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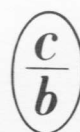
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AUTOMATION AND REMOTE CONTROL

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(АВТОМАТИКА I ТЕЛЕМЕХАНИКА)**

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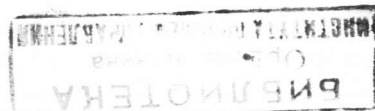
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to have taken place reasonably soon thereafter.*



The analysis and synthesis of digital control systems carried out on the basis of different equations can cause a loss of information about the behavior of a system between the quantization times. The differential-difference approach to describing the digital systems is free of this drawback. This paper deals with an analysis of such systems by use of singular perturbation methods.

1. Introduction

The introduction of microprocessor control systems in industry makes it possible to realize a wide range of control laws from the traditional proportional integral control to the complex algorithms of multiconnected and self-adapting control. A typical digital control system consists of a continuous part (reduced object) and a discrete part (controller) and is described by a system of differential-difference equations. The analysis of digital systems is basically carried out by use of a reduction of the initial problem formulated in terms of "discrete controller-continuous object to an approximate [1] or quasi-equivalent [2] problem of the form "discrete controller-discrete object." In such a case we explicitly or implicitly deal with a partial or total loss of the information about the behavior of the system between the quantization times which calls for an additional analysis in many cases.

This paper suggests an asymptotic method of treating a differential-difference model which enables us to carry out an analysis at all points of the time span under consideration.

This approach is based on ideas used in [3, 4].

2. Statement of the Problem

The quantization effect neglected, the free motion of a linear control system with a Π -controller in the feedback is described as

$$\dot{X}(t) = AX(t) + B \sum_{n=0}^{[t/T]} [H(t-nT) - H(t-(n+1)T)] U(nT), \quad (1)$$

$$U(nT) = -KX(nT), \quad (2)$$

where X is an m -dimensional state vector, U is a k -dimensional control vector; A , B , and K are constant matrices of the corresponding dimensions, $H(\cdot)$ is the Heaviside function; T is a quantization period; by $[\cdot]$ is denoted the greatest integer smaller than or equal to a given number.

This paper aims at constructing a simplified model of the system with a small quantization period.

3. General Scheme of an Asymptotic Analysis

For an asymptotic analysis it is convenient to write system (1), (2) in the following form:

$$\dot{X}(t) = AX(t) + DX \left(t - \varepsilon \left\{ \frac{t}{\varepsilon} \right\} \right), \quad (3)$$

where $D = BK$, $\{\cdot\}$ denotes the fractional part of a number, i.e., $\{t\} = t - [t]$, the quantization period $\varepsilon = T$ plays the role of a small parameter.

Using the change of variables

$$X(t+\xi) = [I + h(t, \xi, \varepsilon)]X(t), \quad \xi \in [-\varepsilon, 0] \quad (4)$$

we can reduce Eq. (3) to

$$\dot{X}(t) = \left[A + D \left(I + h \left(t, -\varepsilon \left\{ \frac{t}{\varepsilon} \right\}, \varepsilon \right) \right) \right] X(t), \quad (5)$$

where I is a unit $m \times n$ -matrix and $h(t, \xi, \varepsilon)$ is a matrix function of three variables. It is easily seen that this change provides a separation of the object matrix and the feedback matrix into components corresponding to a continuous system and a correction which accounts for the pulse nature of the system $h(t, \xi, \varepsilon)$.

It is easily shown (see Appendix) that (3) and (4) imply the following partial differential equation for $h(t, \xi, \varepsilon)$

$$\frac{\partial h(t, \xi, \varepsilon)}{\partial t} + [I + h(t, \xi, \varepsilon)] \left[A + D \left(I + h \left(t_1 - \varepsilon \left\{ \frac{t}{\varepsilon} \right\}, \varepsilon \right) \right) \right] = \frac{\partial h(t, \xi, \varepsilon)}{\partial \xi}. \quad (6)$$

It will be shown in the Appendix [see (A.5) and (A.6)] that $h(t, \xi, \varepsilon)$ is a function depending only of the high-speed time and its asymptotic expansion can be represented in the form

$$h(\varepsilon\tau, \varepsilon\theta, \varepsilon) = \varepsilon h_1(\tau, \theta) + \varepsilon^2 h_2(\tau, \theta) + \dots, \quad (7)$$

where $\tau \in R$ ($\tau = t/\varepsilon$) is the "high-speed time," $\theta \in [-1, 0]$, h_i are continuous in all arguments and uniformly bounded matrix functions.

On substituting (7) into (6) and equating the coefficients of the same powers of ε , we obtain the equations for h_1 and h_2 :

$$\begin{aligned} \frac{\partial h_1(\tau, \theta)}{\partial \tau} + A + D &= \frac{\partial h_1(\tau, \theta)}{\partial \theta}, \\ \frac{\partial h_2(\tau, \theta)}{\partial \tau} + h_1(\tau, \theta)(A + D) + Dh_1(\tau, -\{\tau\}) &= \frac{\partial h_2(\tau, \theta)}{\partial \theta}, \end{aligned} \quad (8)$$

hence, in view of the condition $h_1(\tau, 0) = 0$ we subsequently find

$$h_1 = (A + D)\theta, \quad h_2 = -D(A + D) \int_0^\theta \{\tau + S\} dS + \frac{\theta^2}{2} (A + D)^2. \quad (9)$$

When $\theta = -\{\tau\}$, we obtain

$$h_1(\tau_1 - \{\tau\}) = -\{\tau\}(A + D), \quad h_2(\tau, -\{\tau\}) = \frac{\{\tau\}^2}{2} (2D + A)(A + D). \quad (10)$$

Therefore, Eq. (4) can be represented in the following form up to the terms of order $O(\varepsilon^2)$ inclusively:

$$\dot{X}(t) = \left[A + D \left[I - \varepsilon \left\{ \frac{t}{\varepsilon} \right\} (A + D) + \frac{\varepsilon^2}{2} \left\{ \frac{t}{\varepsilon} \right\}^2 (2D + A)(A + D) \right] \right] X(t). \quad (11)$$

As a result, a complex equation (3) reduces to a simpler equation (11).

If we need to find a solution to the Cauchy problem (3) with an initial condition $X(0) = X$, then it is convenient to represent the solution in the form of an asymptotic expansion

$$X(t, \varepsilon) = X_0(t) + \varepsilon X_1(t, \varepsilon) + \varepsilon^2 X_2(t, \varepsilon) + \dots,$$

where X_i are continuous and bounded functions.

From (11) we obtain equations for X_i :

$$\begin{aligned} X_0 &= (A+D)X_0, \quad X_0(0) = X_0, \\ \dot{X}_1 &= (A+D)X_1 - D(A+D)\left\{\frac{t}{\varepsilon}\right\}X_0, \quad X_1(0, \varepsilon) = 0, \\ \dot{X}_2 &= (A+D)X_2 - D(A+D)\left\{\frac{t}{\varepsilon}\right\}X_1 + \\ &+ \frac{1}{2}D(2D+A)(A+D)\left\{\frac{t}{\varepsilon}\right\}^2 X_0, \quad X_2(0, \varepsilon) = 0 \end{aligned}$$

and so on.

The first term of the expansion of X_0 has an apparent physical meaning of free motion of a continuous system with the matrices A and D . Note that the relation for X_1 can be obtained by use of a formal expansion $X(t - \varepsilon\{t/\varepsilon\})$ in the initial equation (3). However, the calculation of subsequent terms along this way encounters fundamental analytic difficulties.

The proposed method can obviously be extended to the case of forced motion and to the problems of analytic design of a discrete controller.

APPENDIX

Consider Eq. (3) with the initial condition

$$X(0) = X_0. \quad (\text{A.1})$$

By a solution to the Cauchy problem (3) and (A.1), we mean a function $X(t)$ continuous for all $t \geq 0$ and satisfying Eq. (3) for $t > 0$, $t \neq \lambda_\varepsilon$ ($\lambda = 1, 2, \dots$), and the initial condition (A.1). Using the step method [5], we can easily show that problem (3) and (A.1) has a unique solution.

Let H be the class of $m \times m$ -matrix functions $h(t, \xi, \varepsilon)$ ($t \in \mathbb{R}$, $\xi \in [-\varepsilon, 0]$, $\varepsilon \in [0, \varepsilon_0]$, $\varepsilon_0 > 0$) satisfying the equality

$$h(t, 0, \varepsilon) = 0 \quad (\text{A.2})$$

for all $t \in \mathbb{R}$, $\varepsilon \in (0, \varepsilon_0)$ and not exceeding εK ($K > 0$) in norm. We introduce the supremum norm in H . Obviously, H is a complete metric space. We denote by H_1 the set of the functions in H which have continuous partial derivatives with respect to ξ for $t + \xi \neq \lambda_\varepsilon$ ($\lambda = 0, \pm 1, \dots$) and with respect to t for $t \neq \lambda_\varepsilon$, $t + \xi \neq \lambda_\varepsilon$.

We will find a solution to the Cauchy problem (3) and (A.1) which is defined for all $t \in \mathbb{R}$ and can be represented in the form (4) where h is a function in H_1 . Then on substituting the right-hand side of (4) for $X(t - \varepsilon\{t/\varepsilon\})$ in (3) with $\xi = -\varepsilon\{t/\varepsilon\}$, we obtain Eq. (5).

We now find the partial differential equation for the function h . To this end we substitute the right-hand side in (4) in the obvious equality

$$\frac{\partial X(t+\xi)}{\partial t} = \frac{\partial X(t+\xi)}{\partial \xi}. \quad (\text{A.3})$$

Taking (5) into account, we obtain

$$\frac{\partial h(t, \xi, \varepsilon)}{\partial t} + [I + h(t, \xi, \varepsilon)] \left[A + D \left(I + h \left(t - \varepsilon \left\{ \frac{t}{\varepsilon} \right\}, \varepsilon \right) \right) \right] X(t) = \frac{\partial h(t, \xi, \varepsilon)}{\partial \xi} X(t). \quad (\text{A.4})$$

We now consider Eq. (6). By a solution to (6) we will mean a function $h \in H_1$ satisfying Eq. (6) for $t \neq \lambda_\varepsilon$, $t + \xi \neq \lambda_\varepsilon$ ($\lambda = 0, \pm 1, \dots$).

We now prove that Eq. (6) has a solution. Let $U(s, t)$ ($s \in \mathbb{R}$, $t \in \mathbb{R}$) be the fundamental matrix of the solutions to Eq. (5). The function U satisfies the equation

$$U(s, t) = \int_t^s \left[A + D \left(I + h \left(\tau - \varepsilon \left(\frac{\tau}{\varepsilon} \right), \varepsilon \right) \right) \right] U(\tau, t) d\tau + I. \quad (\text{A.5})$$

We consider an auxiliary equation

$$h(t, \xi, \varepsilon) = \int_t^{t+\xi} \left[A + D \left(I + h \left(s, -\varepsilon \left\{ \frac{s}{\varepsilon} \right\}, \varepsilon \right) \right) \right] U(s, t) ds. \quad (\text{A.6})$$

Here $t \in \mathbb{R}$, $\xi \in [-\varepsilon, 0]$, and $h \in H$. Using the contraction principle, we can easily show that the system (A.5) and (A.6) has a unique solution for sufficiently small ε .

LEMMA 1. A function h is a solution to (6) if and only if h satisfies system (A.5) and (A.6).

Proof. Let h be a solution to (6) and let U be a solution to (A.5). We denote the integral on the right of (A.6) by J . Making the change $\tau + t = s$, we obtain

$$J = \int_0^\xi \left[A + D \left(I + h \left(\tau, -\varepsilon \left\{ \frac{t+\tau}{\varepsilon} \right\}, \varepsilon \right) \right) \right] U(t+\tau, t) d\tau.$$

Using (A.5) to express the latter integrand with $t = t + \tau$, $\xi = \xi - \tau$, we have

$$\begin{aligned} J &= \int_0^\xi \left\{ \frac{\partial h(t+\tau, \xi-\tau, \varepsilon)}{\partial \xi} - \frac{\partial h(t+\tau, \xi-\tau, \varepsilon)}{\partial t} - A - D \left(I + h \left(t+\tau, -\varepsilon \left\{ \frac{t+\tau}{\varepsilon} \right\}, \varepsilon \right) \right) \right\} U(t+\tau, t) d\tau = \\ &= - \int_0^\xi \frac{\partial}{\partial \tau} \{ h(t+\tau, \xi-\tau, \varepsilon) U(t+\tau, t) \} d\tau = -h(t+\xi, 0, \varepsilon) U(t+\xi, t) + h(t, \xi, \varepsilon). \end{aligned}$$

This and (A.2) imply (A.6).

Conversely, let $h \in H$ satisfy (A.5) and (A.6). On differentiating (A.6) with respect to t and ξ , we see that $h \in H_1$ and satisfies Eq. (6), since

$$\frac{\partial U(s, t)}{\partial t} = -U(s, t) \left[A + D \left(I + h \left(t, -\varepsilon \left\{ \frac{t}{\varepsilon} \right\}, \varepsilon \right) \right) \right].$$

The lemma is proved.

Lemma 1 and the above remark on the solubility of system (A.5) and (A.6) imply that there is a unique solution to Eq. (6).

LEMMA 2. Let h be a solution to Eq. (6) and let $X(t)$ be a solution to the Cauchy problem (5) and (A.1). Then $X(t)$ is a solution to the Cauchy problem (3) and (A.2) for all $t \in \mathbb{R}$, and (4) holds true.

Proof. Let h be a solution to (6) and let $X(t)$ be a solution to (5) and (A.1). Consider the function

$$Z(t, \xi, \varepsilon) = [I + h(t, \xi, \varepsilon)] X(t).$$

By (5) and (6) we have

$$\frac{\partial Z}{\partial t} = \frac{\partial Z}{\partial \xi}. \quad (\text{A.7})$$

Furthermore, it follows from (A.2) that

$$Z(t, 0, \varepsilon) = X(t). \quad (\text{A.8})$$

On solving (A.7) and (A.8), we obtain $Z(t, \xi, \varepsilon) = X(t + \xi)$, that is, (4) holds true. Finally, replacing $(I + h(t, -\varepsilon\{t/\varepsilon\}, \varepsilon))X(t)$ in (5) by $X(t - \varepsilon\{t/\varepsilon\})$, we obtain (3). Consequently, $X(t)$ is a solution to problem (3) and (A.2).

The lemma is proved.

We now construct an asymptotic expansion of the function h .

LEMMA 3. For sufficiently small ε the function h can be represented in the form

$$h(\varepsilon\tau, \varepsilon\theta, \varepsilon) = \varepsilon h_1(\tau, \theta) + \varepsilon^2 h_2(\tau, \theta) + \dots + \varepsilon^n h_n(\tau, \theta, \varepsilon), \quad (\text{A.9})$$

where $\tau \in \mathbb{R}$, $\theta \in [-1, 0]$, and t_i ($i = 1, \dots, n$) are continuous in all arguments and uniformly bounded $m \times m$ -matrix functions vanishing when $\theta = 0$; n is any natural number.

Proof. On substituting (A.9) into (6), we obtain

$$\begin{aligned} & \frac{\partial}{\partial \tau} [h_1(\tau, \theta) + \dots + \varepsilon^{n-1} h_n(\tau, \theta, \varepsilon)] \pm [I + \varepsilon h_1(\tau, \theta) + \dots + \varepsilon^n h_n(\tau, \theta, \varepsilon)] \times \\ & \times [A + D(I + \varepsilon h_1(\tau, -\{\tau\}) + \dots + \varepsilon^n h_n(\tau, \theta, \varepsilon))] = \frac{\partial}{\partial \theta} [h_1(\tau, \theta) + \dots + \varepsilon^{(n-1)} h_n(\tau, \theta, \varepsilon)]. \end{aligned}$$

On equating the coefficients of the same powers of ε in the latter equation, we obtain Eqs. (8). Solving Eqs. (3) with the initial conditions $h_i(\tau, 0) = 0$ ($i = 1, \dots, n-1$), we uniquely determine the functions h_1, \dots, h_{n-1} .

For h_n we have

$$\begin{aligned} & \frac{\partial}{\partial \tau} h_n(\tau, \theta, \varepsilon) + \varepsilon h_n(\tau, \theta, \varepsilon) [A + D(I + \varepsilon h_1(\tau - \{\tau\}) + \dots \\ & \dots + \varepsilon^n h_n(\tau, -\{\tau\}, \varepsilon))] = \frac{\partial}{\partial \theta} h_n(\tau, \theta, \varepsilon) + \varepsilon B_n(\tau, \theta) h_n(\tau, \theta, \varepsilon) + C_n(\tau, \theta), \\ & h_n(\tau, 0, \varepsilon) = 0, \end{aligned} \quad (\text{A.10})$$

where B_n and C_n are known $m \times m$ -matrix functions, h_n is defined by (A.10) if and only if h_n satisfies the system

$$\begin{aligned} U(s, \tau) &= \int_{\tau}^s [A + D(I + \varepsilon h_1(p, -\{p\}) + \dots + \varepsilon^n h_n(p, -\{p\}, \varepsilon))] U(p, \tau) d\tau + I, \\ h_n(\tau, \theta, \varepsilon) &= - \int_{\tau}^{\tau+\theta} [\varepsilon B_n(s, \theta) h_n(s, \theta, \varepsilon) + C_n(s, \theta)] U(s, \tau) ds. \end{aligned} \quad (\text{A.11})$$

This fact can be proved similarly to Lemma 1. Using the contraction principle, we can show that there is a unique solution to system (A.11) and the function h_n is uniformly bounded. The lemma is proved.

The construction of an asymptotic expansion of a solution to the Cauchy problem (3) and (A.1) has reduced to the construction of an asymptotic expansion of a solution to the problem (5) and (A.1).

LEMMA 4. For sufficiently small ε solution to the Cauchy problem (5) and (A.1) can be expressed as

$$X(t) = X_0(t) + \varepsilon X_1(t, \varepsilon) + \dots + \varepsilon^n X_n(t, \varepsilon), \quad (\text{A.12})$$

where the functions X_i ($i = 1, \dots, n$) are continuous and uniformly bounded for $t \in [0, t_1]$ (with any t_1).

Proof. Substituting (A.12) into (5) and equating coefficients of the same powers of ε [X_i ($i = 1, \dots, n-1$) are assumed independent of ε], we obtain equations like (12). From these equations we uniquely determine X_0, \dots, X_{n-1} . For X_n we obtain

$$\begin{aligned} \dot{X}_n &= \left[A + D \left(I + \varepsilon h_1 \left(\frac{t}{\varepsilon}, - \left\{ \frac{t}{\varepsilon} \right\} \right) + \dots + \varepsilon^n h_n \left(\frac{t}{\varepsilon}, - \left\{ \frac{t}{\varepsilon} \right\}, \varepsilon \right) \right) \right] X_n + E_n(t, \varepsilon), \\ X_n(0, \varepsilon) &= 0, \end{aligned} \quad (\text{A.13})$$

where for $t \in [0, t_1]$ ($t \in \mathbb{R}$) E_n is some known piecewise continuous and uniformly bounded function with the points of discontinuity $t = \ell_\varepsilon$ ($\ell = 1, 2, \dots$). The Cauchy problem (A.13) has a unique solution with the properties given in the lemma. Lemma 4 is proved.

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