ACADEMIC PRESS

J. Math. Anal. Appl. 273 (2002) 24–44

Journal of MATHEMATICAL ANALYSIS AND APPLICATIONS

www.academicpress.com

Stability of linear descriptor systems with delay: a Lyapunov-based approach ☆

Emilia Fridman

Department of Electrical Engineering Systems, Tel Aviv University, Tel Aviv 69978, Israel

Received 12 September 2000

Submitted by William F. Ames

Abstract

The Lyapunov second method is developed for linear coupled systems of delay differential and functional equations. By conventional approaches such equations may be reduced to the neutral systems and the known results for the latter may be exploited. In the present paper we introduce a new approach by constructing a Lyapunov–Krasovskii functional that corresponds directly to the descriptor form of the system. Moreover, by representing a neutral system in the descriptor form we obtain new stability criteria for neutral systems which are less conservative than the existing results. Sufficient conditions for delay-dependent/delay-independent stability and for robustness of stability with respect to small delays are given in terms of linear matrix inequalities. Illustrative examples show the effectiveness of the method.

1. Introduction

Delay differential–algebraic equations, which have both delay and algebraic constraints, often appear in various engineering systems, including aircraft stabilization, chemical engineering systems, lossless transition lines, etc. (see, e.g., [3,13,15,18,25,29] and references therein). Depending on the area of application, these models are called singular or implicit or descriptor systems with delay. As

☆ This work was supported by the Ministry of Absorption of Israel.
E-mail address: emilia@eng.tau.ac.il.

0022-247X/02/$ – see front matter
PII: S0022-247X(02)00202-0

has been pointed out in [4,5], descriptor systems with delay may in fact be systems of advanced type. Being a more general class than neutral systems, descriptor systems may be destabilized by small delay in the feedback [22].

A particular case of such systems (the so-called lossless propagation models)

$$\dot{x}_1(t) = Ax_1(t) + Bx_2(t-h), \qquad x_2(t) = Cx_1(t) + Dx_2(t-h) \tag{1}$$

has been treated as a special class of neutral systems either by letting $x_2(t) = \dot{z}(t)$ [26] or by writing the second equation as [16,25]

$$\frac{d}{dt}\big[x_2(t) - Cx_1(t) - Dx_2(t-h)\big] = 0. \tag{2}$$

Stability of a general neutral type descriptor equations with a single delay

$$E\dot{x}(t) + Ax(t) + B\dot{x}(t-h) + Cx(t-h) = 0 \tag{3}$$

and singular matrix E has been studied in [29] by analyzing its characteristic equation

$$\det\big[sE + A + (sB + C)\exp(-hs)\big] = 0$$

and finding frequency domain conditions which guarantee that all roots of the latter equation have negative real parts bounded away from 0.

In the present paper we consider a descriptor system with multiple and distributed delays. We construct a Lyapunov–Krasovskii functional that corresponds directly to the descriptor system. We derive delay-independent and delay-dependent conditions in terms of linear matrix inequalities (LMIs). For information on the LMI approach to delay-independent and delay-dependent stability criteria for linear retarded and neutral type systems see [2,11,17,20,21,23,24,28]. Note that LMIs give only sufficient conditions, which are more conservative than those obtained by analysis of the characteristic equation. However, the method is better adapted for robust stability of systems with uncertainties (see, e.g., [20]) and for other control problems.

A Lyapunov–Krasovskii functional for descriptor system with delay was suggested (as a conjecture) in [10] on the basis of the traditional (for delay-dependent stability) transformation of the delay system

$$\dot{x}(t) = Ax(t) + Bx(t-h)$$

in the form (see, e.g., [15, p. 156])

$$\dot{x}(t) = (A+B)x(t) - B\int_{-h}^{0} Ax(t-h+s)\,ds.$$

The conservatism of conditions based on this transformation is twofold: the transformed system is not equivalent to the original one having a double distributed delay (see [8]) and bounds should be obtained (completion of the squares) for certain terms.

Recently, for linear differential systems a new Lyapunov–Krasovskii functional has been introduced in [11]. It is based on equivalent augmented model—a "descriptor form" representation of the system and it leads to less conservative conditions. In the present paper we adopt this approach to descriptor systems with delay. We develop the second method of Lyapunov for descriptor equations with delay. By defining appropriate descriptor Lyapunov–Krasovskii functionals we derive stability conditions in terms of LMIs.

Notations. Let R^n be Euclidean space with vector norm $|\cdot|$, $C_n[a,b]$ be the space of continuous functions $\phi:[a,b]\to R^n$ with the supremum norm $|\cdot|$, and $L_2[a,b]$ be the space of square integrable functions. Denote by $x_t(\theta)=x(t+\theta)$ $(\theta\in[-h,0])$. The notation $P>0$ for $(n\times n)$-matrix P means that P is symmetric and positive definite. Symmetric terms in symmetric matrices are denoted by $*$; e.g.,

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix} = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}.$$

2. A class of descriptor system with delay. Existence and uniqueness of solution

2.1. Preliminaries on descriptor systems

Consider a linear autonomous system without delay:

$$E\dot{x}(t)=A_0x(t), \tag{4}$$

where $x(t)\in R^n$, E and A_0 are $n\times n$ matrices, $\operatorname{rank} E=n_1<n$. We assume that (4) is regular, i.e., the characteristic polynomial $\det(sE-A_0)$ does not vanish identically in s. It is well known that descriptor system may have impulsive solutions. The existence of the latter solutions is usually studied in terms of the Weierstrass canonical form and the *index* of the system which are defined as follows [6,7,22]: there exist nonsingular matrices $P, Q\in R^{n\times n}$ such that

$$QEP=\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}, \qquad QA_0P=\begin{bmatrix} J & 0 \\ 0 & I_{n_2} \end{bmatrix}, \tag{5}$$

and (4) for the new variable $y=\operatorname{col}\{y_1,y_2\}=P^{-1}x$ has the canonical form

$$\dot{y}_1(t)=Jy_1(t), \qquad N\dot{y}_2(t)=y_2(t), \tag{6}$$

where $n_1+n_2=n$, $N\in R^{n_2\times n_2}$ and $J\in R^{n_1\times n_1}$ are in Jordan form. The matrix N is nilpotent of index ν, i.e., $N^\nu=0$, $N^{\nu-1}\neq 0$. The index of (4) is the index of nilpotency ν of N.

It is well known that (6) admits impulsive solutions iff $\nu > 1$:

$$y_1(t) = e^{Jt} y_1(0), \qquad y_2(t) = -\sum_{i=0}^{\nu-2} \delta^i(t) N^{i+1} y_2(0^-),$$

where δ is the Dirac delta-function and superscript i denotes the ith distributional derivative.

In the case of system with delay

$$E\dot{x}(t) = A_0 x(t) + A_1 x(t-h), \tag{7}$$

for P and Q as above and $y = P^{-1}x$ we obtain the following canonical form:

$$\begin{aligned} \dot{y}_1(t) &= J y_1(t) + C_1 y_1(t-h) + C_2 y_2(t-h), \\ N\dot{y}_2(t) &= y_2(t) + C_3 y_1(t-h) + C_4 y_2(t-h), \end{aligned} \tag{8}$$

where

$$QA_1P = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}.$$

By the index of descriptor system with delay (7) we mean the index of the corresponding descriptor system without delay (4).

As in the case without delay (8) admits impulsive solutions for $\nu > 1$. Thus for $t \in [0, h)$ we obtain

$$\begin{aligned} y_1(t) &= e^{Jt} y_1(0) + \int_0^t e^{J(t-s)} \big[C_1 y_1(s-h) + C_2 y_2(s-h)\big]\, ds, \\ y_2(t) &= -\sum_{i=0}^{\nu-2} \delta^i(t) N^{i+1} y_2(0^-) - \sum_{i=0}^{\nu-1} N^i \big[C_3 y_1^{(i)}(t-h) + C_4 y_2^{(i)}(t-h)\big]. \end{aligned}$$

That is why for stability analysis we restrict ourselves to descriptor systems of index one.

2.2. Descriptor systems with delay

In the present paper we analyze the stability of the following system:

$$E\dot{x}(t) = \sum_{i=0}^{m} A_i x(t-h_i) + \int_{-h}^{0} B(s) x(t+s)\, ds, \tag{9}$$

where $x(t) = \mathrm{col}\{x_1(t), x_2(t)\}$, $x_1(t) \in \mathcal{R}^{n_1}$, $x_2(t) \in \mathcal{R}^{n_2}$, $h_0 = 0$, $h_i > 0$, $h > 0$, $i = 1, \ldots, m$, E and A_i are constant $(n_1+n_2) \times (n_1+n_2)$-matrices, $B(s)$ is a piecewise-continuous and uniformly bounded $(n_1+n_2)\times(n_1+n_2)$-matrix-function. Denote $n \triangleq n_1+n_2$. We assume that the matrices in (9) have the following structure:

$$
\begin{aligned}
&E=\begin{bmatrix} I_{n_1} & 0\\ 0 & 0\end{bmatrix},\\
&A_i=\begin{bmatrix} A_{i1} & A_{i2}\\ A_{i3} & A_{i4}\end{bmatrix},\quad i=0,\dots,m,\quad \det A_{04}\neq 0,\\
&B=\begin{bmatrix} B_1 & B_2\\ B_3 & B_4\end{bmatrix}.
\end{aligned}\tag{10}
$$

If (4) has index one then (9) can be put in the form of (9) and (10) (e.g., as described in Section 2.1).

Note that (9) and (10) includes a class of neutral descriptor system

$$
\begin{bmatrix} \dot x_1(t)-\sum_{i=1}^m D_i\dot x_1(t-h_i)\\ 0\end{bmatrix}
=\sum_{i=0}^m A_i x(t-h_i)+\int_{-h}^0 B(s)x(t+s)\,ds,\tag{11}
$$

where E and A_0 are given by (10). Really, considering an augmented system

$$
\begin{aligned}
&\dot x_1(t)=y,\\
&\begin{bmatrix} y(t)-\sum_{i=1}^m D_i y(t-h_i)\\ 0\end{bmatrix}
=\sum_{i=0}^m A_i x(t-h_i)+\int_{-h}^0 B(s)x(t+s)\,ds,
\end{aligned}\tag{12}
$$

we obtain a particular case of (9) and (10).

System (9) is the system of functional differential equations:

$$
\dot x_1(t)=\sum_{j=1}^2\sum_{i=0}^m A_{ij}x_j(t-h_i)+\sum_{j=1}^2\int_{-h}^0 B_j x_j(t+s)\,ds,\tag{13a}
$$

$$
0=\sum_{j=1}^2\sum_{i=0}^m A_{i,j+2}x_j(t-h_i)+\sum_{j=1}^2\int_{-h}^0 B_{j+2}x_j(t+s)\,ds.\tag{13b}
$$

Consider the following initial conditions for (9):

$$
x_1(t)=\phi_1(t),\quad x_2(t)=\phi_2(t),\quad t\in[-h,0].\tag{14}
$$

Substituting initial function $\phi = \mathrm{col}\{\phi_1, \phi_2\}$ into the second (functional) equation of (13) we have

$$\sum_{j=1}^{2}\sum_{i=0}^{m} A_{i,j+2}\phi_j(-h_i) + \sum_{j=1}^{2}\int_{-h}^{0} B_{j+2}\phi_j(s)\,ds = 0. \tag{15}$$

Proposition 1. *For any continuous* $\phi = \mathrm{col}\{\phi_1, \phi_2\}$ *that satisfies* (15) *there exists a unique function* $x(t)$ *defined and continuous on* $[-h, \infty)$ *that satisfies system* (9) *on* $[0, \infty)$ *and initial conditions* (14).

Proof. Differentiating the second equation of (13) with respect to t and taking into account that $x_1(t)$ is differentiable we obtain the neutral type system (13a) and

$$\frac{d}{dt}\left[A_{04}x_2(t) + \sum_{j=1}^{2}\sum_{i=1}^{m} A_{i,j+2}x_j(t-h_i) + \sum_{j=1}^{2}\int_{-h}^{0} B_{j+2}x_j(t+s)\,ds\right]$$
$$+ A_{03}\left[\sum_{j=1}^{2}\sum_{i=0}^{m} A_{ij}x_j(t-h_i) + \sum_{j=1}^{2}\int_{-h}^{0} B_j x_j(t+s)\,ds\right] = 0. \tag{16}$$

The latter system has a unique continuous on $[-h, \infty)$ solution satisfying initial conditions (14) [15]. If additionally (15) holds, then this solution is a unique solution of (13), (14). □

3. Second Lyapunov method for descriptor systems with delay

We define stability of the trivial solution of (9) similarly to stability in the case of non-descriptor system with delay [9,15,18]:

Definition 1. The trivial solution of (9) is said to be stable if for any $\varepsilon > 0$ there exists a $\delta(\varepsilon)$ such that for all continuous $\phi = \mathrm{col}\{\phi_1, \phi_2\}$, with ϕ satisfying (15) and $|\phi| < \delta$, the solution to (9), (14) $x(\phi)(t)$ satisfies inequality $|x(\phi)(t)| < \varepsilon$ for all $t \geqslant 0$. The trivial solution of (9) is said to be asymptotically stable if it is stable and furthermore

$$\lim_{t\to\infty} x(\phi)(t) = 0.$$

In the latter case the system (9) is said to be asymptotically stable.

Consider the operator $\mathcal{D}: C_{n_2}[-h,0] \to R^{n_2}$:

$$\mathcal{D}(x_{2t}) = x_2(t) + \sum_{i=1}^{m} A_{04}^{-1} A_{i4} x_2(t-h_i) + \int_{-h}^{0} A_{04}^{-1} B_4(s) x_2(t+s)\, ds.$$

We assume:

(A1) The operator $\mathcal{D}$ is stable (i.e., equation $\mathcal{D}x_{2t} = 0$ is asymptotically stable).

Sufficient condition for (A1) is given by

(A1′) Let

$$\sum_{i=1}^{m} \left| A_{04}^{-1} A_{i4} \right| + \int_{-h}^{0} \left| A_{04}^{-1} B_4(s) \right| ds < 1.$$

For a continuous functional $V: C_n[-h,0] \to R$ define

$$\dot{V}(\phi) = \lim \sup_{h \to 0^+} \frac{1}{h} \left[V\left(x_{t+h}(t,\phi)\right) - V(\phi) \right],$$

where $x_t(t_0,\phi)$ is a solution to (9) such that $x_{t_0} = \phi$.

Lemma 1. *Under* (A1), *if there exist positive numbers α, β, γ and a continuous functional $V: C_n[-h,0] \to R$ such that*

$$\beta \left| \phi_1(0) \right|^2 \leqslant V(\phi) \leqslant \gamma |\phi|^2, \tag{17a}$$

$$\dot{V}(\phi) \leqslant -\alpha \left| \phi(0) \right|^2, \tag{17b}$$

and the function $\bar{V}(t) = V(x_t)$ is absolutely continuous for x_t satisfying (9), *then* (9) *is asymptotically stable.*

Proof. Integrating (17b), where $\phi = x_s$, with respect to s from 0 to t we have

$$V(x_t) - V(\phi) \leqslant -\alpha \int_0^t \left| x(s) \right|^2 ds. \tag{18}$$

From (17a) and (18) it follows that

$$\beta \left| x_1(t) \right|^2 \leqslant V(x_t) \leqslant V(\phi) \leqslant \gamma |\phi|^2. \tag{19}$$

Hence, x_1 is bounded and small for small $|\phi|$. From (13b) we find that $x_2(t)$ is a solution of

$$\mathcal{D}x_{2t} = h(t),$$
$$h(t) = -\sum_{i=0}^{m} A_{04}^{-1} A_{i3} x_1(t - h_i) - \int_{-h}^{0} A_{04}^{-1} B_3(s) x_1(t + s)\, ds, \tag{20}$$

where $h(t)$ is bounded and small for small $|\phi|$. Then under (A1) by Theorem 3.5 [15, p. 275] $x_2(t)$ is bounded and small for small $|\phi|$. Hence, (9) is stable.

To prove asymptotic stability we use Barbalat's lemma [1,12]. We note that the right-hand side of (13a) and thus $\dot{x}_1$ are bounded. Hence, $x_1(t)$ is uniformly continuous on $[0, \infty)$. Moreover, $x_2(t)$ is a bounded solution of (20), where $h(t)$ is uniformly continuous and bounded. Then under (A1) by Lemma 7.1 [15, p. 291] $x_2(t)$ is uniformly continuous on $[0, \infty)$. From (18) it follows that $x(t) \in L_2[0, \infty)$. Therefore by Barbalat's lemma $x(t) \to 0$ for $t \to \infty$. $\square$

4. Delay-independent with respect to discrete delays stability

4.1. Main results

The descriptor type Lyapunov–Krasovskii functional for the system (9) has the following form:

$$V(x_t) = x^T(t) E P x(t) + V_1 + V_2, \tag{21}$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad P_1 = P_1^T > 0, \tag{22}$$

$$V_1 = \sum_{i=1}^{m} \int_{t-h_i}^{t} x^T(s) Q_i x(s)\, ds, \quad Q_i > 0, \tag{23}$$

and

$$V_2 = \int_{-h}^{0} \int_{t+\theta}^{t} x^T(s) B^T(\theta) R B(\theta) x(s)\, ds\, d\theta, \quad R > 0. \tag{24}$$

The first term of (21) corresponds to the descriptor system (see, e.g., [27]), V_1 corresponds to the delay-independent stability with respect to the discrete delays and V_2 to delay-dependent stability with respect to the distributed delays (see [17, 19]). The functional (21) is *degenerate* (i.e., nonpositive definite) as it is usual for descriptor systems.

We obtain the following:

Theorem 1. *Under* (A1′), (9) *is asymptotically stable if there exist* $(n \times n)$*-matrix* P *of* (22), *with* $(n_1 \times n_1)$*-matrix* P_1 *and* $(n_2 \times n_2)$*-matrix* P_3, *and* $(n \times n)$*-matrices* $Q_i = Q_i^T$, $i = 1, \dots, m$, $R = R^T$ *that satisfy the following LMI*:

$$\begin{bmatrix} \begin{matrix} P^T A_0 + A_0^T P + \sum_{i=1}^m Q_i \\ + \int_{-h}^0 B^T(s) R B(s)\, ds \end{matrix} & P^T A_1 & \dots & P^T A_m & h P^T \\ * & -Q_1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & -Q_m & 0 \\ * & * & * & * & -hR \end{bmatrix} < 0. \tag{25}$$

Proof. Note that $x^T E P x = x_1^T P_1 x_1$ and, hence,

$$\frac{d}{dt}\left[x^T(t) E P x(t)\right] = 2x_1^T(t) P_1 \dot{x}_1(t) = 2x^T(t) P^T \begin{bmatrix} \dot{x}_1(t) \\ 0 \end{bmatrix}. \tag{26}$$

Differentiating (21) in t and substituting (9) in (26) we obtain

$$\dot{V}(x_t) = \xi^T \begin{bmatrix} \begin{matrix} P^T A_0 + A_0^T P + \sum_{i=1}^m Q_i \\ + \int_{-h}^0 B^T(s) R B(s)\, ds \end{matrix} & P^T A_1 & \dots & P^T A_m \\ * & -Q_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & * & -Q_m \end{bmatrix} \xi$$

$$+ \eta(t) - \int_{-h}^{0} x^T(t+\theta) B^T(\theta) R B(\theta) x(t+\theta)\, d\theta, \tag{27}$$

where $\xi \triangleq \mathrm{col}\{x(t), x(t-h_1), \dots, x(t-h_m)\}$,

$$\eta(t) \triangleq -2 \int_{t-h}^{t} x^T(t) P^T B(s) x(t+s)\, ds. \tag{28}$$

For any $(n \times n)$-matrices $R > 0$

$$\eta(t) \leqslant h x^T P^T R^{-1} P x + \int_{t-h}^{t} x^T(t+s) B^T(s) R B(s) x(t+s)\, ds. \tag{29}$$

Equations (27) and (29) yield (by Schur complements) that $\dot{V} < 0$ if (25) holds. Therefore functional V satisfies (17) and by Lemma 1 system (9) is asymptotically stable. □

4.2. LMI condition for delay-independent stability of the difference operator

LMI (25) yields the following inequality:

$$\begin{bmatrix} A_{04}^T P_3 + P_3^T A_{04} + \sum_{i=1}^m Q_{i4} & P_3^T A_{14} & \dots & P_3^T A_{m4} \\ * & -Q_{14} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \dots & -Q_{m4} \end{bmatrix} < 0, \tag{30}$$

where Q_{i4} is a $(2,2)$ block of Q_i. If there exists a solution to (25), then there exists a solution to (30). We shall show that (30) guarantees delay-independent stability of $\mathcal{D}$ with $B_4 = 0$, which is equivalent to the following condition (see [15, p. 286, Theorem 6.1]):

(A1$''$) If $\sigma(B)$ is the spectral radius of matrix B, then $\sigma_0 < 1$ where

$$\sigma_0 \triangleq \sup\left\{\sigma\left(\sum_{k=1}^m A_{04}^{-1} A_{k4} e^{i\theta_k}\right):\ \theta_k \in [0, 2\pi],\ k = 1, \dots, m\right\}. \tag{31}$$

Lemma 2. *If there exist $(n_2 \times n_2)$-matrices P_3, $Q_{14}, \dots, Q_{m4}$ that satisfy* (30), *then A_{04} is nonsingular and*

(i) (A1$''$) *holds*;
(ii) *the difference operator*

$$\mathcal{D}(x_{2t}) = x_2(t) + \sum_{i=1}^m A_{04}^{-1} A_{i4} x_2(t - h_i)$$

is stable for all $h_i > 0$;
(iii) *under additional assumption that $P_3 > 0$, the "fast system"*

$$\dot{x}_2(t) = A_{04} x_2(t) + \sum_{i=1}^m A_{i4} x_2(t - h_i) \tag{32}$$

is asymptotically stable for all $h_i > 0$.

Proof. (iii) is well known (see, e.g., [17]) and (ii) is equivalent to (i). To prove (i) note that

$$f(\theta_1, \dots, \theta_m) \triangleq \sigma\left(\sum_{k=1}^m A_{04}^{-1} A_{k4} e^{i\theta_k}\right)$$

is a continuous function on the compact set $[0, 2\pi] \times \cdots \times [0, 2\pi]$ and thus it achieves it maximum value on this set. Therefore it is sufficient to prove that

$$f(\theta_1, \dots, \theta_m) < 1 \quad \forall \theta_1, \dots, \theta_m \in [0, 2\pi] \times \cdots \times [0, 2\pi].$$

By Schur complements (30) implies

$$P_3^T A_{04} + A_{04}^T P_3 + \sum_{k=1}^{m} \left(Q_{k4} + P_3^T A_{k4} Q_{k4}^{-1} A_{k4}^T P_3 \right) < 0. \tag{33}$$

Since $Q_{14}, \ldots, Q_{m4}$ must be positive definite, it follows from (33) that A_{04} is nonsingular. Multiplying (33) by $y \in \mathcal{R}^n$ from the right and by y^T from the left we have

$$-2\left| y^T A_{04}^T P_3 y \right| + \sum_{k=1}^{m} \left[\left| Q_{k4}^{1/2} y \right|^2 + \left| Q_{k4}^{-1/2} A_{k4}^T A_{04}^{-T} A_{04}^T P_3 y \right|^2 \right] < 0,$$

where $A_{04}^{-T} = (A_{04}^{-1})^T$. Since

$$\left| Q_{k4}^{1/2} y \right|^2 + \left| Q_{k4}^{-1/2} A_{k4}^T A_{04}^{-T} A_{04}^T P_3 y \right|^2 \geqslant 2\left| y^T A_{k4}^T A_{04}^{-T} A_{04}^T P_3 y \right|,$$

we obtain from the previous inequality that

$$-\left| y^T A_{04}^T P_3 y \right| + \sum_{k=1}^{m} \left| y^T A_{k4}^T A_{04}^{-T} A_{04}^T P_3 y \right| < 0.$$

Choose y to be an eigenvector of $(\sum_{k=1}^{m} A_{04}^{-1} A_{k4} e^{i\theta_k})$, $\theta_k \in [0, 2\pi]$, that corresponds to the eigenvalue λ. From the latter inequality and the inequality

$$\sum_{k=1}^{m} \left| y^T A_{k4}^T A_{04}^{-T} A_{04}^T P_3 y \right| \geqslant \left| y^T \left(\sum_{k=1}^{m} A_{k4}^T A_{04}^{-T} e^{i\theta_k} \right) A_{04}^T P_3 y \right|$$

we conclude that

$$-\left| y^T A_{04}^T P_3 y \right| + |\lambda| \left| y^T A_{04}^T P_3 y \right| < 0$$

and thus $|\lambda| < 1$. Hence, $f(\theta_1, \ldots, \theta_m) < 1$ $\forall \theta_1, \ldots, \theta_m \in [0, 2\pi] \times \cdots \times [0, 2\pi]$. $\square$

Remark 1. From Lemma 2 it follows that in the case of $B_4 = 0$, Theorem 1 holds without assumption (A1′).

Remark 2. In the scalar case delay-independent stability of the fast system implies that $\sum_{i=1}^{m} |A_{04}^{-1} A_{i4}| \leqslant 1$ [14]. From Lemma 2 it follows that (30) implies (A1″) and thus $\sum_{i=1}^{m} |A_{04}^{-1} A_{i4}| < 1$.

4.3. A special form of term with distributed delay

Consider (9) with a special form of term with distributed delay,

$$\int_{-h}^{0} B(s) x(t+s)\, ds = \sum_{i=1}^{m} h_i B_i \int_{-h_i}^{0} x(t+s)\, ds, \tag{34}$$

i.e., a system of the form

$$E\dot{x}(t) = \sum_{i=0}^{m} A_i x(t-h_i) + \sum_{i=1}^{m} h_i B_i \int_{-h_i}^{0} x(t+s)\,ds, \tag{35}$$

where B_i are constant matrices. Lyapunov–Krasovskii functional for (35) has a form of (21), where P and V_1 are given by (22) and (23), while V_2 is defined by

$$V_2 = \sum_{i=1}^{m} \int_{-h_i}^{0} \int_{t+\theta}^{t} x^T(s) R_i x(s)\,ds\,d\theta, \qquad R_i > 0.$$

Assumption (A1′) for this case has the following form:

($\bar{\text{A}}$1) Let

$$\sum_{i=1}^{m} \big[|A_{i4}| + h_i|B_{i4}|\big] < 1.$$

We obtain the following result:

Theorem 2. *Under* ($\bar{\text{A}}$1), (35) *is stable if there exist* $(n \times n)$*-matrix* P *of* (22), *with* $(n_1 \times n_1)$*-matrix* P_1 *and* $(n_2 \times n_2)$*-matrix* P_3, *and* $(n \times n)$*-matrices* $R_i = R_i^T$, $Q_i = Q_i^T$, $i = 1, \dots, m$, *that satisfy the following LMI*:

$$\begin{bmatrix} P^T A_0 + A_0^T P + \sum_{i=1}^{m} Q_i + \sum_{i=1}^{m} h_i R_i & P^T A_1 & \dots & P^T A_m & h_1 P^T B_1 & \dots & h_m P^T B_m \\ * & -Q_1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & -Q_m & 0 & \dots & 0 \\ * & * & * & * & -h_1 R_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \dots & -h_m R_m \end{bmatrix} < 0. \tag{36}$$

Proof is similar to that of Theorem 1, where in the second line of (27) we obtain

$$\eta - \sum_{i=1}^{m} \int_{-h_i}^{0} x^T(t+\theta) R_i x(t+\theta)\,dt$$

and

$$\eta = \sum_{i=1}^{m} 2 \int_{-h_i}^{0} x^T(s) P^T B_i x(t+s)\,ds$$

$$\leqslant \sum_{i=1}^{m}\left[h_i x^T(t)P^T B_i R_i^{-1} B_i^T x(t) + \int_{-h_i}^{0} x^T(t+s)R_i x(t+s)\,ds\right].$$

From Lemma 2 it follows that in the case of $B_{i4} = 0$, $i = 1, \ldots, m$, Theorem 2 holds without assumption ($\bar{\text{A}}$1).

4.4. *The case of neutral type descriptor systems*

For system (11) we consider a continuous initial function $\phi = \mathrm{col}\{\phi_1, \phi_2\}$ with continuously differentiable ϕ_1. Stability of (11) is defined similar to Definition 1 with the only difference that $\dot{\phi}_1 \in C_{n_1}[-h, 0]$ and instead of $|\phi| < \delta$ we have $|\phi| + |\dot{\phi}_1| < \delta$. By representing (11) in two different retarded type descriptor forms we obtain two different criteria.

First we put (11) in the retarded type descriptor form (12) that we rewrite as

$$\bar{E}\dot{\bar{x}}(t) = \sum_{i=0}^{m} \bar{A}_i \bar{x}(t-h_i) + \int_{-h_i}^{0} \bar{B}(s)\bar{x}(t+s)\,ds, \tag{37}$$

where

$$\bar{x} = \begin{bmatrix} x_1 \\ y \\ x_2 \end{bmatrix}, \qquad \bar{E} = \begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & 0_{n_1} & 0 \\ 0 & 0 & 0_{n_2} \end{bmatrix},$$

$$\bar{A}_0 = \begin{bmatrix} 0 & I & 0 \\ A_{01} & -I_{n_1} & A_{02} \\ A_{03} & 0 & A_{04} \end{bmatrix},$$

$$\bar{A}_i = \begin{bmatrix} 0 & 0 & 0 \\ A_{i1} & D_i & A_{i2} \\ A_{i3} & 0 & A_{i4} \end{bmatrix}, \quad i = 1, \ldots, m,$$

$$\bar{B} = \begin{bmatrix} 0 & 0 & 0 \\ B_1 & 0 & B_2 \\ B_3 & 0 & B_4 \end{bmatrix}. \tag{38}$$

Operator $\bar{\mathcal{D}}$ for (37) has a "triangular" form:

$$\bar{\mathcal{D}}(y_t, x_{2t}) = \begin{bmatrix} y(t) - \sum_{i=1}^{m} D_i y(t-h_i) - \sum_{i=1}^{m} A_{i2} x_2(t-h_i) \\ \quad - \int_{i=1}^{m} B_2(s)x_2(t+s)\,ds \\ x_2(t) + \sum_{i=1}^{m} A_{04}^{-1} A_{i4} x_2(t-h_i) \\ \quad + \int_{-h}^{0} A_{04}^{-1} B_4(s)x_2(t+s)\,ds \end{bmatrix}.$$

To guarantee stability of $\bar{\mathcal{D}}$ independently with respect to $h_1, \ldots, h_m$ we assume (A1$'$) ((A1$''$) for the case of $B_4 = 0$) and

(A2) Let the difference operator $\mathcal{D}_0 y_t = y(t) - \sum_{i=1}^m D_i y(t-h_i)$ be stable for all $h_i > 0$.

Due to Lemma 2 (A2) holds if there exist $(n_1 \times n_1)$ functions $P_0, U_1, \ldots, U_m$ that satisfy the following LMI:

$$\begin{bmatrix} -P_0 - P_0^T + \sum_{i=1}^m U_i & P_0^T D_1 & \ldots & P_0^T D_m \\ * & -U_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \ldots & -U_m \end{bmatrix} < 0. \tag{39}$$

Under (A1′) and (A2) a counterpart of Lemma 1 for (37) holds true and Theorem 1 implies the following result:

Corollary 1. *Under* (A1′) ((A1″) *for the case of* $B_4 = 0$) *and* (A2), (11) *is asymptotically stable if there exist a matrix* P *of* (22), *with* $(n_1 \times n_1)$*-matrix* P_1 *and* $(n \times n)$*-matrix* P_3, *and* $(n_1 + n) \times (n_1 + n)$*-matrices* $R = R^T$, $Q_i = Q_i^T$, $i = 1, \ldots, m$, *that satisfy the following LMI*:

$$\begin{bmatrix} \begin{array}{l} P^T \bar{A}_0 + \bar{A}_0^T \bar{P} + \sum_{i=1}^m Q_i \\ \quad + \int_{-h}^0 \bar{B}^T(s) R \bar{B}(s)\,ds \end{array} & P^T \bar{A}_1 & \ldots & P^T \bar{A}_m & h P^T \\ * & -Q_1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & -Q_m & 0 \\ * & * & * & * & -hR \end{bmatrix} < 0. \tag{40}$$

In the case of (34), (37) has the form

$$\bar{E}\dot{\bar{x}}(t) = \sum_{i=0}^m \bar{A}_i \bar{x}(t-h_i) + \sum_{i=1}^m h_i \bar{B}_i \int_{-h_i}^0 \bar{x}(t+s)\,ds, \tag{41}$$

where $\bar{E}$, $\bar{x}$ and $\bar{A}_i$ are defined as above and

$$\bar{B}_i = \begin{bmatrix} 0 & 0 & 0 \\ B_{i1} & 0 & B_{i2} \\ B_{i3} & 0 & B_{i4} \end{bmatrix}.$$

From Theorem 2 the following result follows:

Corollary 2. *Under* ($\bar{\text{A}}$1) ((A1″) *for the case of* $B_{4i} = 0$) *and* (A2), (11) *with* (34) *is asymptotically stable if there exist a matrix* P *of* (22), *with* $(n_1 \times n_1)$*-matrix* P_1 *and* $(n \times n)$*-matrix* P_3, *and* $(n_1 + n) \times (n_1 + n)$*-matrices* $R_i = R_i^T$, $Q_i = Q_i^T$, $i = 1, \ldots, m$, *that satisfy the following LMI*:

$$\begin{bmatrix} P^T\bar{A}_0+\bar{A}_0^T\bar{P}+\sum_{i=1}^m Q_i + \sum_{i=1}^m h_i R_i & P^T\bar{A}_1 & \dots & P^T\bar{A}_m & h_1P^T\bar{B}_1 & \dots & h_mP^T\bar{B}_m \\ * & -Q_1 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & -Q_m & 0 & \dots & 0 \\ * & * & * & * & -h_1R_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \dots & -h_mR_m \end{bmatrix} < 0. \tag{42}$$

The second representation of neutral system (11) is obtained by introducing a new variable $y = x_1 - \sum_{i=1}^m D_i x_1(t-h_i)$ and it has a form (37) (or (41) for B of (34)), where

$$\bar{x} = \begin{bmatrix} y \\ x_1 \\ x_2 \end{bmatrix}, \qquad \bar{E} = \begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & 0_{n_1} & 0 \\ 0 & 0 & 0_{n_2} \end{bmatrix},$$
$$\bar{A}_0 = \begin{bmatrix} 0 & A_{01} & A_{02} \\ I_{n_1} & -I_{n_1} & 0 \\ 0 & A_{03} & A_{04} \end{bmatrix},$$
$$\bar{A}_i = \begin{bmatrix} 0 & A_{i1} & A_{i2} \\ 0 & D_i & 0 \\ 0 & A_{i3} & A_{i4} \end{bmatrix}, \quad i = 1, \dots, m,$$
$$\bar{B} = \begin{bmatrix} 0 & B_1 & B_2 \\ 0 & 0 & 0 \\ 0 & B_3 & B_4 \end{bmatrix}, \qquad \bar{B}_i = \begin{bmatrix} 0 & B_{i1} & B_{i2} \\ 0 & 0 & 0 \\ 0 & B_{i3} & B_{i4} \end{bmatrix}. \tag{43}$$

Corollaries 1 and 2 hold also with matrices given by (43), i.e., for the second representation.

Remark 3. The case of non-descriptor system (11), where $n_2 = 0$, with a special distributed delay term (34) has been considered in [11] by applying the first descriptor form representation. For this case results of Corollary 2 coincide with results of [11].

The two different descriptor representations of neutral systems may lead to complementary results: for some systems conditions of Corollary 1 (or 2) for one of representations hold and for another do not. We illustrate this by two examples of non-descriptor neutral systems.

Example 1. Consider a non-descriptor neutral system

$$\dot{x}(t) - D_1\dot{x}(t-g) = A_0x(t) + A_1x(t-h), \tag{44}$$

where

$$A_0=\begin{bmatrix}-2 & 0\\ 0 & -15\end{bmatrix},\qquad A_1=\begin{bmatrix}1 & 3\\ -3 & 1\end{bmatrix},$$
$$D_1=\begin{bmatrix}-0.8 & 0\\ 0.2 & -0.8\end{bmatrix}. \tag{45}$$

Since there is no distributed delay term we may apply both corollaries. We apply Corollary 1. We use an LMI Toolbox of Matlab for solution of LMIs. The stability conditions of [28] and of Corollary 1 for the second representation do not hold for this system. Applying Corollary 1 for the first representation we find that this system is stable for all delays.

Example 2. Considering the system (44) with

$$A_0=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix},\qquad A_1=\begin{bmatrix}0 & 0.9\\ -1.3 & -1.9\end{bmatrix},$$

and D_1 as above, we find that stability conditions of Corollary 1 for the first representation do not hold. By [28] and by Corollary 1 for the second representation the system is asymptotically stable for all delays.

5. Delay-dependent stability. Effects of small delays on stability of descriptor systems

5.1. Delay-dependent stability conditions

We are looking for delay-dependent conditions with respect to slow variable x_1. With respect to discrete delays in the fast variables, the results will be delay-independent. The latter guarantees robust stability with respect to small changes of delay. We apply to (11) a descriptor representation introduced in [11] for non-descriptor case:

$$\begin{aligned}
&\dot{x}_1(t)=y(t),\\
&\begin{bmatrix} y(t)-\sum_{i=1}^m D_i y(t-h_i)\\ 0\end{bmatrix}\\
&\quad=\begin{bmatrix}\sum_{i=0}^m A_{i1} & A_{02}\\ \sum_{i=0}^m A_{i3} & A_{04}\end{bmatrix}x(t)+\sum_{i=1}^m\begin{bmatrix}A_{i2}\\ A_{i4}\end{bmatrix}x_2(t-h_i)\\
&\qquad-\sum_{i=1}^m\begin{bmatrix}A_{i1}\\ A_{i3}\end{bmatrix}\int_{-h_i}^{0} y(t+s)\,ds+\int_{-h}^{0}B(s)x(t+s)\,ds.
\end{aligned}\tag{46}$$

The latter system can be represented in the form

$$\bar{E}\dot{\bar{x}}(t) = \sum_{i=0}^{m} \bar{A}_i \bar{x}(t-h_i) + \sum_{i=1}^{m} h_i \bar{B}_i \int_{-h_i}^{0} \bar{x}(t+s)\,ds + \int_{-h}^{0} \bar{B}(s)\bar{x}(t+s)\,ds, \tag{47}$$

where

$$\bar{x} = \begin{bmatrix} x_1 \\ y \\ x_2 \end{bmatrix}, \qquad \bar{E} = \begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & 0_{n_1} & 0 \\ 0 & 0 & 0_{n_2} \end{bmatrix},$$

$$\bar{A}_0 = \begin{bmatrix} 0 & I & 0 \\ \sum_{i=0}^{m} A_{i1} & -I_{n_1} & A_{02} \\ \sum_{i=0}^{m} A_{i3} & 0 & A_{04} \end{bmatrix},$$

$$\bar{A}_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & D_i & A_{i2} \\ 0 & 0 & A_{i4} \end{bmatrix}, \quad i = 1, \dots, m,$$

$$\bar{B}_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -A_{i1} & 0 \\ 0 & -A_{i3} & 0 \end{bmatrix}, \qquad \bar{B} = \begin{bmatrix} 0 & 0 & 0 \\ B_1 & 0 & B_2 \\ B_3 & 0 & B_4 \end{bmatrix}. \tag{48}$$

From Theorems 1 and 2 the following result follows:

Theorem 3. *Under* (A1′) ((A1″) *for the case of* $B_4 = 0$) *and* (A2), (11) *is asymptotically stable if there exist a matrix* P *of* (22), *with* $(n_1 \times n_1)$*-matrix* P_1 *and* $(n \times n)$*-matrix* P_3, *and* $(n_1 + n) \times (n_1 + n)$*-matrices* $R_i = R_i^T$, $Q_i = Q_i^T$, $i = 1, \dots, m$, $R = R^T$ *that satisfy the following LMI*:

$$\begin{bmatrix} \Psi & P^T\bar{A}_1 & \dots & P^T\bar{A}_m & h_1P^T\bar{B}_1 & \dots & h_mP^T\bar{B}_m & hP^T \\ * & -Q_1 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & -Q_m & 0 & \dots & 0 & 0 \\ * & * & * & * & -h_1R_1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & * & \dots & -h_mR_m & 0 \\ * & * & * & * & * & \dots & * & -hR \end{bmatrix} < 0, \tag{49}$$

where

$$\Psi = P^T\bar{A}_0 + \bar{A}_0^T\bar{P} + \sum_{i=1}^{m} Q_i + \sum_{i=1}^{m} h_iR_i + \int_{-h}^{0} \bar{B}^T(s)R\bar{B}(s)\,ds.$$

For the case of special distributed delay term (34) we obtain (47) with $\bar{E}$, $\bar{x}$, $\bar{A}_i$ given by (48), and

$$\bar{B} = 0, \qquad \bar{B}_i = \begin{bmatrix} 0 & 0 & 0 \\ B_{i1} & -A_{i1} & B_{i2} \\ B_{i3} & -A_{i3} & B_{i4} \end{bmatrix}.$$

For this case Theorem 3 holds under ($\bar{\text{A}}$1) and (A2).

Remark 4. In the case of non-descriptor system (11) with a special distributed delay term (34) Theorem 3 gives the same delay-dependent conditions as [11].

5.2. *Sufficient conditions for robustness of stability with respect to small delays*

It is well known [22] that small delays may change the stability of a descriptor system. Necessary conditions and sufficient conditions for robust stability of descriptor systems with respect to small delays are given in [22] in terms of the spectral radius of a certain transfer matrix. Theorem 3 yields the following effective LMI criterion for robust stability:

Corollary 3. *Assume that* (A2) *holds. Then,* (11) *is asymptotically stable for all small enough $h_i \geqslant 0$ and $h \geqslant 0$ if there exist a matrix P of* (22), *with $(n_1 \times n_1)$-matrix P_1 and $(n \times n)$-matrix P_3, and $(n_1 + n) \times (n_1 + n)$-matrices $Q_i = Q_i^T$, $i = 1, \dots, m$, that satisfy the following LMI*:

$$\begin{bmatrix} P^T \bar{A}_0 + \bar{A}_0^T \bar{P} + \sum_{i=1}^m Q_i & P^T \bar{A}_1 & \dots & P^T \bar{A}_m \\ * & -Q_1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & * & -Q_m \end{bmatrix} < 0. \tag{50}$$

Proof. If (50) holds, then for $R_i = R = I_{n_1+n}$ and small enough delays (49) holds and result follows from Theorem 3. □

Note that (50) guarantees delay-independent stability for the following descriptor system without terms with distributed delay and with zero-delay in the slow variable:

$$\begin{bmatrix} \dot{x}_1(t) - \sum_{i=1}^m D_i \dot{x}_1(t-h_i) \\ 0 \end{bmatrix} = \sum_{i=0}^m A_i \begin{bmatrix} x_1(t) \\ x_2(t-h_i) \end{bmatrix}.$$

5.3. *Illustrative examples*

We consider two simple examples from [22] and [10].

Example 3 [22]. Consider the system

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t),$$
$$u(t) = [1\ -1]x(t-h), \tag{51}$$

where $x(t) = \text{col}\{x_1(t), x_2(t)\} \in R^2$. The closed-loop system (51) has a following canonical form:

$$\dot{x}_1(t) = 0.5x_1(t) - x_1(t-h), \tag{52a}$$
$$0 = -x_1(t) - x_2(t). \tag{52b}$$

Stability of (52) is equivalent to stability of the first retarded type equation and thus stability is robust with respect to small delays. By well-known frequency domain results (see, e.g., [9, Chapter 3, Section 3]), (52a) is asymptotically stable for $h < h^*$ and unstable for $h > h^*$, where $h^* = \arccos 0.5/\sqrt{(3/4)} \approx 1.2092$. Applying to (52a) more conservative LMI criteria of [17,20] we obtain that the system is stable for $h \leqslant 0.33$. By LMI criterion of Theorem 3 we obtain the less restrictive result: (51) is asymptotically stable for $h \leqslant 1$.

Example 4 [10]. Consider the system

$$\begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} x(t-h), \tag{53}$$

where $\varepsilon = 0$. By applying LMI of [10] we find that the system is asymptotically stable for $h \leqslant 0.14$. By applying delay-independent stability criterion of Theorem 1 we find that the corresponding LMI has a solution. Therefore the system is asymptotically stable for all delays.

Note that solving the algebraic equation of (53) with respect to x_1 and substituting the resulting expression into the differential equation of (53), we obtain the following decoupled system of equations:

$$\dot{x}_2(t) - 0.5\dot{x}_2(t-h) = -2x_2(t) + x_2(t-h) - 0.5x_2(t-2h),$$
$$x_1(t) = -0.5x_2(t) + 0.25x_2(t-h).$$

By well-known results [14] the first scalar equation of the latter system is asymptotically stable for all delays. Hence, the latter system and system (53) are asymptotically stable for all delays.

References

[1] I. Barbalat, Systems d'equations differentielles d'oscillations nonlinearies, Rev. Roumaine Math. Pures Appl. 4 (1959) 267–270.

[2] S. Boyd, L. Ghaoui, E. Feron, V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, in: SIAM Studies in Applied Mathematics, Vol. 15, SIAM, Philadelphia, 1994.
[3] R. Brayton, Small signal stability criterion for electrical networks containing lossless transmission lines, IBM J. Res. Develop. 12 (1968) 431–440.
[4] S. Campbell, Singular linear systems of differential equations with delays, Appl. Anal. 2 (1980) 129–136.
[5] S. Campbell, 2-D (differential-delay) implicit systems, in: Proceedings of IMACS World Congress on Scientific Computation, Dublin, 1991, pp. 1828–1829.
[6] S. Campbell, R. Nikoukhah, F. Delebecque, Nonlinear descriptor systems, in: P. Frank (Ed.), Advances in Control, Springer-Verlag, London, 1999, pp. 247–282.
[7] L. Dai, Singular Control Systems, Springer-Verlag, Berlin, 1989.
[8] V. Charitonov, D. Melchor-Aguilar, On delay-dependent stability conditions, Systems Control Lett. 40 (2000) 71–76.
[9] L. El'sgol'ts, S. Norkin, Introduction to the Theory and Applications of Differential Equations with Deviating Arguments, in: Mathematics in Science and Engineering, Vol. 105, Academic Press, New York, 1973.
[10] E. Fridman, Effects of small delays on stability of singularly perturbed systems, Automatica 38 (2002) 897–902.
[11] E. Fridman, New Lyapunov–Krasovskii functionals for stability of linear retarded and neutral type systems, Systems Control Lett. 43 (2001) 309–319.
[12] K. Gopalsamy, Stability and Oscillations in Delay Differential Equations of Population Dynamics, Kluwer Academic, Dordrecht, 1992.
[13] A. Halanay, V. Rasvan, Stability radii for some propagation models, IMA J. Math. Control Inform. 14 (1997) 95–107.
[14] J. Hale, E. Infante, F. Tsen, Stability in linear delay equations, J. Math. Anal. Appl. 105 (1985) 533–555.
[15] J. Hale, S. Lunel, Introduction to Functional Differential Equations, Springer-Verlag, New York, 1993.
[16] J. Hale, P. Martinez Amores, Stability in neutral equations, Nonlinear Anal. 1 (1977) 161–172.
[17] V. Kolmanovskii, S.-I. Niculescu, J.P. Richard, On the Liapunov–Krasovskii functionals for stability analysis of linear delay systems, Internat. J. Control 72 (1999) 374–384.
[18] V. Kolmanovskii, V. Nosov, Stability of Functional Differential Equations, Academic Press, New York, 1986.
[19] V. Kolmanovskii, J.-P. Richard, Stability of some linear systems with delays, IEEE Trans. Automat. Control 44 (1999) 984–989.
[20] X. Li, C. de Souza, Criteria for robust stability and stabilization of uncertain linear systems with state delay, Automatica 33 (1997) 1657–1662.
[21] C.-H. Lien, K.-W. Yu, J.-G. Hsieh, Stability conditions for a class of neutral systems with multiple time delays, J. Math. Anal. Appl. 245 (2000) 20–27.
[22] H. Logemann, Destabilizing effects of small time delays on feedback-controlled descriptor systems, Linear Algebra Appl. 272 (1998) 131–153.
[23] M. Mahmoud, Robust Control and Filtering for Time-Delay Systems, Marcel Decker, New York, 2000.
[24] S.-I. Niculescu, Further remarks on delay-dependent stability of linear neutral systems, in: Proceedings of MTNS 2000, Perpignan, France, 2000.
[25] S.-I. Niculescu, V. Rasvan, Delay-independent stability in lossless propagation models with applications, in: Proceedings of MTNS 2000, Perpignan, France, 2000.
[26] V. Rasvan, Absolute Stability of Time Lag Control Systems, Ed. Academiei, Bucharest, 1975, in Romanian.
[27] K. Takaba, N. Morihira, T. Katayama, A generalized Lyapunov theorem for descriptor system, Systems Control Lett. 24 (1995) 49–51.

[28] E. Verriest, S.-I. Niculescu, Delay-independent stability of linear neutral systems: A Riccati equation approach, in: L. Dugard, E. Verriest (Eds.), Stability and Control of Time-Delay Systems, in: Lecture Notes in Control and Inform. Sci., Vol. 227, Springer-Verlag, London, 1997, pp. 92–100.

[29] W. Zhu, L. Petzold, Asymptotic stability of linear delay differential–algebraic equations and numerical methods, Appl. Numer. Math. 24 (1997) 247–264.