrices

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{36}$$

satisfy Assumption 2.1. It should be remarked that $\mathrm{eig}(A) = \{-1, 1\}$ and thus the open-loop system is exponentially unstable. The nonlinearity $g(x, t)$ can be decomposed as the product of $g^t(t) = \sin t$ and $g^x(x) = x_1^2$. Clearly, $|g^t(t_1) - g^t(t_2)| \leq |t_2 - t_1|$, $\forall t_1, t_2 \geq 0$. Also, $|g^x(y) - g^x(z)| = |(y_1 + z_1)(y_1 - z_1)| \leq \beta |[1,\ 0,\ 0](y - z)|, \forall |y_1 + z_1| \leq \beta$. Therefore, Assumption 2.2 is satisfied with $c_1 = 1$, $c_2 = 1$ and $l^{\mathrm{T}} = [\beta,\ 0,\ 0]$ for all $|x_1| \leq \beta/2$. The value of β should be selected according to some design requirements and it is here arbitrarily chosen as $\beta = 3$, which makes Assumption 2.2 to hold locally in the region $\mathcal{D} = \{x \in \mathbb{R}^n : |x_1| \leq 1.5\}$. The disturbance is considered to be constant, which satisfies Assumption 2.3 with $G = 0$ and $H = 1$. The observer (8)–(9) and the control law (14) are implemented with

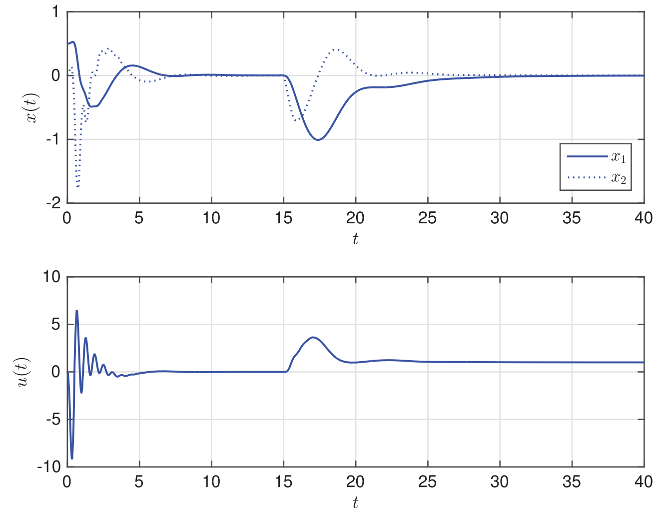

Figure 1. Example 1: state (top) starting from $x(0) = [0.5, 0]^{\mathrm{T}}$, with the observer starting from zero initial conditions and a disturbance $w(t) = -1, \forall t \geq 15$; control action (bottom).

$m = 2$. The gains K and L are designed using Theorem 4.2. The problem

$$\min_{\gamma > 0} \gamma \quad \text{subject to} \quad (32) - (33) \tag{37}$$

is solved using the Yalmip toolbox for Matlab. There are two parameters left to adjust, namely, the decay rate $\alpha \geq 0$ and auxiliary variable $\epsilon > 0$. For a given value of α, the problem (37) is solved for different values of ϵ and the one leading to the minimum γ is taken as the optimal solution. This procedure is repeated for increasing values of α until the problem becomes unfeasible. In this example, we obtained $\gamma = 63$, for $\alpha = 0.15$ and $\epsilon = 0.23$, leading to the gains

$$K = \begin{bmatrix} 2.54 & 1.18 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 8.92 \\ 32.54 \\ 10.50 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 6.65 & 0.01 & 0.51 \\ 22.22 & 0.27 & 1.81 \\ 4.15 & 0.21 & 0.41 \end{bmatrix}.$$

A simulation with the system starting from $x(0) = [0.5, 0]^{\mathrm{T}}$ and a disturbance signal $w(t) = -1, \forall t \geq 15$ is carried out. The evolution of the system state and the control action are depicted in Figure 1, where it can be seen that the state converges asymptotically to zero, as expected from Theorem 4.2. The observer error is also shown in Figure 2. It should be remarked that, in the example here considered, the previous work in Sanz et al. (2017) fails to produce any stabilising controller.

5.2 Example 2

Another example is considered, which consists of a servo positioning system governed by (Yao, Jiao, & Ma, 2014)

$$\theta_1 \ddot{y}(t) = -F(\dot{y}) - \theta_4 \dot{y}(t) + \tau(t - h) + \tau_L(t), \tag{38}$$

where $F(s) = \theta_2 \tanh(c_1 s) + \theta_3[\tanh(c_2 s) - \tanh(c_3 s)]$ is the friction model, y is the motor rotation angle, τ is the motor

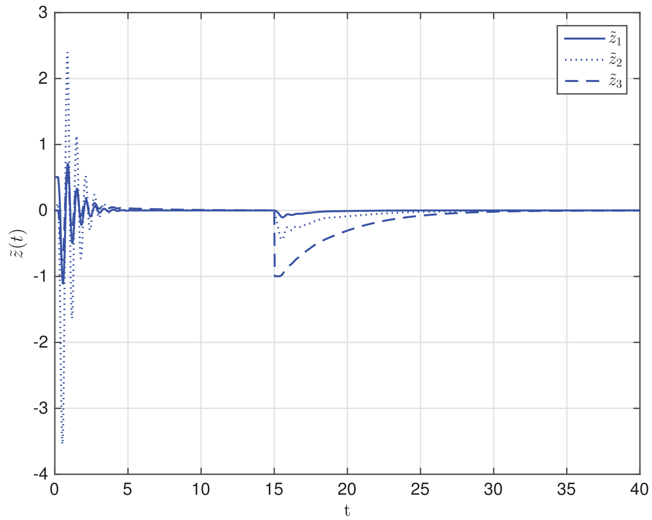

Figure 2. Example 1: observer error with the state starting from $x(0) = [0.5, 0]^T$, the observer starting from zero initial conditions and a disturbance $w(t) = -1, \forall t \geq 15$.

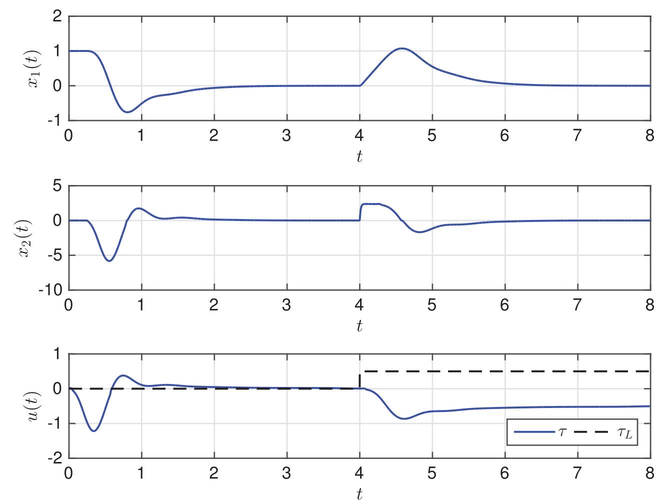

Figure 3. Example 2: State (top and centre) starting from $x(0) = [1, 0]^T$, with the observer starting from zero initial conditions and a disturbance $w(t) = 0.5, \forall t \geq 4$; control action and disturbance (bottom).

torque, τ_L is a load disturbance torque and $\theta_1, \theta_2, \theta_3, \theta_4$ is a set of physical parameters. The following parameters $c_1 = 700$, $c_2 = 15$, $c_3 = 1.5$, $\theta_1 = 2.5 \times 10^{-3}$, $\theta_2 = 0.02$, $\theta_3 = 0.01$ and $\theta_4 = 0.205$ are given in Yao et al. (2014). Only x_1 is assumed to be measured here and the model is modified to include an input delay of $h = 0.1$ s. The system (38) is then written in the form of (1)–(2) with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\theta_4}{\theta_1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{\theta_1} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

and $g(t, x) = -F(x_2)$, $w(t) = \tau_L(t)$. Assumption 2.1 is fulfilled by the triple (A, B, C) and the nonlinearity satisfies Assumption 2.2 with $c_1 = 1$, $c_2 = 0$ and $l^T = [0, l_2]$, where l_2 is the Lipschitz constant that is computed numerically as $l_2 = \sup_s |F'(s)| \approx 14.13$. The disturbance is assumed to be a constant, which simulates a load attached to the motor shaft. Such disturbance satisfies Assumption 2.3 with $G = 0$ and $H = 1$.

Using Theorem 4.2 with $m = 2$, $\alpha = 2$ and $\epsilon = 0.1$ yields the following gains:

$$L_1 = \begin{bmatrix} 7.49 \\ -1.10 \\ 2.71 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 6.47 & -0.33 & 1.55 \\ 3.64 & -0.91 & 15.40 \\ 0.97 & -0.11 & 2.51 \end{bmatrix}$$

and $K = [0.180 \quad -0.157]$. A simulation with the system starting from $x = [1, 0]^T$ is shown in Figure 3. A constant load disturbance is introduced at $t = 4$ s. One can see that system performance is fairly good in spite of the time delay and how the load disturbance is successfully identified and rejected.

5.3 Example 3

The aim of this example is to illustrate the rejection of a time-varying disturbance. The following model is an approximation of the longitudinal dynamics of A4D aircraft at a flight condition of 15,000 ft and 0.9 Mach (Guo & Chen, 2005):

$$A = \begin{bmatrix} -0.0605 & 32.3700 & 0 & 32.2000 \\ -0.0001 & -1.4750 & 1.0000 & 0 \\ -0.0111 & -34.7200 & -2.7930 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ -0.1064 \\ -33.8000 \\ 0 \end{bmatrix},$$

where the state $x = [x_1, x_2, x_3, x_4]$ is assumed to be measurable, x_1 is the forward velocity (ft/s), x_2 is the angle of attack (rad), x_3 is the pitching rate *rad*/*s*, x_4 is the pitch angle (rad) and u is the elevator deflection (deg). As in Guo and Chen (2005), the external disturbance is assumed to be a sinusoidal signal with frequency 5 rad/s, described by Assumption 2.3 with

$$H = \begin{bmatrix} 25 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}.$$

The example is modified by including an input delay of $h = 0.1$ s, which could be caused by a slow actuator dynamics. For the sake of comparison, the same K as in Guo and Chen (2005) is chosen, which is given by

$$K = \begin{bmatrix} 2.32 & 9.94 & 4.00 & 13.85 \end{bmatrix}.$$

The observer gains were then chosen using Theorem 4.2, as explained in the previous example. Selecting $\alpha = 0.5$ and $\epsilon = 0.5$, the following gains were obtained:

$$L_1 = \begin{bmatrix} 10.3 & 150.3 & 0.3 & -116.8 \\ 0.2 & 17.3 & 0.9 & -12.2 \\ 4.1 & 154.4 & 4.7 & -150.6 \\ 0.4 & 17.5 & 0.9 & -11.7 \\ 0.008 & -0.07 & -0.01 & 0.1 \\ 0.003 & 0.06 & 0.02 & 0.1 \end{bmatrix}$$

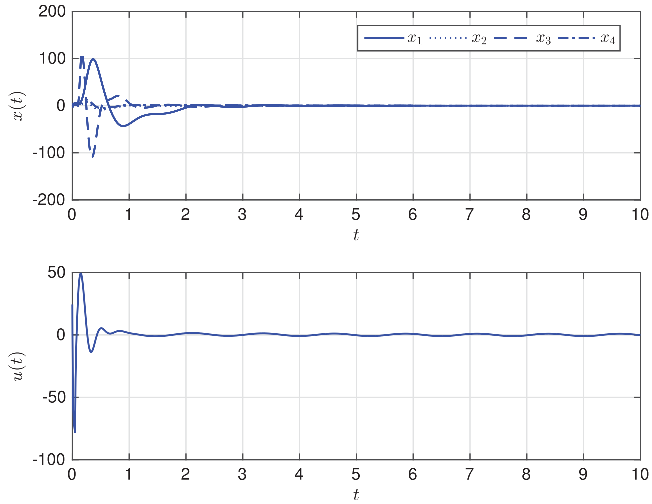

Figure 4. Example 3: State (top) starting from $x(0) = [2, -2, 3, 2]^{\mathrm{T}}$, with the observer starting from zero initial conditions and a disturbance $w(t) = \sin 5t, \forall t \geq 0$; control action (bottom).

and

$$L_2 = \begin{bmatrix} 16.8 & 444.9 & -2.9 & -426.9 & -60.6 & 26.1 \\ 0.5 & 35.2 & -0.5 & -22.5 & -23.5 & 5.7 \\ 1.2 & 168.9 & -0.05 & -122.8 & -10.6 & 35.4 \\ 0.8 & 41.5 & -0.5 & -28.6 & -24.5 & 6.1 \\ -0.005 & -0.5 & 0.004 & 0.3 & 0.4 & -0.06 \\ 0.01 & 0.4 & -0.005 & -0.3 & -0.3 & 0.1 \end{bmatrix}.$$

A simulation shows that the strategy reported in Guo and Chen (2005) becomes unstable when the input delay $h = 0.1$ s is introduced. The results of the strategy proposed in this paper are shown in Figure 4. One can see that, although the performance is obviously degraded, stability is preserved and the sinusoidal disturbance is rejected in spite of the delay.

6. Conclusions

A robust control strategy for a class of nonlinear systems with time-varying input delay was proposed. This strategy makes use of sequential predictors whose implementation is straightforward, in contrast to prediction-based controllers. Nonlinearities have been considered, which is an open problem in the context of sequential predictors. Furthermore, a design methodology by means of linear matrix inequalities has been derived. The design procedure has been illustrated with a numerical example.

Simulations show that the LMI design conditions are quite conservative. This is due to the restrictions imposed in the decision variables in order to derive computable design criteria. Therefore, other design procedures and/or Lyapunov–Krasovskii functionals that introduce less conservatism could be investigated in the future. Simulations also point out the so-called peaking phenomenon. This is a well-known behaviour that can cause instability of nonlinear systems. Therefore, future research may also be focused on mitigating this effect by using saturation functions.

Note

1. The equality $z_{j-1}(t - h_j) - \bar{z}(t - h/m - h_j) = \tilde{z}_j(t - h/m) - \tilde{z}_{j-1}(t - h/m)$ was used to derive (12), which can be obtained by subtracting $\tilde{z}_j(t)$ and $\tilde{z}_{j-1}(t)$ as defined in (10), delaying the resulting expression by h/m units of time, and using the fact that $h_{j-1} + h/m = h_j$.

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Appendix. Proof of Proposition 4.2

Taking the norm of (13) and using Lemma 2.1 with $t_1 = t_2 = t$, $x_1 = x(t)$ and $x_2 = \bar{x}_j(t - h_j)$ yields

$$|\delta g_j| \le c_1 \left| m^{\mathrm{T}} \left(x(t) - \bar{x}_j(t - h_j) \right) \right| = c_1 |m^{\mathrm{T}} \tilde{x}_j(t)| = c_1 |m^{\mathrm{T}} \Phi_1^{\mathrm{T}} \tilde{z}_j(t)|. \tag{A1}$$

Taking squares on both sides of (A1) yields $|\delta g_j|^2 \le \tilde{z}_j^{\mathrm{T}}(t) \bar{M}_1 \tilde{z}_j(t)$. Since $|\delta g|^2 = \sum_{j=1}^m |\delta g_j|^2 \le \sum_{j=1}^m \tilde{z}_j^{\mathrm{T}}(t) M_1 \tilde{z}_j(t)$ and recalling that $\mu(t) = [x^{\mathrm{T}}(t), \tilde{z}_1^{\mathrm{T}}(t), \ldots, \tilde{z}_m^{\mathrm{T}}(t)]^{\mathrm{T}}$, then $\mathcal{S}_1 \ge 0$ follows.

Now, using Lemma 2.1 with $t_1 = t$, $t_2 = t - \eta(t)$, $x_1 = \bar{x}_m(t - h)$ and $x_2 = \bar{x}_m(t - \tau(t))$ yields

$$|\Delta g| \le c_1 |m^{\mathrm{T}} [\bar{x}_m(t-h) - \bar{x}_m(t - \tau(t))]| + c_2 \eta |m^{\mathrm{T}} \bar{x}_m(t - \tau(t))|. \tag{A2}$$

Let us rewrite

$$\bar{x}_m(t-h) - \bar{x}_m(t - \tau(t)) = \int_{t-\tau(t)}^{t-h} \dot{\bar{x}}_m(\theta)\, \mathrm{d}\theta = \int_{t-\eta(t)}^{t} \dot{\bar{x}}_m(s-h)\, \mathrm{d}s, \tag{A3}$$

where the change of variable $\theta = s - h$ was performed. From (10), noting that $h = h_m$, we have that

$$\bar{x}_m(t-h) = x(t) - \tilde{x}_m(t) = x(t) - {\Phi_1}^{\mathrm{T}} \tilde{z}_m(t) = {\Phi_2}^{\mathrm{T}} \mu(t). \tag{A4}$$

Plugging (A4) into (A3), it follows that

$$\bar{x}_m(t-h) - \bar{x}_m(t - \tau(t)) = \int_{t-\eta(t)}^{t} {\Phi_2}^{\mathrm{T}} \dot{\mu}(s)\, \mathrm{d}s. \tag{A5}$$

On the other hand, delaying (A4) by $\eta(t)$ units of time leads to

$$\bar{x}_m(t - \tau(t)) = {\Phi_2}^{\mathrm{T}} \mu(t - \eta(t)). \tag{A6}$$

Plugging (A5)–(A6) into (A2) yields

$$|\Delta g| \le c_1 \left| m^{\mathrm{T}} \Phi_2^{\mathrm{T}} \int_{t-\eta(t)}^{t} \dot{\mu}(\theta)\, \mathrm{d}\theta \right| + c_2 \eta \left| m^{\mathrm{T}} \Phi_2^{\mathrm{T}} \mu(t - \eta(t)) \right|. \tag{A7}$$

Squaring both sides of (A7) and using Young's inequality to bound the cross term leads to

$$\begin{aligned} |\Delta g|^2 \le & \left(\int_{t-\eta(t)}^{t} \dot{\mu}^{\mathrm{T}}(\theta)\, \mathrm{d}\theta \right) M_2 \left(\int_{t-\eta(t)}^{t} \dot{\mu}(\theta)\, \mathrm{d}\theta \right) \\ & + \mu^{\mathrm{T}}(t - \eta(t)) M_3 \mu(t - \eta(t)). \end{aligned} \tag{A8}$$

Finally, replacing $\mu(t - \eta(t))$ by $\mu(t) - \int_{t-\eta(t)}^{t} \dot{\mu}(s)\, \mathrm{d}\theta$ in (A8) yields $\mathcal{S}_2 \ge 0$, which completes the proof.