Observer-Based Decentralized Predictor Control for Large-Scale Interconnected Systems With Large Delays

Yang Zhu and Emilia Fridman, *Fellow, IEEE*

***Abstract*—This article proposes a decentralized predictor-based output feedback to compensate for large delays in large-scale interconnected systems under the premise that the interconnections between subsystems are not strong. The full state of each subsystem is assumed to be unmeasurable and the observer-based output feedback is employed. Two methods are used to tackle the large delays: the backstepping-based partial differential equation (PDE) approach and the reduction-based ordinary differential equation (ODE) approach. The PDE method is used for continuous-time control and manages with larger delays, whereas the ODE method is applicable to sampled-data implementation under discrete-time measurement.**

***Index Terms*—Delay, large scale, predictor.**

I. Introduction

By virtue of rapidly developed communication and digital technologies, networked control systems (NCSs) show great potential in modern control. However, the development of NCSs is also full of challenges. Among many technical difficulties, an important and popular topic is the time delay, which renders the controlled system unstable when disregarded. A large body of existing literature on NCSs concentrates on the robust stability analysis with respect to "small" delays in the feedback loop via communication network. In other words, the delays are not compensated and the largest values of the delays that preserve the performance are investigated in terms of Linear Matrix Inequality (LMI) condition [3], [7]

To compensate for large delays, a key tool is the predictor feedback, which has found a widespread application in practice since it was developed 60 years ago [1]. However, most results assume a single plant with a centralized controller [2], [10], [11]. The decentralized networked control for large-scale time-delay systems [4]–[6], [17]–[19] applies the predictor-free feedback, but the delays are not compensated in the control design so that the delay length is limited to be "small." The eigenvalue-based approach [14]–[16] provides a necessary and sufficient condition for the stability analysis of time-delay systems, which reduces the conservativeness. The recent paper [9] considers predictor-based stabilization for two interconnected systems, but the results are based on state feedback and restricted to continuous-time control.

Manuscript received March 28, 2020; revised July 9, 2020; accepted July 19, 2020. Date of publication July 23, 2020; date of current version May 27, 2021. This work was supported by Postdoctoral Program of the Planning and Budgeting Committee, Council for Higher Education in Israel and Israel Science Foundation under Grant 673/19. Recommended by Associate Editor W. Michiels. *(Corresponding author: Yang Zhu.)*

Yang Zhu is with the State Key Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China, and also with the School of Electrical Engineering, Tel Aviv University, Tel Aviv 6997801, Israel (e-mail: zhuyang@iipc.zju.edu.cn).

Emilia Fridman is with the School of Electrical Engineering, Tel Aviv University, Tel Aviv 6997801, Israel (e-mail: emilia@tauex.tau.ac.il).

Color versions of one or more of the figures in this article are available online at https://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TAC.2020.3011396

This article extends the predictor feedback to decentralized control for large-scale interconnected systems with large input delays. The local control network of each subsystem operates independently without employing information from other neighbors provided that the interactions in large-scale systems are not strong. Different from our preliminary work [12] where the state-feedback is considered, this article addresses a more challenging problem where the full-state of each subsystem is assumed to be unmeasured. We propose two output feedback approaches for the delay compensation: the backstepping-based partial differential equation (PDE) method and the reduction-based ordinary differential equation (ODE) method. The PDE-based predictor is capable to derive simpler LMI conditions and withstand larger delays, whereas the ODE-based method is applicable to both continuous-time and sampled-data stabilization. In our conference paper [13], the sampled-data control with continuous-time measurement is taken into account, whereas this article addresses a more realistic case of the sampled-data control with discrete-time measurement.

II. Continuous-Time Feedback: PDE Approach

A. PDE Framework

Consider large-scale interconnected linear systems with input delays as follows:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t-r_i) + \sum_{i\neq j} F_{ij} x_j(t) \tag{1}$$

$$y_i(t) = C_i x_i(t) \tag{2}$$

where $i = 1, 2, \ldots, M$ is the subsystem index, $x_i(t) \in \mathbb{R}^{n_i}$, $y_i(t) \in \mathbb{R}^{q_i}$, and $u_i(t) \in \mathbb{R}^{m_i}$ are the state, output, and local control input of the ith plant, respectively. We denote F_{ij} to be the interaction between the ith plant $x_i(t)$ and the jth plant $x_j(t)$. The control input is subject to a large constant and known input delay $r_i > 0$. We assume that the plant state $x_i(t)$ is unmeasurable, the pair (A_i, B_i) is stabilizable and (A_i, C_i) is detectable, which means there exist matrices of appropriate dimensions K_i and L_i such that $A_i + B_i K_i$ and $A_i - L_i C_i$ are Hurwitz.

In this section, we deal with the case of continuous-time feedback by the PDE-based framework [8].

We introduce a multivariable function

$$v_i(\sigma, t) = u_i(t+\sigma-r_i), \quad \sigma \in [0, r_i] \tag{3}$$

to represent the control input $u_i(\theta)$ over the time interval $\theta \in [t-r_i, t]$. With (3), the system (1)–(2) is represented by the ODE-PDE cascade as follows:

$$\dot{x}_i(t) = A_i x_i(t) + B_i v_i(0, t) + \sum_{i\neq j} F_{ij} x_j(t) \tag{4}$$

$$y_i(t) = C_i x_i(t) \tag{5}$$

$$\partial_t v_i(\sigma,t) = \partial_\sigma v_i(\sigma,t), \quad \sigma \in [0, r_i] \tag{6}$$

$$v_i(r_i,t) = u_i(t). \tag{7}$$

It is apparent that (3) is a solution of the transport PDE (6)–(7). We denote $\hat{x}_i(t)$ to be an estimate of the unmeasured state $x_i(t)$ and the observer is designed as

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_i v_i(0,t) + L_i\left(y_i(t) - C_i\hat{x}_i(t)\right) \tag{8}$$

with the estimation error $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ satisfying

$$\dot{\tilde{x}}_i(t) = (A_i - L_iC_i)\tilde{x}_i(t) + \sum_{i\neq j} F_{ij}x_j(t). \tag{9}$$

The predictor-based boundary controller is designed as

$$u_i(t) = v_i(r_i,t) = K_i\left(e^{A_ir_i}\hat{x}_i(t) + \int_0^{r_i} e^{A_i(r_i-\delta)}B_iv_i(\delta,t)d\delta\right). \tag{10}$$

For convenience of stability analysis, we bring in the invertible backstepping transformation

$$w_i(\sigma,t) = v_i(\sigma,t) - K_ie^{A_i\sigma}\hat{x}_i(t) - K_i\int_0^{\sigma} e^{A_i(\sigma-\delta)}B_iv_i(\delta,t)d\delta \tag{11}$$

$$v_i(\sigma,t) = w_i(\sigma,t) + K_ie^{(A_i+B_iK_i)\sigma}\hat{x}_i(t) + K_i\int_0^{\sigma} e^{(A_i+B_iK_i)(\sigma-\delta)}B_iw_i(\delta,t)d\delta \tag{12}$$

through which the transport PDE (6)–(7), the observer, and its error (8)–(9) are converted into the closed-loop target system as follows:

$$\dot{\hat{x}}_i(t) = (A_i + B_iK_i)\hat{x}_i(t) + B_iw_i(0,t) + L_iC_i\tilde{x}_i(t) \tag{13}$$

$$\dot{\tilde{x}}_i(t) = (A_i - L_iC_i)\tilde{x}_i(t) + \sum_{i\neq j} F_{ij}\left(\hat{x}_j(t) + \tilde{x}_j(t)\right) \tag{14}$$

$$\partial_t w_i(\sigma,t) = \partial_\sigma w_i(\sigma,t) - K_ie^{A_i\sigma}L_iC_i\tilde{x}_i(t), \quad \sigma\in[0,r_i] \tag{15}$$

$$w_i(r_i,t) = 0. \tag{16}$$

Theorem 1: Consider the closed-loop system consisting of the plant (1)–(2), observer (8), and controller (10). Given tuning parameters $0 < \epsilon < \alpha$, let a parameter $\lambda_i > 0$, matrices $P_i, R_i \in \mathbb{R}^{n_i\times n_i} > 0$, $U_i \in \mathbb{R}^{m_i\times m_i} > 0$, $P_j, R_j \in \mathbb{R}^{n_j\times n_j} > 0$, for $j = 1,\ldots,M$ and $j \neq i$, satisfy the LMIs

$$\Phi_i = \begin{pmatrix} \phi_{11}^i & P_iL_iC_i & P_iB_i & 0 & 0 & 0 \\ * & \phi_{22}^i & 0 & -\lambda_iC_i^TL_i^T & R_i\mathcal{F}_i & R_i\mathcal{F}_i \\ * & * & -U_i & 0 & 0 & 0 \\ * & * & * & -\lambda_iI_{n_i} & 0 & 0 \\ * & * & * & * & -\phi_{55}^i & 0 \\ * & * & * & * & * & -\phi_{66}^i \end{pmatrix} < 0 \tag{17}$$

$$\Psi_i = \begin{pmatrix} U_i & U_iK_i \\ K_i^TU_i^T & \frac{\lambda_i}{r_i}e^{-2(1+2\alpha)r_i-2|A_i|r_i}I_{n_i} \end{pmatrix} > 0 \tag{18}$$

where Φ_i is a symmetric matrix, $I_{n_i} \in \mathbb{R}^{n_i\times n_i}$ is a unit matrix, $|A_i| = \sqrt{\lambda_{\max}(A_i^TA_i)}$, and

$$\begin{aligned} \phi_{11}^i &= (A_i + B_iK_i)^TP_i + P_i(A_i + B_iK_i) + 2\alpha P_i \\ \phi_{22}^i &= (A_i - L_iC_i)^TR_i + R_i(A_i - L_iC_i) + 2\alpha R_i \\ \phi_{55}^i &= \mathrm{diag}_{j=1,\ldots,M}\left\{\tfrac{2\epsilon}{M-1}P_j, j\neq i\right\} \\ \phi_{66}^i &= \mathrm{diag}_{j=1,\ldots,M}\left\{\tfrac{2\epsilon}{M-1}R_j, j\neq i\right\} \\ \mathcal{F}_i &= \mathrm{row}_{j=1,\ldots,M}\left\{F_{ij}, j\neq i\right\}. \end{aligned}$$

Then, the closed-loop large-scale system is exponentially stable with a decay rate $\rho = \alpha - \epsilon$.

Proof: The Lyapunov–Krasovskii functional (LKF) is selected as $V_i(t) = V_{P_i}(t) + V_{R_i}(t) + V_{U_i}(t)$ where

$$V_{P_i}(t) = \hat{x}_i^T(t)P_i\hat{x}_i(t), \quad P_i > 0 \tag{19}$$

$$V_{R_i}(t) = \tilde{x}_i^T(t)R_i\tilde{x}_i(t), \quad R_i > 0 \tag{20}$$

$$V_{U_i}(t) = \int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iw_i(\sigma,t)d\sigma, \quad U_i > 0. \tag{21}$$

Taking the time derivative of (19) along (13), we have

$$\begin{aligned} &\dot{V}_{P_i}(t) + 2\alpha V_{P_i}(t) \\ &= \hat{x}_i^T(t)\left(2P_i(A_i + B_iK_i) + 2\alpha P_i\right)\hat{x}_i(t) \\ &\quad + 2\hat{x}_i^T(t)P_iB_iw_i(0,t) + 2\hat{x}_i^T(t)P_iL_iC_i\tilde{x}_i(t). \end{aligned} \tag{22}$$

Taking the time derivative of (20) along (14), we get

$$\begin{aligned} &\dot{V}_{R_i}(t) + 2\alpha V_{R_i}(t) \\ &= \tilde{x}_i^T(t)\left(2R_i(A_i - L_iC_i) + 2\alpha R_i\right)\tilde{x}_i(t) \\ &\quad + 2\tilde{x}_i^T(t)R_i\sum_{i\neq j}F_{ij}\left(\hat{x}_j(t) + \tilde{x}_j(t)\right). \end{aligned} \tag{23}$$

Taking the time derivative of (21) along (15)–(16) and using the integration by parts in σ, we obtain

$$\begin{aligned} &\dot{V}_{U_i}(t) + 2\alpha V_{U_i}(t) \\ &= 2\int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_i\partial_\sigma w_i(\sigma,t)d\sigma \\ &\quad - 2\int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iK_ie^{A_i\sigma}d\sigma L_iC_i\tilde{x}_i(t) \\ &\quad + 2\alpha\int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iw_i(\sigma,t)d\sigma \\ &= -w_i^T(0,t)U_iw_i(0,t) - 2\xi_i^T(t)L_iC_i\tilde{x}_i(t) \\ &\quad - \int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iw_i(\sigma,t)d\sigma \end{aligned} \tag{24}$$

where $\xi_i^T(t) = \int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iK_ie^{A_i\sigma}d\sigma$.

Utilizing Jensen's inequality, $\xi_i^T(t)$ satisfies

$$\begin{aligned} \left|\xi_i^T(t)\right|^2 &= \left|\int_0^{r_i} e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iK_ie^{A_i\sigma}d\sigma\right|^2 \\ &\le r_i\int_0^{r_i}\left|e^{(1+2\alpha)\sigma}w_i^T(\sigma,t)U_iK_ie^{A_i\sigma}\right|^2d\sigma \\ &\le \underbrace{r_ie^{2(1+2\alpha)r_i+2|A_i|r_i}}_{\mu_i}\int_0^{r_i}\left|w_i^T(\sigma,t)U_iK_i\right|^2d\sigma. \end{aligned} \tag{25}$$

From (22)–(25), we have

$$\begin{aligned} &\dot{V}_i(t) + 2\alpha V_i(t) - \frac{2\epsilon}{M-1}\sum_{i\neq j}V_j(t) \\ &\quad + \frac{1}{\lambda_i}\left(\mu_i\int_0^{r_i}\left|w_i^T(\sigma,t)U_iK_i\right|^2d\sigma - \left|\xi_i^T(t)\right|^2\right) \\ &\le -\int_0^{r_i} w_i^T(\sigma,t)\left(U_i - \frac{\mu_i}{\lambda_i}U_iK_iK_i^TU_i^T\right)w_i(\sigma,t)d\sigma \end{aligned}$$

$$+\eta_i^T(t)\text{diag}\left\{I,I,I,\frac{1}{\lambda_i}I,I,I\right\}$$
$$\Phi_i\text{diag}\left\{I,I,I,\frac{1}{\lambda_i}I,I,I\right\}\eta_i(t)\le 0 \tag{26}$$

where $\lambda_i > 0$ and $\eta_i(t) = \text{col}\{\hat{x}_i(t), \tilde{x}_i(t), w_i(0,t), \xi_i(t), \text{col}_{j=1,\dots,M}\{\hat{x}_j(t), j\neq i\}, \text{col}_{j=1,\dots,M}\{\tilde{x}_j(t), j\neq i\}\}$ and I is a unit matrix of appropriate dimension.

Applying Schur complement lemma in [3, Sec. 3.2.3], inequality (26) is implied by LMI condition (17)–(18). From (26), we conclude the LKF candidate along the solution of closed-loop system (13)–(16) satisfies $\dot{V}_i(t) + 2\alpha V_i(t) \le \frac{2\epsilon}{M-1}\sum_{j\neq i} V_j(t)$, then we have $\dot{V}(t) + 2(\alpha-\epsilon)V(t)\le 0$ where $V(t) = \sum_{i=1}^M V_i(t)$, which implies the exponential stability of the closed-loop system by the comparison principle. ■

Remark 1: Given any large delays, as long as the couplings among the large-scale systems are not strong, the PDE-based LMIs (17)–(18) are always feasible. When $F_{ij} = 0$ in (1), which implies there is no interaction among subsystems, applying Schur complement to (18), the LMIs (17)–(18) are reduced to

$$\bar{\phi}_i = \begin{pmatrix} \phi_{11}^i & P_iL_iC_i & P_iB_i & 0 \\ * & \phi_{22}^i & 0 & -\lambda_i C_i^T L_i^T \\ * & * & -U_i & 0 \\ * & * & * & -\lambda_i I_{n_i}\end{pmatrix} < 0$$
$$U_i - \frac{\mu_i}{\lambda_i}U_iK_iK_i^TU_i^T > 0. \tag{27}$$

Let λ_i $(i = 1,\dots,M)$ be large scalars. Then, there exist U_i that satisfy the second inequality in (27). Since $(A_i + B_iK_i)$ in ϕ_{11}^i and $(A_i - L_iC_i)$ in ϕ_{22}^i are assumed to be Hurwitz, for some $\alpha > 0$, there exist P_i and R_i such that $\bar{\phi}_i < 0$. Fix next $\epsilon \in (0,\alpha)$. Applying Schur complement to (17) with P_i, R_i, U_i subject to (27), we obtain

$$\bar{\phi}_i + \begin{pmatrix} 0 \\ R_i\mathcal{F}_i \\ 0 \\ 0\end{pmatrix}\left({\phi_{55}^i}^{-1} + {\phi_{66}^i}^{-1}\right)\left(0\ \mathcal{F}_i^T R_i^T\ 0\ 0\right)$$
$$= \bar{\phi}_i + \frac{M-1}{2\epsilon}\sum_{j\neq i}\begin{pmatrix} 0 & 0 & 0\ 0 \\ 0\ R_iF_{ij}\left(P_j^{-1}+R_j^{-1}\right)F_{ij}^TR_i^T & & 0\ 0 \\ 0 & 0 & 0\ 0 \\ 0 & 0 & 0\ 0\end{pmatrix} < 0. \tag{28}$$

By selecting F_{ij} such that

$$F_{ij}F_{ij}^T \le \omega_j\lambda_{\max}^{-1}\left({P_j}^{-1} + {R_j}^{-1}\right)I \tag{29}$$

where ω_j are small scalars and $\lambda_{\max}(\cdot)$ denote the maximum eigenvalue of the matrix, we achieve the feasibility of LMIs (17)–(18).

B. Stability Analysis: Decentralized Versus Centralized

In this section, treating the large-scale system as an overall system, we apply a full-order LKF to stability analysis, and compare the conventionally centralized method with the decentralized one of Theorem 1.

Please note the delay r_i of each subsystem appears in the limit of integration of (21). In order to construct a global LKF, this should be avoided. Thus, we apply the rescaled unity-interval notation to (3) so that (3) is replaced by $v_i(\sigma,t) = u_i(t + r_i(\sigma-1))$, $\sigma\in[0,1]$. Accordingly, the target system (15)–(16) becomes

$$r_i\partial_t w_i(\sigma,t) = \partial_\sigma w_i(\sigma,t) - r_iK_ie^{A_ir_i\sigma}L_iC_i\tilde{x}_i(t) \tag{30}$$
$$w_i(1,t) = 0,\quad \sigma\in[0,1]. \tag{31}$$

TABLE I
COMPARISON BETWEEN DECENTRALIZED AND CENTRALIZED LMIS

Theorem	No. of Decision Variables	No. of Lines	Time
Decentralized (M=2)	44	52	0.195
Centralized (M=2)	140	52	0.228
Decentralized (M=3)	66	102	0.484
Centralized (M=3)	307	78	0.497

To analyze the large-scale systems as an overall system, we integrate (13)–(14) and (30)–(31) for $i = 1,2,\dots,M$ into

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}w(0,t) \tag{32}$$
$$\bar{R}\partial_t w(\sigma,t) = \partial_\sigma w(\sigma,t) - \bar{K}\bar{E}(\sigma)\bar{L}\bar{x}(t) \tag{33}$$
$$w(1,t) = 0 \tag{34}$$

where $\bar{x}(t) = [\hat{x}_1(t)^T,\dots,\hat{x}_M(t)^T,\tilde{x}_1(t)^T,\dots,\tilde{x}_M(t)^T]^T$, $w(\sigma,t) = [w_1(\sigma,t)^T,\dots,w_M(\sigma,t)^T]^T$, and as also given in the unnumbered equation shown at the bottom of the next page.

Proposition 1: Consider the closed-loop system consisting of the plant (4)–(7), observer (8), and controller (10). Given tuning parameters $\rho > 0$ and $\mu > \max\{r_1,\dots,r_M\}$, let parameter $\lambda > 0$, matrices $\bar{P}\in\mathbb{R}^{2(n_1+\cdots+n_M)\times 2(n_1+\cdots+n_M)} > 0$, $\bar{U}\in\mathbb{R}^{(m_1+\cdots+m_M)\times(m_1+\cdots+m_M)} > 0$, satisfy the LMIs

$$\Phi = \begin{pmatrix} \bar{A}^T\bar{P} + \bar{P}\bar{A} + 2\rho\bar{P} & \bar{P}\bar{B} & -\lambda\bar{L}^T \\ * & -\bar{R}\bar{U} & 0 \\ * & * & -\lambda I\end{pmatrix} < 0 \tag{35}$$
$$\Psi = \begin{pmatrix} 2\rho\bar{R}\bar{U}\left(\mu I - \bar{R}\right) & \bar{R}\bar{U}\bar{K} \\ \bar{K}^T\bar{U}\bar{R} & \frac{\lambda}{\gamma}I\end{pmatrix} > 0 \tag{36}$$

where Φ is a symmetric matrix, I is a unit matrix with appropriate dimension, $\gamma = e^{4\rho\mu}|\text{diag}\{e^{2|A_1|r_1},\dots,e^{2|A_M|r_M}\}|$. Then, the closed-loop system is exponentially stable with a decay rate ρ.

Proof: The global LKF is selected as $V(t) = V_{\bar{P}}(t) + V_{\bar{U}}(t)$ where

$$V_{\bar{P}}(t) = \bar{x}^T(t)\bar{P}\bar{x}(t),\quad \bar{P} > 0 \tag{37}$$
$$V_{\bar{U}}(t) = \int_0^1 e^{2\rho\mu\sigma}w^T(\sigma,t)\bar{R}\bar{U}\bar{R}w(\sigma,t)d\sigma,\quad \bar{U} > 0. \tag{38}$$

Note that the tuning parameter $\mu > 0$ is inserted into $V_{\bar{U}}$ of (38) to address the asymmetric structure of (30) with r_i multiplying $\partial_t w_i$. Following a similar argument of the proof of Theorem 1, we conclude that the inequality $\dot{V}(t) + 2\rho V(t)\le 0$ is implied by (35) and (36). ■

Remark 2: Comparing Theorem 1 with Proposition 1, we have the following.

1) Decentralized analysis: The number of lines of the coupled matrices (Φ_i,Ψ_i) of all subsystems is $(3n_i + m_i + 2\sum_{j\neq i} n_j)M + (m_i + n_i)M$. The number of decision variables $(P_i, R_i, U_i, \lambda_i)$ is $\sum_{i=1}^M(\frac{n_i^2+n_i}{2} + \frac{n_i^2+n_i}{2} + \frac{m_i^2+m_i}{2} + 1)$. The number of tuning parameters (α,ϵ) is 2.
2) Centralized analysis: The number of lines of the matrix (Φ,Ψ) is $4\sum_{i=1}^M n_i + \sum_{i=1}^M m_i + \sum_{i=1}^M m_i + 2\sum_{i=1}^M n_i$. The number of decision variables $(\bar{P},\bar{U},\lambda)$ is $\frac{(2\sum_{i=1}^M n_i)^2 + 2\sum_{i=1}^M n_i}{2} + \frac{(\sum_{i=1}^M m_i)^2 + \sum_{i=1}^M m_i}{2} + 1$. The number of tuning parameters (ρ,μ) is 2.

It is seen that the decentralized LMIs possess less decision variables but more lines than the centralized LMI as the number of subsystems increases (see Table I). In other words, there is a tradeoff between the number of decision variables and lines via distinct LMIs.

What is more important, when delays and sampling intervals vary from subsystem to subsystem, the decentralized analysis under decentralized predictors is performed in a simpler way. Note that, for decentralized analysis, there is no need to group the closed-loop subplants [which is in a decentralized manner (13)–(16)] into a complex global body like (32)–(34). This simplicity is more evident in the sampled-data case where the centralized closed-loop system is more complicated since it contains asynchronous sampling terms and multiple inverse Artstein transformations due to distinct delays [see (67) and (68)]. Correspondingly, multiple integral terms [see V_{U_i}, V_{W_i} in (69)] and additional inequalities [see (71)] should be inserted into the LKF-based centralized analysis to take care of multiple samplings and inverse Artstein transformations, which complicates the analysis.

III. Continuous-Time Feedback: ODE Approach

A. ODE Framework

In this section, we still address the case of continuous-time control. To lay a foundation for the sampled-data implementation in later sections, we employ the ODE scheme here.

Concentrating on the system (1)–(2), the variable $\hat{x}_i(t)$ is used to denote the estimate of the unmeasured state $x_i(t)$ with the estimation error $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$. The observer is designed as

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_i u_i(t - r_i) + L_i\left(y_i(t) - C_i\hat{x}_i(t)\right). \tag{39}$$

We introduce the observer-based predictor

$$\hat{z}_i(t) = e^{A_i r_i}\hat{x}_i(t) + \int_{t-r_i}^{t} e^{A_i(t-s)}B_i u_i(s)ds. \tag{40}$$

If the term of estimation error $L_i(y_i(t) - C_i\hat{x}_i(t)) = 0$ in (39), it is evident that (40) is an exact prediction of the future state $\hat{z}_i(t) = \hat{x}_i(t + r_i)$.

The predictor control law is selected as

$$u_i(t) = K_i\hat{z}_i(t) \tag{41}$$

$$= K_i\left(e^{A_i r_i}\hat{x}_i(t) + \int_{t-r_i}^{t} e^{A_i(t-s)}B_i u_i(s)ds\right). \tag{42}$$

For stability analysis, taking the time derivative of (40) along (39), the dynamics of $\hat{z}_i(t)$ is calculated as

$$\begin{aligned}\dot{\hat{z}}_i(t) &= A_i\hat{z}_i(t) + B_i u_i(t) + e^{A_i r_i}L_i\left(y_i(t) - C_i\hat{x}_i(t)\right)\\ &= (A_i + B_i K_i)\hat{z}_i(t) + e^{A_i r_i}L_i C_i\tilde{x}_i(t).\end{aligned} \tag{43}$$

Making use of (41), the inverse conversion of (40) for the jth plant is brought in as

$$\begin{aligned}\hat{x}_j(t) &= e^{-A_j r_j}\hat{z}_j(t) - \int_{t-r_j}^{t} e^{A_j(t-r_j-s)}B_j u_j(s)ds\\ &= e^{-A_j r_j}\hat{z}_j(t) - \int_{t-r_j}^{t} e^{A_j(t-r_j-s)}B_j K_j\hat{z}_j(s)ds\\ &= e^{-A_j r_j}\hat{z}_j(t) - \underbrace{\int_{-r_j}^{0} e^{-A_j(\theta+r_j)}B_j K_j\hat{z}_j(t+\theta)d\theta}_{\xi_j(t)}.\end{aligned} \tag{44}$$

Subtracting (39) from (1) and substituting (44), the estimation error $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ satisfies

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A_i\tilde{x}_i(t) - L_i\left(y_i(t) - C_i\hat{x}_i(t)\right) + \sum_{i\neq j} F_{ij}x_j(t)\\ &= (A_i - L_i C_i)\tilde{x}_i(t) + \sum_{i\neq j} F_{ij}\left(\tilde{x}_j(t) + \hat{x}_j(t)\right)\\ &= (A_i - L_i C_i)\tilde{x}_i(t) + \sum_{i\neq j} F_{ij}\left(\tilde{x}_j(t) + e^{-A_j r_j}\hat{z}_j(t) - \xi_j(t)\right).\end{aligned} \tag{45}$$

$$\bar{A} = \begin{bmatrix} A_1+B_1K_1 & 0 & \cdots & 0 & L_1C_1 & 0 & \cdots & 0\\ 0 & A_2+B_2K_2 & \cdots & 0 & 0 & L_2C_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & A_M+B_MK_M & 0 & 0 & \cdots & L_MC_M\\ 0 & F_{12} & \cdots & F_{1M} & A_1-L_1C_1 & F_{12} & \cdots & F_{1M}\\ F_{21} & 0 & \cdots & F_{2M} & F_{21} & A_2-L_2C_2 & \cdots & F_{2M}\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots\\ F_{M1} & F_{M2} & \cdots & 0 & F_{M1} & F_{M2} & \cdots & A_M-L_MC_M \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} B_1 & & \\ & \ddots & \\ & & B_M\\ 0_{n_1\times m_1} & & \\ & \ddots & \\ & & 0_{n_M\times m_M} \end{bmatrix} \quad \bar{R} = \begin{bmatrix} r_1 I_{m_1} & & \\ & \ddots & \\ & & r_M I_{m_M} \end{bmatrix} \quad \bar{K} = \begin{bmatrix} 0_{m_1\times n_1} & & & r_1K_1 & & \\ & \ddots & & & \ddots & \\ & & 0_{m_M\times n_M} & & & r_MK_M \end{bmatrix}$$

$$\bar{E}(\sigma) = \begin{bmatrix} 0_{n_1\times n_1} & & & & & \\ & \ddots & & & & \\ & & 0_{n_M\times n_M} & & & \\ & & & e^{A_1 r_1\sigma} & & \\ & & & & \ddots & \\ & & & & & e^{A_M r_M\sigma} \end{bmatrix} \quad \bar{L} = \begin{bmatrix} 0_{n_1\times n_1} & & & & & \\ & \ddots & & & & \\ & & 0_{n_M\times n_M} & & & \\ & & & L_1C_1 & & \\ & & & & \ddots & \\ & & & & & L_MC_M \end{bmatrix}$$

Remark 3: In [10] where a single plant is considered, besides the observer predictor (40), the plant predictor is also introduced such that $z_i(t) = e^{A_i r_i} x_i(t) + \int_{t-r_i}^{t} e^{A_i(t-s)} B_i u_i(s) ds$. If the method of [10] is applied to large-scale systems, an alternative version of the closed-loop system (43) and (45) is of the form: $\dot{\hat{z}}_i(t) = (A_i + B_i K_i)\hat{z}_i(t) + e^{A_i r_i} L_i C_i e^{-A_i r_i} \tilde{z}_i(t)$, $\dot{\tilde{z}}_i(t) = (A_i - e^{A_i r_i} L_i C_i e^{-A_i r_i})\tilde{z}_i(t) + e^{A_i r_i} \sum_{i\neq j} F_{ij}(e^{-A_j r_j} \tilde{z}_j(t) + e^{-A_j r_j} \hat{z}_j(t) - \xi_j(t))$, where $\tilde{z}_i(t) = z_i(t) - \hat{z}_i(t)$. It is apparent that (43) and (45) proposed in the present article is simpler and the redundant change of variable $z_i(t)$ is avoided. The similar analysis can be extended to later sections.

Theorem 2: Consider the closed-loop system consisting of the plant (1)–(2), observer (39), and controller (42). Given tuning parameters $0 < \epsilon < \alpha$, let matrices $P_i, R_i, W_i \in \mathbb{R}^{n_i \times n_i} > 0$, $P_j, R_j, W_j \in \mathbb{R}^{n_j \times n_j} > 0$, for $j = 1, \ldots, M$ and $j \neq i$, satisfy the LMIs

$$\Phi_i = \begin{pmatrix} \phi_{11}^i & P_i e^{A_i r_i} L_i C_i & 0 & 0 & 0 \\ & \phi_{22}^i & R_i \mathcal{F}_i & R_i \mathcal{F}_i^{\hat{z}} & -R_i \mathcal{F}_i \\ & * & -\phi_{33}^i & 0 & 0 \\ & * & * & -\phi_{44}^i & 0 \\ & * & * & * & -\phi_{55}^i \end{pmatrix} < 0 \tag{46}$$

where Φ_i is a symmetric matrix, and

$$\phi_{11}^i = (A_i + B_i K_i)^T P_i + P_i(A_i + B_i K_i) + 2\alpha P_i + \bar{W}_i$$
$$\bar{W}_i = r_i K_i^T B_i^T \left(\int_{-r_i}^{0} e^{-A_i^T(\theta + r_i)} W_i e^{-A_i(\theta + r_i)} d\theta \right) B_i K_i$$
$$\phi_{22}^i = (A_i - L_i C_i)^T R_i + R_i(A_i - L_i C_i) + 2\alpha R_i$$
$$\phi_{33}^i = \mathrm{diag}_{j=1,\ldots,M} \left\{ \frac{2\epsilon}{M-1} R_j, j \neq i \right\}$$
$$\phi_{44}^i = \mathrm{diag}_{j=1,\ldots,M} \left\{ \frac{2\epsilon}{M-1} P_j, j \neq i \right\}$$
$$\phi_{55}^i = \mathrm{diag}_{j=1,\ldots,M} \left\{ \frac{1}{M-1} e^{-2\alpha r_j} W_j, j \neq i \right\}$$
$$\mathcal{F}_i = \mathrm{row}_{j=1,\ldots,M} \{F_{ij}, j \neq i\}$$
$$\mathcal{F}_i^{\hat{z}} = \mathrm{row}_{j=1,\ldots,M} \{F_{ij} e^{-A_j r_j}, j \neq i\}.$$

Then, the closed-loop system is exponentially stable with a decay rate $\rho = \alpha - \epsilon$. ■

Proof: The LKF is constructed as $V_i(t) = V_{P_i}(t) + V_{R_i}(t) + V_{W_i}(t)$ where

$$V_{P_i}(t) = \hat{z}_i^T(t) P_i \hat{z}_i(t), \quad P_i > 0 \tag{47}$$
$$V_{R_i}(t) = \tilde{x}_i^T(t) R_i \tilde{x}_i(t), \quad R_i > 0 \tag{48}$$
$$V_{W_i}(t) = r_i \int_{-r_i}^{0} \int_{t+\theta}^{t} e^{2\alpha(s-t)} \hat{z}_i^T(s) K_i^T B_i^T e^{-A_i^T(\theta + r_i)} W_i \times e^{-A_i(\theta + r_i)} B_i K_i \hat{z}_i(s) ds d\theta, \quad W_i > 0. \tag{49}$$

Please note that $V_{W_i}(t)$ is used to handle the distributed delay $\xi_j(t)$ in (45).

Taking the time derivative of (47) along (43), we have

$$\begin{aligned} &\dot{V}_{P_i}(t) + 2\alpha V_{P_i}(t) \\ &= \hat{z}_i^T(t)\left(2P_i(A_i + B_i K_i) + 2\alpha P_i\right)\hat{z}_i(t) \\ &\quad + 2\hat{z}_i^T(t) P_i e^{A_i r_i} L_i C_i \tilde{x}_i(t). \end{aligned} \tag{50}$$

Taking the time derivative of (48) along (45), we get

$$\begin{aligned} &\dot{V}_{R_i}(t) + 2\alpha V_{R_i}(t) \\ &= \tilde{x}_i^T(t)\left(2R_i(A_i - L_i C_i) + 2\alpha R_i\right)\tilde{x}_i(t) \\ &\quad + 2\tilde{x}_i^T(t) R_i \sum_{i\neq j} F_{ij}\left(\tilde{x}_j(t) + e^{-A_j r_j}\hat{z}_j(t) - \xi_j(t)\right). \end{aligned} \tag{51}$$

Taking the time derivative of (49) and using Jensen's inequality, we have

$$\begin{aligned} &\dot{V}_{W_i}(t) + 2\alpha V_{W_i}(t) = r_i \hat{z}_i^T(t) K_i^T B_i^T \\ &\times \left(\int_{-r_i}^{0} e^{-A_i^T(\theta + r_i)} W_i e^{-A_i(\theta + r_i)} d\theta \right) B_i K_i \hat{z}_i(t) \\ &- r_i \int_{-r_i}^{0} e^{2\alpha\theta} \hat{z}_i^T(t+\theta) K_i^T B_i^T e^{-A_i^T(\theta + r_i)} \\ &\times W_i e^{-A_i(\theta + r_i)} B_i K_i \hat{z}_i(t+\theta) d\theta \\ &\leq \hat{z}_i^T(t) \bar{W}_i \hat{z}_i(t) \\ &- e^{-2\alpha r_i} \left(\int_{-r_i}^{0} \hat{z}_i^T(t+\theta) K_i^T B_i^T e^{-A_i^T(\theta + r_i)} d\theta \right) \\ &\times W_i \left(\int_{-r_i}^{0} e^{-A_i(\theta + r_i)} B_i K_i \hat{z}_i(t+\theta) d\theta \right) \\ &= \hat{z}_i^T(t) \bar{W}_i \hat{z}_i(t) - e^{-2\alpha r_i} \xi_i^T(t) W_i \xi_i(t) \end{aligned} \tag{52}$$

where $\bar{W}_i$ has been given underneath (46).

From (50)–(52), we get

$$\begin{aligned} &\dot{V}_i(t) + 2\alpha V_i(t) - \frac{2\epsilon}{M-1} \sum_{j\neq i} V_j(t) + e^{-2\alpha r_i} \xi_i^T(t) W_i \xi_i(t) \\ &- \frac{1}{M-1} \sum_{j\neq i} e^{-2\alpha r_j} \xi_j^T(t) W_j \xi_j(t) \\ &\leq \eta_i^T(t) \Phi_i \eta_i(t) \leq 0 \end{aligned} \tag{53}$$

where $\eta_i(t) = \mathrm{col}\{\hat{z}_i(t), \tilde{x}_i(t), \mathrm{col}_{j=1,\ldots,M}\{\tilde{x}_j(t), j \neq i\}, \mathrm{col}_{j=1,\ldots,M}\{\hat{z}_j(t), j \neq i\}, \mathrm{col}_{j=1,\ldots,M}\{\xi_j(t), j \neq i\}\}$. It is apparent that inequality (53) is suggested by LMI condition (46). Thus, we derive $\dot{V}(t) + 2(\alpha - \epsilon)V(t) \leq 0$ from (53) where $V(t) = \sum_{i=1}^{M} V_i(t)$, which implies the exponential stability of the closed-loop system. ■

Remark 4: Similar to Remark 1, we check the feasibility of the ODE-based LMI (46) for nonstrong coupling. When $F_{ij} = 0$ in (1), which implies there is no interaction among subsystems, the LMI (46) is reduced to

$$\bar{\phi}_i = \begin{pmatrix} \phi_{11}^i & P_i e^{A_i r_i} L_i C_i \\ * & \phi_{22}^i \end{pmatrix} < 0. \tag{54}$$

Given $\alpha > \epsilon > 0$, applying Schur complement to (46) with P_i and R_i subject to (54), we obtain

$$\begin{aligned} &\bar{\phi}_i + \begin{pmatrix} 0 \\ R_i \mathcal{F}_i \end{pmatrix} \left({\phi_{33}^i}^{-1} + {\phi_{55}^i}^{-1} \right) \left(0 \ \mathcal{F}_i^T R_i^T \right) \\ &\quad + \begin{pmatrix} 0 \\ R_i \mathcal{F}_i^{\hat{z}} \end{pmatrix} {\phi_{44}^i}^{-1} \left(0 \ \mathcal{F}_i^{\hat{z}\,T} R_i^T \right) = \\ &\bar{\phi}_i + (M-1) \sum_{j\neq i} \begin{pmatrix} 0 & 0 \\ 0 & R_i F_{ij} \left(\frac{1}{2\epsilon} {R_j}^{-1} + e^{2\alpha r_j} W_j^{-1} \right) F_{ij}^T R_i^T \end{pmatrix} \\ &+ \frac{(M-1)}{2\epsilon} \sum_{j\neq i} \begin{pmatrix} 0 & 0 \\ 0 & R_i F_{ij} e^{-A_j r_j} {P_j}^{-1} e^{-A_j^T r_j} F_{ij}^T R_i^T \end{pmatrix} < 0. \end{aligned} \tag{55}$$

By selecting F_{ij} such that

$$\begin{aligned} F_{ij} F_{ij}^T &\leq \omega_j \lambda_{\max}^{-1} \left(\tfrac{1}{2\epsilon} {R_j}^{-1} + e^{2\alpha r_j} W_j^{-1} \right) I \\ F_{ij} F_{ij}^T &\leq \omega_j \lambda_{\max}^{-1} \left(e^{-A_j r_j} {P_j}^{-1} e^{-A_j^T r_j} \right) I \end{aligned} \tag{56}$$

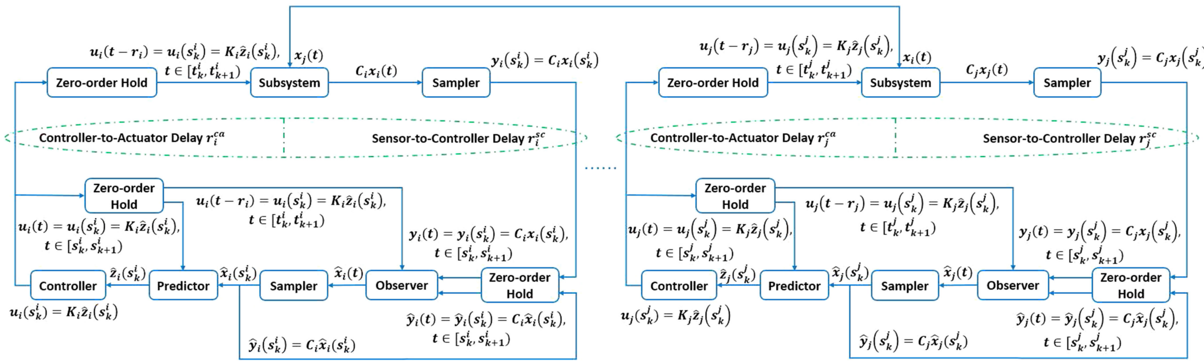

Fig. 1. Sampled-data control with discrete-time measurement for large-scale systems with large delays.

where ω_j are small scalars and $\lambda_{\max}(\cdot)$ denote the maximum eigenvalue of the matrix, we achieve the feasibility of LMI (46).

B. Time Delay in Interconnections

In this section, we consider a more complicated case where the system is subject to a time delay $d_j(t) \in [0, \bar{d}_j]$ in interactions between the subsystems. In this case, the system (1)–(2) is of the form

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t-r_i) + \textstyle\sum_{i\neq j} F_{ij} x_j(t-d_j(t)) \\ y_i(t) &= C_i x_i(t). \end{aligned} \tag{57}$$

The observer, predictor, and controller are identical with (39)–(42). As a result, the dynamics of $\hat{z}_i(t)$ takes the same form as (43).

Substituting $t - d_j(t)$ into (44), under the interconnection time delay, the estimation error is govern by

$$\begin{aligned} \dot{\tilde{x}}_i(t) = (A_i - L_i C_i)\tilde{x}_i(t) + \sum_{i\neq j} F_{ij} \big(\tilde{x}_j(t-d_j(t)) \\ + e^{-A_j r_j} \hat{z}_j(t-d_j(t)) - \xi_j(t)\big) \end{aligned} \tag{58}$$

where $\xi_j(t) = \int_{t-d_j(t)-r_j}^{t-d_j(t)} e^{A_j(t-d_j(t)-r_j-s)} B_j K_j \hat{z}_j(s) ds$.

Proposition 2: Consider the closed-loop system consisting of the plant (57), observer, and controller (39)–(42). Given tuning parameters $0 < (\epsilon_1 + \epsilon_2) < \alpha$, let matrices $P_i, R_i \in \mathbb{R}^{n_i \times n_i} > 0$ and $P_j, R_j \in \mathbb{R}^{n_j \times n_j} > 0$, and parameters $\lambda_j > 0$ for $j = 1, \ldots, M$ and $j \neq i$, satisfy the LMIs

$$\Phi_i < 0, \quad P_j - \lambda_j \gamma_j I_{n_j} > 0 \tag{59}$$

where Φ_i has the same structure as (46) without $\bar{W}_i$ in ϕ_{11}^i, $\gamma_j = r_j \int_0^{r_j} |e^{-A_j\theta} B_j K_j|^2 d\theta$ and

$$\begin{aligned} \phi_{33}^i &= \mathrm{diag}_{j=1,\ldots,M} \left\{ \tfrac{2\epsilon_1}{M-1} R_j, j \neq i \right\} \\ \phi_{44}^i &= \mathrm{diag}_{j=1,\ldots,M} \left\{ \tfrac{2\epsilon_1}{M-1} P_j, j \neq i \right\} \\ \phi_{55}^i &= \mathrm{diag}_{j=1,\ldots,M} \left\{ \tfrac{2\epsilon_2}{M-1} \lambda_j I_{n_j}, j \neq i \right\}. \end{aligned}$$

Then, the closed-loop system is exponentially stable with a decay rate ρ, which is a unique positive solution of the equation $\rho = \alpha - (\epsilon_1 + \epsilon_2) e^{2\rho(d+r)}$ with $d = \max_i \{\bar{d}_i\}$ and $r = \max_i \{r_i\}$.

Proof: Making use of Halanay's inequality, the proof is similar to that of Theorem 3. ∎

IV. Sampled-Data Feedback

In this section, we deal with the most challenging case of sampled-data feedback with discrete-time measurement. As shown in Fig. 1, let $\{s_k^i\}$ with $k \in \mathbb{Z}_0^+$ be sampling instants of the ith subsystem such that

$$0 = s_0^i < s_1^i < s_2^i < \cdots, \quad \lim_{k\to\infty} s_k^i = \infty, \quad s_{k+1}^i - s_k^i \le h_i. \tag{60}$$

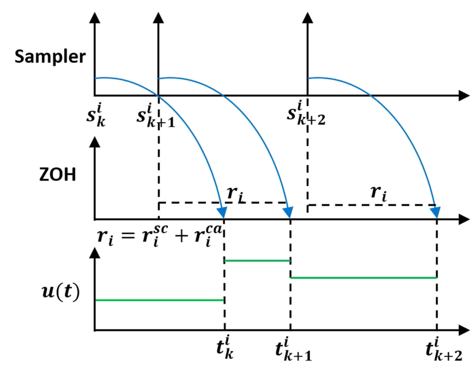

Fig. 2. Timing diagram of the NCS.

The system output $C_i x_i(t)$ is sampled at the instant s_k^i, and the sampled data are transmitted to the observer-based controller by the communication network subject to a known constant sensor-to-controller delay $r_i^{sc} > 0$. A control signal is calculated by the controller and transmitted to the zero-order hold through the communication network subject to a known constant controller-to-actuator delay $r_i^{ca} > 0$. Both r_i^{sc} and r_i^{ca} could be much larger than the sampling interval. The controller and zero-order hold are assumed to be event driven, which means they update their outputs as soon as they receive new data. Therefore the updating instants of the zero-order hold on the plant side satisfy $t_k^i = s_k^i + r_i$, where $r_i = r_i^{sc} + r_i^{ca}$. For the convenience, the timing diagram of the sampled-data feedback is revealed in Fig. 2.

Under the feedback loop of the NCS, the subsystem is of the form

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(s_k^i) + \textstyle\sum_{i\neq j} F_{ij} x_j(t), \quad t \in [t_k^i, t_{k+1}^i) \\ y_i(t) &= y_i(s_k^i) = C_i x_i(s_k^i), \quad t \in [s_k^i, s_{k+1}^i). \end{aligned} \tag{61}$$

For (61), in order to build a predictor-based controller to compensate delay, we define a piecewise function such that

$$u_i(t) = u_i(s_k^i), \quad t \in [s_k^i, s_{k+1}^i) \tag{62}$$

where $u_i(t) = 0$ for $t < 0$, and $u_i(s_k^i)$ denotes the control law employing the information sampled at the time instant s_k^i. Thus, (61) recovers the form of (1) and (2) such that

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t-r_i) + \textstyle\sum_{i\neq j} F_{ij} x_j(t), \quad t \in [t_k^i, t_{k+1}^i) \\ y_i(t) &= y_i(s_k^i) = C_i x_i(s_k^i), \quad t \in [s_k^i, s_{k+1}^i). \end{aligned} \tag{63}$$

Based on (61)–(63), the observer is designed as

$$\begin{aligned}\dot{\hat{x}}_i(t) &= A_i\hat{x}_i(t) + B_iu_i(s_k^i) + L_i\left(y_i(t) - \hat{y}_i(t)\right), \quad t \in [t_k^i, t_{k+1}^i)\\ &= A_i\hat{x}_i(t) + B_iu_i(t-r_i) + L_i\left(y_i(t) - \hat{y}_i(t)\right), \quad t \in [t_k^i, t_{k+1}^i)\\ \hat{y}_i(t) &= \hat{y}_i(s_k^i) = C_i\hat{x}_i(s_k^i), \quad t \in [s_k^i, s_{k+1}^i).\end{aligned} \tag{64}$$

Here, we assume that the clocks of the plant, observer, and controller are synchronized and the time stamp s_k^i is transmitted together with the signal so that $t_k^i = s_k^i + r_i$ can be calculated by the controller and observer. Moreover, similar to [10], the sampled data $y_i(s_k^i)$ and the time stamp s_k^i are available to the observer at the time $s_k^i + r_i^{sc}$; therefore, $\hat{x}_i(s_k^i)$ can be calculated by solving (64) on s_k^i, at the time $s_k^i + r_i^{sc}$. The benefit of such an observer design is that the subsequent estimation error (68) has a delay due to sampled data only (does not undergo additional delays).

According to (64), the observer-based predictor is introduced as

$$\hat{z}_i(t) = e^{A_ir_i}\hat{x}_i(t) + \int_{t-r_i}^{t} e^{A_i(t-s)}B_iu_i(s)ds \tag{65}$$

and the control law is designed as

$$\begin{aligned}u_i(s_k^i) &= K_i\hat{z}_i(s_k^i)\\ &= K_i\left(e^{A_ir_i}\hat{x}_i(s_k^i) + \int_{s_k^i-r_i}^{s_k^i} e^{A_i(s_k^i-s)}B_iu_i(s)ds\right).\end{aligned} \tag{66}$$

For stability analysis, taking the time derivative of $\hat{z}_i(t)$ in (65) along (64), we have

$$\begin{aligned}\dot{\hat{z}}_i(t) &= A_i\hat{z}_i(t) + B_iu_i(t) + e^{A_ir_i}L_i\left(y_i(t) - \hat{y}_i(t)\right), t \in [t_k^i, t_{k+1}^i)\\ &= A_i\hat{z}_i(t) + B_iK_i\hat{z}_i(s_k^i) + e^{A_ir_i}L_iC_i\tilde{x}_i(s_k^i)\\ &= (A_i + B_iK_i)\hat{z}_i(t) + B_iK_iv_{\hat{z}_i}(t) + e^{A_ir_i}L_iC_i\tilde{x}_i(t)\\ &\quad + e^{A_ir_i}L_iC_iv_{\tilde{x}_i}(t), \quad t \in [s_k^i, s_{k+1}^i) \cap [t_0^i, +\infty)\end{aligned} \tag{67}$$

where $v_{\hat{z}_i}(t) = \hat{z}_i(s_k^i) - \hat{z}_i(t)$ and $v_{\tilde{x}_i}(t) = \tilde{x}_i(s_k^i) - \tilde{x}_i(t)$.

Subtracting (64) from (63) and using the inverse transformation of (65), it is evident that the dynamics of the estimation error $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ is a "control-free" system such that

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A_i\tilde{x}_i(t) - L_i\left(y_i(t) - \hat{y}_i(t)\right) + \textstyle\sum_{i\neq j} F_{ij}x_j(t), t \in [t_k^i, t_{k+1}^i)\\ &= A_i\tilde{x}_i(t) - L_iC_i\tilde{x}_i(s_k^i) + \textstyle\sum_{i\neq j} F_{ij}x_j(t)\\ &= (A_i - L_iC_i)\tilde{x}_i(t) + \textstyle\sum_{i\neq j} F_{ij}\left(\tilde{x}_j(t) + e^{-A_jr_j}\hat{z}_j(t) - \xi_j(t)\right),\\ &\quad - L_iC_iv_{\tilde{x}_i}(t), \quad t \in [s_k^i, s_{k+1}^i) \cap [t_0^i, +\infty)\end{aligned} \tag{68}$$

where $\xi_j(t) = \int_{t-r_j}^{t} e^{A_j(t-r_j-s)}B_ju_j(s)ds = \int_0^{r_j} e^{-A_j\theta}B_ju_j(t - r_j + \theta)d\theta$.

To compensate the distributed input term $\xi_j(t)$ on the right-hand side of (68), utilizing (62) and (66), we get the following inequality:

$$\begin{aligned}|\xi_j(t)|^2 &= \left|\int_0^{r_j} e^{-A_j\theta}B_ju_j(t-r_j+\theta)d\theta\right|^2\\ &\le r_j\int_0^{r_j}\left|e^{-A_j\theta}B_ju_j(t-r_j+\theta)\right|^2 d\theta\\ &\le r_j\int_0^{r_j}\left|e^{-A_j\theta}B_j\right|^2 d\theta \cdot \sup_{\theta\in[-h_j-r_j,0]}\left|K_j\hat{z}_j(t+\theta)\right|^2\\ &\le \underbrace{r_j\int_0^{r_j}\left|e^{-A_j\theta}B_j\right|^2 d\theta\,|K_j|^2}_{\gamma_j}\sup_{\theta\in[-h_j-r_j,0]}\left|\hat{z}_j(t+\theta)\right|^2.\end{aligned} \tag{69}$$

Theorem 3: Consider the closed-loop system consisting of the plant (63), observer (64), and controller (62), (66). Given positive tuning parameters ϵ_1, ϵ_2 and α such that $\epsilon_1 + \epsilon_2 < \alpha$, let matrices $P_i, R_i, U_i, W_i \in \mathbb{R}^{n_i\times n_i} > 0$, $P_j, R_j \in \mathbb{R}^{n_j\times n_j} > 0$, and parameters $\lambda_j > 0$, for $j = 1, \ldots, M$ and $j \neq i$, satisfy the LMIs

$$\begin{pmatrix}\Phi_i & \Psi_i^{\hat{z}} & \Psi_i^{\tilde{x}}\\ * & -H_i^{\hat{z}} & 0\\ * & * & -H_i^{\tilde{x}}\end{pmatrix} < 0, \quad P_j - \lambda_j\gamma_jI_{n_j} > 0 \tag{70}$$

TABLE II
RESULTS WITH DIFFERENT FEEDBACK SCHEMES

	under the decay rate $\rho = 0.0001$	
	input delay r	sampled interval h
Continuous-time Predictor-free	0.058	-
Sampled-data Predictor-free	0.049	0.001
Theorem 1	0.2	-
Theorem 2	0.1	-
Theorem 3	0.082	0.001

where

$$\Psi_i^{\hat{z}} = \begin{pmatrix}(A_i + B_iK_i)^TH_i^{\hat{z}}\\ K_i^TB_i^TH_i^{\hat{z}}\\ C_i^TL_i^Te^{A_i^Tr_i}H_i^{\hat{z}}\\ C_i^TL_i^Te^{A_i^Tr_i}H_i^{\hat{z}}\\ 0\\ 0\\ 0\end{pmatrix}, \quad \Psi_i^{\tilde{x}} = \begin{pmatrix}0\\ 0\\ (A_i - L_iC_i)^TH_i^{\tilde{x}}\\ -C_i^TL_i^TH_i^{\tilde{x}}\\ \mathcal{F}_i^TH_i^{\tilde{x}}\\ \mathcal{F}_i^{\hat{z}\,T}H_i^{\tilde{x}}\\ -\mathcal{F}_i^TH_i^{\tilde{x}}\end{pmatrix}$$

$$H_i^{\hat{z}} = h_i^2e^{2\alpha h_i}U_i, \quad H_i^{\tilde{x}} = h_i^2e^{2\alpha h_i}W_i$$

and Φ_i is a symmetric matrix composed of

$$\begin{aligned}\Phi_{11}^i &= (A_i + B_iK_i)^TP_i + P_i(A_i + B_iK_i) + 2\alpha_iP_i\\ \Phi_{12}^i &= P_iB_iK_i, \quad \Phi_{13}^i = \Phi_{14}^i = P_ie^{A_ir_i}L_iC_i\\ \Phi_{22}^i &= -\tfrac{\pi^2}{4}U_i, \quad \Phi_{44}^i = -\tfrac{\pi^2}{4}W_i\\ \Phi_{33}^i &= (A_i - L_iC_i)^TR_i + R_i(A_i - L_iC_i) + 2\alpha_iR_i\\ \Phi_{34}^i &= -R_iL_iC_i\\ \Phi_{35}^i &= R_i\mathcal{F}_i, \quad \Phi_{37}^i = -R_i\mathcal{F}_i, \quad \mathcal{F}_i = \mathrm{row}_{j=1,\ldots,M}\{F_{ij}, j \neq i\}\\ \Phi_{36}^i &= R_i\mathcal{F}_i^{\hat{z}}, \quad \mathcal{F}_i^{\hat{z}} = \mathrm{row}_{j=1,\ldots,M}\left\{F_{ij}e^{-A_jr_j}, j \neq i\right\}\\ \Phi_{55}^i &= -\mathrm{diag}_{j=1,\ldots,M}\{\tfrac{2\epsilon_1}{M-1}R_j, j \neq i\}\\ \Phi_{66}^i &= -\mathrm{diag}_{j=1,\ldots,M}\{\tfrac{2\epsilon_1}{M-1}P_j, j \neq i\}\\ \Phi_{77}^i &= -\mathrm{diag}_{j=1,\ldots,M}\{\tfrac{2\epsilon_2}{M-1}\lambda_jI_{n_j}, j \neq i\}\end{aligned}$$

with $I_{n_j} \in \mathbb{R}^{n_j\times n_j}$ being an identity matrix. Then, the closed-loop system is exponentially stable with a decay rate ρ, which is a unique positive solution of the equation $\rho = \alpha - \epsilon_1 - \epsilon_2e^{2\rho(h+r)}$ with $h = \max_i\{h_i\}$, $r = \max_i\{r_i\}$.

Proof: The LKF is constructed as $V_i(t) = V_{P_i}(t) + V_{R_i}(t) + V_{U_i}(t) + V_{W_i}(t)$ where

$$\begin{aligned}V_{P_i}(t) &= \hat{z}_i^T(t)P_i\hat{z}_i(t), \quad P_i > 0\\ V_{R_i}(t) &= \tilde{x}_i^T(t)R_i\tilde{x}_i(t), \quad R_i > 0\\ V_{U_i}(t) &= h_i^2e^{2\alpha h_i}\int_{s_k^i}^{t} e^{2\alpha(s-t)}\dot{\hat{z}}_i^T(s)U_i\dot{\hat{z}}_i(s)ds\\ &\quad - \frac{\pi^2}{4}\int_{s_k^i}^{t} e^{2\alpha(s-t)}[\hat{z}_i(s_k^i) - \hat{z}_i(s)]^TU_i\\ &\quad \times [\hat{z}_i(s_k^i) - \hat{z}_i(s)]ds, \quad U_i > 0, \quad t \in [s_k^i, s_{k+1}^i)\\ V_{W_i}(t) &= h_i^2e^{2\alpha h_i}\int_{s_k^i}^{t} e^{2\alpha(s-t)}\dot{\tilde{x}}_i^T(s)W_i\dot{\tilde{x}}_i(s)ds\\ &\quad - \frac{\pi^2}{4}\int_{s_k^i}^{t} e^{2\alpha(s-t)}[\tilde{x}_i(s_k^i) - \tilde{x}_i(s)]^TW_i\\ &\quad \times [\tilde{x}_i(s_k^i) - \tilde{x}_i(s)]ds, \quad W_i > 0, \quad t \in [s_k^i, s_{k+1}^i).\end{aligned} \tag{71}$$

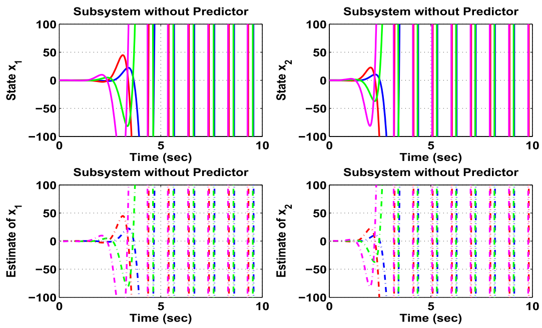

Fig. 3. Predictor-free feedback with small delays $r_1 = r_2 = 0.1$ s.

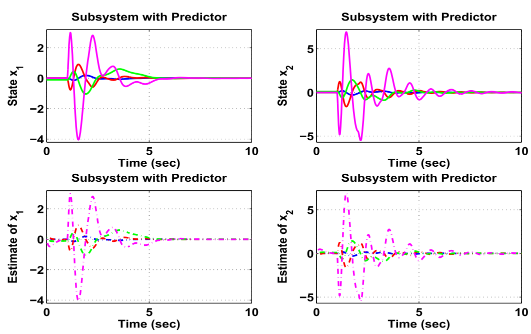

Fig. 4. Predictor-based feedback with large delays $r_1 = r_2 = 1$ s.

Taking the time derivative of (71), we get

$$\begin{aligned}
&\dot V_{P_i}(t)+2\alpha V_{P_i}(t)\\
&=\hat z_i^T(t)\left(2P_i(A_i+B_iK_i)+2\alpha P_i\right)\hat z_i(t)+2\hat z_i^T(t)P_iB_iK_iv_{\hat z_i}(t)\\
&\quad+2\hat z_i^T(t)P_ie^{A_ir_i}L_iC_i\tilde x_i(t)+2\hat z_i^T(t)P_ie^{A_ir_i}L_iC_iv_{\tilde x_i}(t)\\
&\dot V_{R_i}(t)+2\alpha V_{R_i}(t)\\
&=\tilde x_i^T(t)\left(2R_i(A_i-L_iC_i)+2\alpha R_i\right)\tilde x_i(t)-2\tilde x_i^T(t)R_iL_iC_iv_{\tilde x_i}(t)\\
&\quad+2\tilde x_i^T(t)R_i\textstyle\sum_{i\neq j}F_{ij}\left(\tilde x_j(t)+e^{-A_jr_j}\hat z_j(t)-\xi_j(t)\right)\\
&\dot V_{U_i}(t)+2\alpha V_{U_i}(t)\\
&=h_i^2e^{2\alpha h_i}\dot{\hat z}_i^T(t)U_i\dot{\hat z}_i(t)-\tfrac{\pi^2}{4}v_{\hat z_i}^T(t)U_iv_{\hat z_i}(t)\\
&\dot V_{W_i}(t)+2\alpha V_{W_i}(t)\\
&=h_i^2e^{2\alpha h_i}\dot{\tilde x}_i^T(t)W_i\dot{\tilde x}_i(t)-\tfrac{\pi^2}{4}v_{\tilde x_i}^T(t)W_iv_{\tilde x_i}(t).
\end{aligned}\tag{72}$$

Above all, we get

$$\begin{aligned}
&\dot V_i(t)+2\alpha V_i(t)-\tfrac{2\epsilon_1}{M-1}\textstyle\sum_{j\neq i}V_j(t)\\
&\quad-\tfrac{2\epsilon_2}{M-1}\textstyle\sum_{j\neq i}\sup_{\theta\in[-h_j-r_j,0]}V_j(t+\theta)\\
&\quad+\tfrac{2\epsilon_2}{M-1}\textstyle\sum_{j\neq i}\lambda_j\left(\gamma_j\sup_{\theta\in[-h_j-r_j,0]}|\hat z_j(t+\theta)|^2-|\xi_j(t)|^2\right)\\
&\le\eta_i^T(t)\Phi_i\eta_i(t)+\eta_i^T(t)\left(\Psi_i^{\hat z}\ \Psi_i^{\tilde x}\right)\begin{pmatrix}H_i^{\hat z^{-1}} & 0\\ 0 & H_i^{\tilde x^{-1}}\end{pmatrix}\begin{pmatrix}\Psi_i^{\hat z^T}\\ \Psi_i^{\tilde x^T}\end{pmatrix}\eta_i(t)\\
&\quad-\tfrac{2\epsilon_2}{M-1}\textstyle\sum_{j\neq i}\sup_{\theta\in[-h_j-r_j,0]}\hat z_j^T(t+\theta)\left(P_j-\lambda_j\gamma_jI_{n_j}\right)\hat z_j(t+\theta)\\
&\le 0
\end{aligned}\tag{73}$$

where $\eta_i(t)=\text{col}\{\hat z_i(t),v_{\hat z_i}(t),\tilde x_i(t),v_{\tilde x_i}(t),\text{col}_{j=1,\ldots,M}\{\tilde x_j(t),j\neq i\},\text{col}_{j=1,\ldots,M}\{\hat z_j(t),j\neq i\},\text{col}_{j=1,\ldots,M}\{\xi_j(t),j\neq i\}\}$. Applying Schur complement, inequality (73) implies (70) and

$$\dot V(t)+2(\alpha-\epsilon_1)V(t)-2\epsilon_2\sup_{\theta\in[-h-r,0]}V(t+\theta)\le 0 \tag{74}$$

where $V(t)=\sum_{i=1}^{M}V_i(t)$. By Halanay's inequality in [3, Secs. 4.1.2 and 4.7.2], inequality (74) means the exponentially stability of the closed-loop system. ■

V. Simulation

In this section, we use an example of two coupled inverted pendulums on two carts ($M=2$) from [5] and [6] under the decentralized control scheme. The results for three coupled pendulums ($M=3$) under the continuous-time measurements (see in [13]). As revealed in Table I, the decentralized LMIs have less decision variables but more lines comparatively to the centralized LMI. The simulation results under different feedback schemes are shown in Table II and Figs. 3 and 4. It is evident that the predictor-based controller promises a larger delay than the predictor-free controller.

References

[1] O. J. M. Smith, "A controller to overcome dead time," *ISA Trans.*, vol. 6, pp. 28–33, 1959.

[2] Z. Artstein, "Linear systems with delayed controls: A reduction," *IEEE Trans. Autom. Control*, vol. AC-27, no. 4, pp. 869–879, Aug. 1982.

[3] E. Fridman, *Introduction to Time-Delay Systems: Analysis and Control*. Cambridge, MA, USA: Birkhäuser, 2014.

[4] D. Freirich and E. Fridman, "Decentralized networked control of systems with local networks: A time-delay approach," *Automatica*, vol. 69, pp. 201–209, 2016.

[5] V.-S. Dolk, D.-P. Borgers, and W. Heemels, "Output-based and decentralized dynamic event-triggered control with guaranteed L_p-gain performance and zeno-freeness," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 34–49, Jan. 2017.

[6] W. Heemels, D. Borgers, N. van de Wouw, D. Nesic, and A. Teel, "Stability analysis of nonlinear networked control systems with asynchronous communication: A small-gain approach," in *Proc. 52nd IEEE Conf. Decis. Control*, Florence, Italy, Dec. 10–13, 2013, pp. 4631–4637.

[7] K Liu and E Fridman, "Wirtinger's inequality and Lyapunov-based sampled-data stabilization," *Automatica*, vol. 48, pp. 102–108, 2012.

[8] M. Krstic, *Delay Compensation for Nonlinear, Adaptive, and PDE Systems*. Berlin, Germany: Birkhäuser, 2009.

[9] K.-Z. Liu, X.-M. Sun, and M. Krstic, "Distributed predictor-based stabilization of continuous interconnected systems with input delays," *Automatica*, vol. 91, pp. 69–78, 2018.

[10] A. Selivanov and E. Fridman, "Observer-based input-to-state stabilization of networked control systems with large uncertain delays," *Automatica*, vol. 74, pp. 63–70, 2016.

[11] V. Lechappe, E. Moulay, F. Plestan, and Q.-L. Han, "Discrete predictor-based event-triggered control of networked control systems," *Automatica*, vol. 107, pp. 281–288, 2019.

[12] Y. Zhu and E. Fridman, "Predictor methods for decentralized control of large-scale systems with input delays," *Automatica*, vol. 116, pp. 1–9, 2020.

[13] Y. Zhu and E. Fridman, "Decentralized predictor output feedback for large-scale systems with large delays," in *Proc. IFAC World Congr.*, Berlin, Germany, to be published.

[14] Z. Wu and W. Michiels, "Reliably computing all characteristic roots of delay differential equations in a given right half plane using a spectral method," *J. Comput. Appl. Math.*, vol. 236, pp. 2499–2514, 2012.

[15] T. Vyhlidal and P. Zitek, "QPmR-quasi-polynomial root-finder: Algorithm update and examples," in *Delay Systems: From Theory to Numerics and Appl.*. New York, NY, USA: Springer, 2014, pp. 299–312.

[16] W. Michiels and S. I. Niculescu, *Stability and Stabilization of Time-Delay Systems: An Eigenvalue-Based Approach*. Philadelphia, PA, USA: SIAM, 2007.

[17] L. Bakule, "Decentralized control: An overview," *Annu. Rev. Control*, vol. 32, pp. 87–98, 2008.

[18] M. S. Mahmoud, "Decentralized stabilization of interconnected systems with time-varying delays," *IEEE Trans. Autom. Control*, vol. 54, no. 11, pp. 2663–2668, Nov. 2009.

[19] H. Ersin Erol and A. Iftar, "Stabilization of decentralized descriptor-type neutral time-delay systems by time-delay controllers," *Automatica*, vol. 64, pp. 262–269, 2016.