Brief paper

Sampled-data H_∞ control and filtering: Nonuniform uncertain sampling ☆

V. Suplin, E. Fridman, U. Shaked*

School of Electrical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel

Abstract

Sampled-data H_∞ control of linear systems is considered. The measured output is sampled and the only restriction on the sampling is that the distance between sequel sampling times does not exceed a given bound. A novel performance index is introduced which takes into account the sampling rates of the measurement and it is thus related to the energy of the measurement noise. Three types of controllers are designed: a continuous-time controller, a sample and hold controller (synchronized with the sampling of the measurement), and an unsynchronized sampled and hold controller. A novel structure is adopted for these controllers where the dynamics of the controller is affected by the continuous-time state vector and the sampled value of this vector. A new approach, which was recently introduced to sampled-data stabilization is developed: the system is modeled as a continuous-time one, where the measurement output has a piecewise-continuous delay. A simple solution to the H_∞ control problem is derived in terms of linear matrix inequalities (LMIs). This solution is based on a new bounded real lemma (BRL) with state and disturbance delays. The results that are obtained for the output-feedback controller are readily applied to the problem of robust sampled-data H_∞ filtering with time-varying uncertain sampling rate.

1. Introduction

Sampled-data H_∞ control of systems has been studied in a number of papers (see e.g., Bamieh & Pearson, 1992; Basar & Bernard, 1995; Chen & Francis, 1995; Lall & Dullerod, 2001; Sagfors & Toivonen, 1997; Sivashankar & Khargonekar, 1994 and the references therein). Two main approaches have been used. The first one is based on the lifting technique (Bamieh & Pearson, 1992; Yamamoto, 1990) in which the problem is transformed to equivalent finite-dimensional discrete H_∞ control problem. The second, a more direct approach, is based on the representation of the system in the form of hybrid discrete/continuous model and the solution is obtained in terms of differential Riccati equations with jumps (Basar & Bernard, 1995; Sivashankar & Khargonekar, 1994). These approaches provide necessary and sufficient conditions and lead to equivalent solutions.

The LMI solution to sampled-data output-feedback H_∞ control was derived by Lall and Dullerod (2001) for the lifted discrete system when the sampling and the hold operators are periodic and their rates are commensurable. This solution is computationally complicated because it includes the evaluation of the matrices of the lifted system.

The hybrid system approach has been applied recently to robust H_∞ filtering under sampled-data measurements (Xu & Chen, 2003). Sampling interval-independent LMI conditions have been derived there which are quite restrictive.

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* Corresponding author.
E-mail address: shaked@eng.tau.ac.il (U. Shaked).

Modeling of continuous-time systems with digital control as continuous systems with delayed control input was introduced by Mikheev, Sobolev, and Fridman (1988). The digital control law may be represented as delayed control as follows:

$$u(t) = u_d(t_k) = u_d(t - (t - t_k)) = u_d(t - \tau(t)),$$
$$t_k \leqslant t < t_{k+1}, \quad \tau(t) = t - t_k, \tag{1}$$

where u_d is a discrete-time control signal and the time-varying delay $\tau(t) = t - t_k$ is piecewise linear with derivative $\dot{\tau}(t) = 1$ for $t \neq t_k$. Moreover, $\tau < t_{k+1} - t_k$.

Recently, this approach was applied to robust sampled-data stabilization (Fridman, Seuret, & Richard, 2004) and to H_∞ control (Fridman, Shaked, & Suplin, 2005). In Fridman et al. (2005) a conventional performance index (Sagfors & Toivonen, 1997; Xu & Chen, 2003) is considered, which has no physical meaning in the case of non-uniform sampling. The solution of the output-feedback control problem in Fridman et al. (2005) is based on introducing some special filters that precede the sampling of the measurement and the control input and that recover the filtering property of the sample and hold which filters out the high frequency part of the sampled signal. In the present paper we introduce a novel performance index, taking into consideration the energy of the measurement noise. We construct directly the filter by deriving a new BRL for systems with time varying state delay and piecewise constant disturbances. Moreover, an improved stability and BRL conditions for systems with time-varying delay τ, where $\dot{\tau} \leqslant 1$ (for almost all t) are derived. This is achieved by developing the input–output approach (Gu, Kharitonov, & Chen, 2003; Huang & Zhou, 2000; Zhang, Knopse, & Tsiotras, 2001), where stability of systems with constant or slow-varying delays with $\dot{\tau} \leqslant q < 1$ was analyzed, to the BRL and to the fast-varying delay with $\dot{\tau} \leqslant 1$.

Notation: Throughout the paper the superscript 'T' stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space with vector norm $|\cdot|$, $\mathcal{R}^{n \times m}$ is the set of all $n \times m$ real matrices, and the notation $P > 0$, for $P \in \mathcal{R}^{n \times n}$ means that P is symmetric and positive definite. Let $L_2[0, \infty)$ be the space of the square integrable functions with the norm $\|\cdot\|_2$. The vector $[a^T \ b^T]^T$ is denoted by $col\{a, \ b\}$ and $diag\{A, \ B\}$ is a block diagonal matrix with A and B on the diagonal.

2. Problem formulation

Consider the system:

$$\dot{x}(t) = A_0 x(t) + B_1 w(t) + B_2 u(t), \quad x(0) = 0,$$
$$z(t) = C_1 x(t) + D_{12} u(t), \tag{2}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $w(t) \in \mathcal{R}^q$ is the disturbance, $u(t) \in \mathcal{R}^p$ is the control input and $z(t) \in \mathcal{R}^r$ is the signal to be controlled or estimated.

The measurement output $y_k \in \mathcal{R}^m$ is assumed to be available at discrete sampling instants:

$$0 = \sigma_0 < \sigma_1 < \cdots < \sigma_k < \cdots, \quad \lim_{k \to \infty} \sigma_k = \infty$$

and it may be corrupted by $v_k = v(\sigma_k)$, where $v(t)$ is a measurement noise signal:

$$y_k = C_2 x(\sigma_k) + D_{21} v_k, \quad k = 0, 1, 2, \dots . \tag{3}$$

We assume that

A1. $\sigma_{k+1} - \sigma_k \leqslant h_1, \quad \forall k \geqslant 0.$

We formulate below three types of output-feedback control problems and two types of estimation problems.

2.1. The control problems

We define the following performance index for a prescribed scalar $\gamma > 0$:

$$J_c(w) = \int_0^\infty [z^T(s)z(s) - \gamma^2 w^T(s)w(s)] \, ds - \gamma^2 \sum_{k=0}^\infty (\sigma_{k+1} - \sigma_k) v^T(\sigma_k) v(\sigma_k) \tag{4}$$

and we seek the following three types of controllers:

Type 1: The output of this controller is continuous in time. Its dynamics is given by

$$\dot{x}_c(t) = A_{c0} x_c(t) + A_{c1} x_{ck} + B_c y_k, \quad x_{ck} = x_c(\sigma_k), \quad x_c(0) = 0,$$
$$u(t) = C_c x_c(t), \quad \sigma_k \leqslant t < \sigma_{k+1}, \tag{5}$$

where $x_c \in \mathscr{R}^n$. It is required that, for all sampling times satisfying A1 and for a prescribed value of γ, this controller will stabilize the system and will lead to $J_c < 0$, for all nonzero $w \in L_2[0, \infty)$ and $v(\sigma_k) \in l_2[0, \infty)$.

Type 2: The output of this controller is a sampled signal applied to a zero-order-hold. The sampling of this signal is synchronized with the sampling of the measured output y. The corresponding dynamics of this type of controller is given by

$$\dot{x}_c(t) = A_{c0}x_c(t) + A_{c1}x_{ck} + B_c y_k, \quad x_{ck} = x_c(\sigma_k), \quad x_c(0) = 0,$$
$$u(t) = C_c x_{ck}, \quad \sigma_k \leqslant t < \sigma_{k+1}. \tag{6}$$

Type 3: This controller is characterized by a sampled output (through a zero-order-hold) at sampling instants

$$0 = \eta_0 < \eta_1 < \cdots < \eta_j < \cdots, \quad \lim_{j \to \infty} \eta_j = \infty.$$

We assume that:

A2. $\eta_{j+1} - \eta_j \leqslant h_2, \ \forall j \geqslant 0$.

Denoting: $\bar{h} = \max(h_1, h_2)$, this controller is described by

$$\dot{x}_c(t) = A_{c0}x_c(t) + A_{c1}x_{ck} + A_{c2}x_{cj} + B_c y_k, \quad \sigma_k \leqslant t < \sigma_{k+1}, \ \eta_j \leqslant t < \eta_{j+1},$$
$$x_{ck} = x_c(\sigma_k), \quad x_{cj} = x_c(\eta_j), \quad x_c(0) = 0,$$
$$u(t) = C_c x_{cj}, \quad \eta_j \leqslant t < \eta_{j+1}. \tag{7}$$

Remark 1. In the definition of J_c the energies of the signals $w(t)$ and $v(t)$ are properly considered. The last summation in this definition is a rectangular approximation of the energy entailed in the measurement noise. In the past, an attempt has been made to solve the sampling problem by taking the sum of the squared L_2-norm of $w(t)$ and the squared l_2-norm of $\{v(\sigma_k)\}$ (Sagfors & Toivonen, 1997; Xu & Chen, 2003). Unfortunately, the latter summation has a little physical sense since it does not take the sampling rate into account and, in fact, it puts an increasing weight on the measurement noise the shorter the sampling interval becomes.

Remark 2. The above controllers are a generalization of the standard continuous-time nth order controller. Their dynamics is affected not only by the continuous control state $x_c(t)$ but also by the sampled values of this signal at the sampling instants. The additional degree of freedom introduced by A_{c1} and A_{c2} should lead to a lower value of γ for which a solution can be found.

2.2. The filtering problems

In the filtering problem we define the cost function

$$J_f(w) = \int_0^\infty [\tilde{z}^T(s)\tilde{z}(s) - \gamma^2 w^T(s)w(s)]\,ds - \gamma^2 \sum_{k=0}^\infty (\sigma_{k+1} - \sigma_k)v^T(\sigma_k)v(\sigma_k),$$
$$\tilde{z}(t) = C_1 x(t) - z_f(t), \tag{8}$$

where we assume that $u(t) \equiv 0$ in (2), and where we consider the following type 1 nth order filter:

$$\dot{x}_f(t) = A_{f0}x_f(t) + A_{f1}x_{fk} + B_f y_k, \quad x_{fk} = x_f(\sigma_k), \quad x_f(0) = 0,$$
$$z_f(t) = C_f x_f(t), \quad \sigma_k \leqslant t < \sigma_{k+1}. \tag{9}$$

The latter applies the sampled value of the estimation state vector x_f to the dynamics. The more standard full order estimator (denoted here as the type 2 filter) has the following structure:

$$\dot{x}_f(t) = A_{f0}x_f(t) + B_f y_k, \quad x_f(0) = 0,$$
$$z_f(t) = C_f x_f(t), \quad \sigma_k \leqslant t < \sigma_{k+1}. \tag{10}$$

Both filters should guarantee an estimation error level of γ, namely, $J_f < 0$ for all nonzero $w \in L_2[0, \infty)$, and $v(\sigma_k) \in l_2[0, \infty)$.

It will be shown below that whereas the type 1 filter can be derived from the results obtained for the type 1 controller, the solution of the filter of the second type requires some additional manipulations.

3. The output delay model

We consider the following piecewise-constant measurement:

$$y(t) = C_2 x(t - \tau_1(t)) + D_{21}v(t - \tau_1(t)), \tag{11}$$
$$\tau_1(t) = t - \sigma_k, \quad \sigma_k \leqslant t < \sigma_{k+1}. \tag{12}$$

From A1 it follows that $0 \leqslant \tau_1(t) < h_1$ and it is also found from (11) that $(d/dt)\tau_1 = 1$ over (σ_k, σ_{k+1}), for all $k \geqslant 0$.

Defining $\xi = col\{x, x_c\}$ we obtain the following augmented model for the three types of the output-feedback controller.

$$\dot{\xi}(t) = \bar{A}_0\xi(t) + \bar{A}_1\xi(t - \tau_1(t)) + \bar{A}_2\xi(t - \tau_2(t)) + \bar{B}w(t) + \bar{B}_1v(t - \tau_1(t)), \tag{12a}$$

$$\xi(t) = 0, \quad t \in [-\bar{h} \ 0], \tag{12b}$$

$$\bar{z}(t) = \bar{L}_0\xi(t) + \bar{L}_1\xi(t - \tau_1(t)) + \bar{L}_2\xi(t - \tau_2(t)). \tag{12c}$$

In the latter the delay $\tau_2(t)$ is the one that describes the sampling and hold of $u(t)$ in the controller of type 3. Namely, it follows from A2 that $0 \leqslant \tau_2(t) < h_2$ and that $(d/dt)\tau_2 = 1$ over $(\eta_j, \ \eta_{j+1})$, for all $j \geqslant 0$.

In the controller of type 1 the matrices of the above model are:

$$\bar{A}_0 = \begin{bmatrix} A_0 & B_2C_c \\ 0 & A_{c0} \end{bmatrix}, \qquad \bar{A}_1 = \begin{bmatrix} 0 & 0 \\ B_cC_2 & A_{c1} \end{bmatrix}, \qquad \bar{A}_2 = 0,$$

$$\bar{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \qquad \bar{B}_1 = \begin{bmatrix} 0 \\ B_cD_{21} \end{bmatrix}, \qquad \bar{L}_0 = [C_1 \ D_{12}C_c], \quad \bar{L}_1 = \bar{L}_2 = 0. \tag{13}$$

The type 2 controller is described by

$$\bar{A}_0 = \begin{bmatrix} A_0 & 0 \\ 0 & A_{c0} \end{bmatrix}, \qquad \bar{A}_1 = \begin{bmatrix} 0 & B_2C_c \\ B_cC_2 & A_{c1} \end{bmatrix}, \qquad \bar{A}_2 = 0,$$

$$\bar{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \qquad \bar{B}_1 = \begin{bmatrix} 0 \\ B_cD_{21} \end{bmatrix}, \qquad \bar{L}_0 = [C_1 \ 0], \quad \bar{L}_1 = [0 \ D_{12}C_c], \quad \bar{L}_2 = 0, \tag{14}$$

and, under assumption A2 the controller of type 3 is characterized by the following:

$$\bar{A}_0 = \begin{bmatrix} A_0 & 0 \\ 0 & A_{c0} \end{bmatrix}, \qquad \bar{A}_1 = \begin{bmatrix} 0 & 0 \\ B_cC_2 & A_{c1} \end{bmatrix}, \qquad \bar{A}_2 = \begin{bmatrix} 0 & B_2C_c \\ 0 & A_{c2} \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \qquad \bar{B}_1 = \begin{bmatrix} 0 \\ B_cD_{21} \end{bmatrix}, \qquad \bar{L}_0 = [C_1 \ 0], \quad \bar{L}_1 = 0 \quad \text{and} \quad \bar{L}_2 = [0 \ D_{12}C_c]. \tag{15}$$

4. New BRL for systems with state and disturbance delays

For the system (12) a bounded real lemma (BRL) is required which handles delays in the disturbance and in the objective function for $0 \leqslant \tau_i(t) < h_i, \ i = 1, 2$.

We represent (12) in the following form.

$$\dot{\xi}(t) = \left(\sum_{i=0}^{2} \bar{A}_i\right)\xi(t) + \sum_{i=1}^{2} \bar{A}_i \int_{t}^{t-\tau_i(t)} \dot{\xi}(s)\,ds + \bar{B}w(t) + \bar{B}_1v(t - \tau_1(t)),$$

$$\bar{z}(t) = \left(\sum_{i=0}^{2} \bar{L}_i\right)\xi(t) + \sum_{i=1}^{2} \bar{L}_i \int_{t}^{t-\tau_i(t)} \dot{\xi}(s)\,ds. \tag{16}$$

Applying to (16) the input–output approach to stability (see Gu et al., 2003 and the references therein), we introduce the following forward system (Fridman & Shaked, 2006)

$$\dot{\xi}(t) = \left(\sum_{i=0}^{2} \bar{A}_i\right)\xi(t) + h_1\bar{A}_1\omega_1(t) + h_2\bar{A}_2\omega_2(t) + \bar{B}w(t) + \bar{B}_1v(t - \tau_1(t)), \tag{17a}$$

$$\bar{z}(t) = (\bar{L}_0 + \bar{L}_1 + \bar{L}_2)\xi(t) + h_1\bar{L}_1\omega_1(t) + h_2\bar{L}_2\omega_2(t), \tag{17b}$$

$$y_i(t) = \dot{\xi}(t), \quad i = 1, 2 \tag{17c}$$

with feedback

$$\omega_i(t) = -1/h_i \int_{-\tau_i(t)}^{0} y_i(t+s)\,ds. \tag{18}$$

Assume that $\omega_i(t) = 0, \ t \leqslant 0$. Then the following holds for matrix $R_i > 0$ (Fridman & Shaked, 2006):

$$\int_{0}^{\infty} \omega_i^{T}(t)R_i\omega(t)\,dt \leqslant \int_{0}^{\infty} y_i^{T}(t)R_iy_i(t)\,dt. \tag{19}$$

Consider the Lyapunov function $V(t) = \xi^T(t) P_1 \xi(t)$, $P_1 > 0$. We are looking for the conditions that guarantee the following inequality along (17)

$$\mathcal{W} = \frac{d}{dt} V + \sum_{i=1}^{2} h_i y_i^T(t) R_i y_i(t) - \sum_{i=1}^{2} h_i \omega_i^T(t) R_i \omega_i(t) + \bar{z}^T(t)\bar{z}(t)$$
$$- \gamma^2(w^T(t)w(t) + v^T(t - \tau_1(t))v(t - \tau_1(t))) < 0. \tag{20}$$

Integrating (20) in t from 0 to ∞ and taking into account (19) and the equality $v(t - \tau_1(t)) = v(\sigma_k)$, $t \in [\sigma_k, \sigma_{k+1})$ we see that (20) implies $J_c < 0$.

Applying to (17a) the descriptor model transformation (Fridman, 2001) we have

$$\dot{V}(t) = 2\xi^T(t) P_1 \dot{\xi}(t) = 2[\xi^T(t)\ \dot{\xi}^T(t)] P^T[\dot{\xi}^T(t)\ 0]^T$$
$$= 2[\xi^T(t)\ \dot{\xi}^T(t)] P^T \left[\begin{array}{c} \dot{\xi}(t) \\ \left(\sum\limits_{i=0}^{2} \bar{A}_i\right)\xi(t) + h_1\bar{A}_1\omega_1(t) + h_2\bar{A}_2\omega_2(t) + \bar{B}w(t) + \bar{B}_1 v(t - \tau_1(t)) - \dot{\xi}(t) \end{array} \right], \tag{21}$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}. \tag{22}$$

Substituting (21) into $\mathcal{W}$ and applying Schur complements to the term $\sum_{i=1}^{2} h_i y_i^T(t) R_i y_i(t) + \bar{z}^T(t)\bar{z}(t)$ we conclude that (20) is satisfied if

$$\begin{bmatrix} \sum\limits_{i=0}^{2} P_2^T \bar{A}_i + \sum\limits_{i=0}^{2} \bar{A}_i^T P_2 & P_1 - P_2^T + \sum\limits_{i=0}^{2} \bar{A}_i^T P_3 & h_1 P_2^T \bar{A}_1 & h_2 P_2^T \bar{A}_2 & P_2^T \bar{B} & P_2^T \bar{B}_1 & \sum\limits_{i=0}^{2} \bar{L}_i^T \\ * & -P_3^T - P_3 + h_1 R_1 + h_2 R_2 & h_1 P_3^T \bar{A}_1 & h_2 P_3^T \bar{A}_2 & P_3^T \bar{B} & P_3^T \bar{B}_1 & 0 \\ * & * & -h_1 R_1 & 0 & 0 & 0 & h_1 \bar{L}_1^T \\ * & * & * & -h_2 R_2 & 0 & 0 & h_2 \bar{L}_2^T \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0. \tag{23}$$

We proved the following BRL:

Lemma 1. *Consider* (12). *For a prescribed $\gamma > 0$, the cost function* (4) *achieves $J_c(w) < 0$ for all nonzero $w \in L_2[0, \infty)$, $v(\sigma_k) \in l_2[0, \infty)$ and for $\tau_1(t) = t - \sigma_k < h_1$, $t \in [\sigma_k, \sigma_{k+1})$, $\tau_2(t) = t - \eta_j < h_2$, $t \in [\eta_j, \eta_{j+1})$, if there exist matrices $P_1 > 0$, P_2, P_3, $R_1 = R_1^T$ and $R_2 = R_2^T$ that satisfy LMI* (23).

Remark 3. An equivalent BRL may be derived by direct application of the descriptor Lyapunov–Krasovskii functional (Fridman, 2001).

5. Controller design

5.1. Sampled-data state-feedback H_∞ control

The above BRL is required not only for deriving the output-feedback controller but also for obtaining a solution to the sampled state-feedback control problem where a control law

$$u(t) = Kx(\sigma_k), \quad \sigma_k \leqslant t < \sigma_{k+1} \tag{24}$$

is sought that achieves

$$J_{sf}(w) = \int_0^\infty [z^T(s)z(s) - \gamma^2 w^T(s)w(s)]\,ds < 0 \tag{25}$$

for all nonzero $w \in L_2[0, \infty)$. This problem has been solved in the past by Fridman et al. (2005) assuming that D_{12} in (2) is zero. The BRL we obtained in the last section provides a tool by which the case of nonzero weighting on the control effort is considered in the cost function.

The state-feedback law of (24) can be written as $u(t) = Kx(t - \tau_1(t))$ where $\tau_1(t) = t - \sigma_k$, $\sigma_k \leqslant t < \sigma_{k+1}$. We consider therefore the model of (12) with

$$\bar{A}_0 = A, \quad \bar{A}_1 = B_2 K, \quad \bar{A}_2 = 0, \quad \bar{B} = B_1, \quad \bar{B}_1 = 0, \quad \bar{L}_0 = C_1 \quad \text{and} \quad \bar{L}_1 = D_{12} K.$$

Considering then the LMI of (23) we seek a solution that satisfies $P_3 = \varepsilon P_2$ where ε is a nonzero scalar. As such, since $P_3 + P_3^T$ appear on the diagonal, the matrix P_2 is nonsingular. Denoting $Q = P_2^{-1}$ we multiply (23), after deleting the columns and rows that include A_2, R_2 and $\bar{B}_1$, by $\text{diag}\{Q^T, Q^T, Q^T, I, I\}$ and $\text{diag}\{Q, Q, Q, I, I\}$, from the left and the right, respectively. Denoting $V = KQ$, $\bar{R} = Q^T R_1 Q$ and $\bar{P}_1 = Q^T P_1 Q$ we obtain the following:

Lemma 2. *Given the performance level γ and the tuning parameter ε. The control law (24) achieves (25) for all nonzero $w \in L_2[0, \infty)$ is there exist Q, $0 < \bar{P}_1$, $\bar{R} \in \mathscr{R}^{n \times n}$ and $V \in \mathscr{R}^{p \times n}$ that satisfy the following LMI:*

$$\bar{\Phi} = \begin{bmatrix} AQ + Q^T A^T + B_2 V + V^T B_2^T & \bar{P}_1 - Q + \varepsilon(Q^T A^T + V^T B_2^T) & h_1 B_2 V & B_1 & Q^T C_1^T + V^T D_{12}^T \\ * & -\varepsilon(Q^T + Q) + h_1 \bar{R} & h_1 \varepsilon B_2 V & \varepsilon B_1 & 0 \\ * & * & -h_1 \bar{R} & 0 & h_1 V^T D_{12}^T \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \quad (26)$$

If a solution to the latter LMI exists, the state-feedback gain matrix is given by: $K = VQ^{-1}$.

Remark 4. Another method for solving (23) is based on the iterative algorithm developed by Gao and Wang (2003). This method may be preferable in cases of relatively large h_i, $i = 1, 2$, since it can lead to less conservative results. However, the latter requires longer computer time due to the iterative process.

5.2. Output-feedback H_∞ control

In order to obtain a solution to the output-feedback controller we consider (23), with $P_3 = \varepsilon P_2$, where ε is a scalar, and similarly to the method (Scherer, Gahinet, & Chilali, 1997) we denote the partitions:

$$P_2^T = \begin{bmatrix} X & M \\ \bar{M} & U \end{bmatrix} \quad \text{and} \quad P_2^{-T} = \begin{bmatrix} Y & N \\ \bar{N} & \bar{V} \end{bmatrix} \quad (27)$$

and define

$$J = \begin{bmatrix} I & Y^T \\ 0 & N^T \end{bmatrix}, \quad \bar{J} = \text{diag}\{J, J, J, J, J, J\}.$$

Multiplying (23) by $\text{diag}\{\bar{J}, I, I, I\}$ from the right and by $\text{diag}\{\bar{J}^T, I, I, I\}$ from the left we obtain the following two inequalities:
For the controllers of type 1 and 2:

$$\Phi = \begin{bmatrix} XA_0 + A_0^T X^T + SC_2 + C_2^T S^T & \hat{A}_0 + A_0^T & \bar{P}_{11} - X + \varepsilon(A_0^T X^T + C_2^T S^T) \\ * & A_0 Y^T + Y A_0^T + B_2 Z + Z^T B_2^T & \bar{P}_{12} - I + \varepsilon \hat{A}_0^T \\ * & * & -\varepsilon(X + X^T) + h_1 \bar{R}_{11} \\ * & * & * \\ * & * & * \\ * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

$$\begin{matrix} \bar{P}_{12} - T + \varepsilon A_0^T & h_1 SC_2 & h_1 \hat{A}_1 & XB_1 & SD_{21} & C_1^T \\ \bar{P}_{13} - Y^T + \varepsilon(YA_0^T + Z^T B_2^T) & 0 & \Sigma_1 & B_1 & 0 & YC_1^T + Z^T D_{12}^T \\ -\varepsilon(I + T) + h_1 \bar{R}_{12} & \varepsilon h_1 SC_2 & \varepsilon h_1 \hat{A}_1 & \varepsilon XB_1 & \varepsilon SD_{21} & 0 \\ -\varepsilon(Y + Y^T) + h_1 \bar{R}_{13} & 0 & \varepsilon \Sigma_1 & \varepsilon B_1 & 0 & 0 \\ * & -h_1 \bar{R}_{11} & -h_1 \bar{R}_{12} & 0 & 0 & 0 \\ * & * & -h_1 \bar{R}_{13} & 0 & 0 & \Sigma_2 \\ * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -I \end{matrix} < 0, \quad (28a)$$

where

$$\hat{P} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{12}^{\mathrm{T}} & \bar{P}_{13} \end{bmatrix} = J^{\mathrm{T}} P_1 J \tag{28b}$$

and

$$\hat{R}_1 = \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ \bar{R}_{12}^{\mathrm{T}} & \bar{R}_{13} \end{bmatrix} = J^{\mathrm{T}} R_1 J \tag{28c}$$

and where for the controller of type 1

$$\hat{A}_0 = X A_0 Y^{\mathrm{T}} + M(A_{c0} + A_{c1})N^{\mathrm{T}} + X B_2 Z + S C_2 Y^{\mathrm{T}}, \tag{28d}$$

$$\hat{A}_1 = S C_2 Y^{\mathrm{T}} + M A_{c1} N^{\mathrm{T}}, \tag{28e}$$

$$Z = C_c N^{\mathrm{T}}, \tag{28f}$$

$$S = M B_c, \tag{28g}$$

$$T = X Y^{\mathrm{T}} + M N^{\mathrm{T}}, \tag{28h}$$

$$\Sigma_1 = 0, \tag{28i}$$

and

$$\Sigma_2 = 0. \tag{28j}$$

The corresponding matrices in the solution for type 2 are given by (28d), (28f)–(28h) with

$$\hat{A}_1 = S C_2 Y^{\mathrm{T}} + M A_{c1} N^{\mathrm{T}} + X B_2 Z, \tag{29a}$$

$$\Sigma_1 = h_1 B_2 Z \tag{29b}$$

and

$$\Sigma_2 = h_1 Z^{\mathrm{T}} D_{12}^{\mathrm{T}}. \tag{29c}$$

The controller of the third type is obtained by solving the following inequality:

$$\begin{bmatrix} & & & & & & & & & 0 & h_2 \hat{A}_2 & 0 & 0 \\ & & & & & & & & & 0 & h_2 B_2 Z & 0 & 0 \\ & & & & & & & & & 0 & \varepsilon h_2 \hat{A}_2 & h_2 \bar{R}_{21} & h_2 \bar{R}_{22} \\ & & & & & & & & & 0 & \varepsilon h_2 B_2 Z & h_2 \bar{R}_{22}^{\mathrm{T}} & h_2 \bar{R}_{23} \\ & & & & \Phi & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & & & 0 & h_2 D_{12} Z & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -h_2 \bar{R}_{21} & -h_2 \bar{R}_{22} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -h_2 \bar{R}_{23} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -h_2 \bar{R}_{21} & -h_2 \bar{R}_{22} \\ * & * & * & * & * & * & * & * & * & * & * & * & -h_2 \bar{R}_{23} \end{bmatrix} < 0, \tag{30a}$$

where Φ is defined in (28a), with $\Sigma_1 = \Sigma_2 = 0$,

$$\hat{A}_0 = X A_0 Y^{\mathrm{T}} + M(A_{c0} + A_{c1} + A_{c2})N^{\mathrm{T}} + X B_2 Z + S C_2 Y^{\mathrm{T}} \tag{30b}$$

$$\hat{A}_1 = S C_2 Y^{\mathrm{T}} + M A_{c1} N^{\mathrm{T}} \tag{30c}$$

and

$$\hat{A}_2 = X B_2 Z + M A_{c2} N^{\mathrm{T}} \tag{30d}$$

and where

$$\hat{R}_2 = \begin{bmatrix} \bar{R}_{21} & \bar{R}_{22} \\ \bar{R}_{22}^{\mathrm{T}} & \bar{R}_{23} \end{bmatrix} = J^{\mathrm{T}} R_2 J. \tag{30e}$$

For a given value of ε the latter inequalities are linear in the decision variables: X, Y, S, Z, T, $\hat{A}_i$, $i = 1, \ldots, 3$, $\hat{P}$ and $\hat{R}_i$, $i = 1, 2$. This leads to the following.

Theorem 1. *For a prescribed $\gamma > 0$, there exists a controller of the form (5)–(7) that achieves $J_c(w, v) < 0$ for all nonzero $w \in L_2[0, \infty)$, $v(\sigma_k) \in l_2[0, \infty)$ and for $\tau_1(t) = t - \sigma_k < h_1$, $t \in [\sigma_k, \sigma_{k+1})$, and $\tau_2(t) = t - \eta_j < h_2$, $t \in [\eta_j, \eta_{j+1})$, if there exist matrices $X, Y, S, Z, T \hat{P}, \hat{R}_i$, $i = 1, 2$ and $\hat{A}_i$, $i = 1, 2, 3$, with $\hat{P} > 0$, that, for a tuning scalar ε, satisfy (28a) with (28i), (28j), (28a) with (29a)–(29c), or (30a)–(30d), respectively.*

If the corresponding LMI has a solution the controller matrices can be readily obtained from (28d)–(28h), (29a) and (30b)–(30d).

Remark 5. The matrices M and N in, say (28d)–(28h), are not decision variables in the condition of Theorem 1. The question arises how these matrices can be found and whether they are nonsingular. The definition of M and N is given in (27). It follows from this definition that if we arbitrarily choose $N = I_n$ and apply the definition of (28h) the following is obtained:

$$M = T - XY^T. \tag{31}$$

If the resulting M turns out to be singular, a small perturbation added to the matrix T in the (1,4) block in the above LMIs will lead to an invertible M. The invertibility of X and Y is guaranteed by the fact that $X + X^T$ and $Y + Y^T$ are blocks on the diagonal of the above LMIs.

Remark 6. Comparing between the minimum performance level γ that is achievable by applying the three types of controllers to the same system it is clear that the controller of type one should achieve the smallest minimum value since it describes the case where the output of the controller is continuously fed to the control input of the plant without any sampling. A comparison between the type 2 and type 3 controllers depends on h_2 and h_1. When $h_2 \ll h_1$ the value of γ that is achievable by the controller of the third type will be close to (though still bigger than) the value achieved by the type 1 controller. This is clearly seen from (30a) when applying Schur formula. In the limit where h_2 tends to zero the condition becomes the one of (28a).

Example 1 (*Control*). We consider the system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -16 & 4.8 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 16 \end{bmatrix} w(t), \quad z(t) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t). \tag{32}$$

The system is taken to be unstable because in stable systems, when one applies a too large sampling interval, the solutions for the output-feedback controllers of Sections 3 and 5 will lead to a minimum value of γ which is very close to the H_∞-norm of the open-loop. This is achieved by deriving a controller with very small gains.

We begin by considering the case where a sampled version

$$x_k = x(\sigma_k), \quad \sigma_{k+1} - \sigma_k \leqslant h_1 = \pi/25, \quad k = 1, 2, \ldots$$

is available for feedback. Applying Lemma 2 we readily obtain a minimum performance level of $\gamma_{\min} = 24.207$, achieved for $\varepsilon = 1.1$. The corresponding state-feedback gain matrix is $K = [0.8430 \ -0.4781]$.

We next consider the case where the measurement is described by

$$y_k = [1 \ 0] x(\sigma_k) + 0.1 v_k, \quad \sigma_{k+1} - \sigma_k \leqslant h_1 = \pi/25, \quad k = 0, 1, \ldots .$$

Applying Theorem 1 to the above system, a type 1 controller is first obtained which achieves a minimum performance value of $\gamma_{\min} = 19.99$, for $\varepsilon = 1$. Note that γ achieved by the dynamic output-feedback controller of type 1 is smaller than the one achieved by the constant delayed state-feedback.

The type 2 controller should achieve a higher γ since it applies a sampled and hold input to the plant (synchronized with the measurements). Indeed, the minimum achievable value of γ for this controller is $\gamma_{\min} = 339.8$ which is achieved for $\varepsilon = 1.4$.

A controller of type 3 controller is then sought which for unsynchronous sample and hold of the control signal u provides a minimum bound on γ. For sampling rate bound of $h_2 = 0.01$ a minimum value of $\gamma = 20.83$ is achieved for $\varepsilon = 1$. It should be noted that for smaller sampling rate bound the achievable values of γ may become quite large. For, say $h_2 = h_1 = \pi/25$, a minimum achievable value of $\gamma = 339$ is obtained, using $\varepsilon = 1.5$.

We note that the results obtained by the controller of type 3 tend to coincide with those of type 1 in the limit where h_2 tends to zero. In the limit where h_1 tends to zero the result of the controller of type 1 produces the result for the continuous-time output-feedback controller. A value of $\gamma = 0.3237$ is then produced for ε that tends to zero. A result that may look odd, at least at first sight, is the fact that the sampled state-feedback achieves a result that is inferior to the one obtained by the type 1 controller. The reason why a constant feedback gain may be insufficient to obtain the lowest attenuation level may be the high frequency content of the sampling equivalent disturbance (see the input–output approach to delay (Gu et al., 2003)) that requires a low pass component in the controller.

6. Filtering design

The above results have been obtained for the control problem. The solution to the type 1 filtering problem is derived from the result that has been obtained above for the controller of type 1 as follows. Substituting in (28a) $B_2 = 0$ and $D_{12} = -I_r$ we obtain the following result for the sampled data H_∞ filter.

Theorem 2. *For a prescribed $\gamma > 0$, there exits a filter of the form (9) that achieves $J_f(w, v) < 0$ for all nonzero $w \in L_2[0, \infty)$, $v(\sigma_k) \in l_2[0, \infty)$ and for $\tau_1(t) = t - \sigma_k < h_1$, $t \in [\sigma_k, \sigma_{k+1})$, if there exist matrices $X, Y, S, Z, T \hat{P}, \hat{R}_1$ and $\hat{A}_i$, $i = 1, 2$, with $\hat{P} > 0$, that, for a tuning scalar ε, satisfy (28a) for $B_2 = 0$, $D_{12} = -I_r$, $\Sigma_1 = 0$ and $\Sigma_2 = 0$.*
If the LMI has a solution, the filter matrices are obtained by

$$A_{f0} = (T - XY^{\mathrm{T}})^{-1}[\hat{A}_0 - XA_0Y^{\mathrm{T}} - \hat{A}_1], \tag{33a}$$

$$A_{f1} = (T - XY^{\mathrm{T}})^{-1}[\hat{A}_1 - SC_2Y^{\mathrm{T}}], \tag{33b}$$

$$B_f = (T - XY^{\mathrm{T}})^{-1}S, \tag{33c}$$

and

$$C_f = Z. \tag{33d}$$

In the filter of type 2, a zero A_{f1} is applied. If we again rely on (28a) with $B_2 = 0$ and $D_{12} = -I_n$, we find from (28e) that the choice of $A_{f1} = A_{c1} = 0$ implies that the decision variable $\hat{A}_1$ is rank deficient. Replacing then $\hat{A}_1$ in (28a) with $MB_fC_2Y^{\mathrm{T}}$ the resulting inequality is no longer linear. This inequality can, however, be linearized as follows.

Denoting $\bar{Y} = Y^{-1}$ we substitute: $B_2 = 0$, $D_{12} = -I_n$ and $\hat{A}_1 = MB_fC_2Y^{\mathrm{T}}$ in (28a) (with $\Sigma_1 = \Sigma_2 = 0$). We then multiply the second, fourth, sixth and eighth rows of the resulting inequality by $\bar{Y}$ and the corresponding columns by $\bar{Y}^{\mathrm{T}}$. Denoting: $\tilde{P}_{12} = \bar{P}_{12}\bar{Y}^{\mathrm{T}}$, $\check{P}_{12} = \bar{Y}P_{13}\bar{Y}^{\mathrm{T}}$, $\check{R}_{12} = \bar{R}_{12}\bar{Y}^{\mathrm{T}}$, $\check{R}_{13} = \bar{Y}\bar{R}_{13}\bar{Y}^{\mathrm{T}}$ we obtain the following:

Corollary 1. *For a prescribed $\gamma > 0$, there exits a filter of the form (10) that achieves $J_f(w, v) < 0$ for all nonzero $w \in L_2[0, \infty)$, $v(\sigma_k) \in l_2[0, \infty)$ and for $\tau_1(t) = t - \sigma_k < h_1$, $t \in [\sigma_k, \sigma_{k+1})$, if there exist matrices $X, \bar{Y}, S, \tilde{Z}, \tilde{T}, 0 < \bar{P}_{11}, \tilde{P}_{12}, \tilde{P}_{13}, \bar{R}_{11}, \tilde{R}_{12}, \tilde{R}_{13}$ and $\tilde{A}_0$, that, for a tuning scalar ε, satisfy the following LMI:*

$$
\begin{bmatrix}
XA_0 + A_0^{\mathrm{T}}X^{\mathrm{T}} + SC_2 + C_2^{\mathrm{T}}S^{\mathrm{T}} & \tilde{A}_0 + A_0^{\mathrm{T}}\bar{Y}^{\mathrm{T}} & \bar{P}_{11} - X + \varepsilon(A_0^{\mathrm{T}}X^{\mathrm{T}} + C_2^{\mathrm{T}}S^{\mathrm{T}}) & \tilde{P}_{12} - \tilde{T} + \varepsilon A_0^{\mathrm{T}}\bar{Y}^{\mathrm{T}} \\
* & \bar{Y}A_0 + A_0\bar{Y}^{\mathrm{T}} & \tilde{P}_{12}^{\mathrm{T}} - \bar{Y} + \varepsilon\tilde{A}_0 & \tilde{P}_{13} - \bar{Y} + \varepsilon A_0^{\mathrm{T}}\bar{Y}^{\mathrm{T}} \\
* & * & -\varepsilon(X + X^{\mathrm{T}}) + h_1\tilde{R}_{11} & -\varepsilon(\bar{Y}^{\mathrm{T}} + \tilde{T}) + h_1\tilde{R}_{12} \\
* & * & * & -\varepsilon(\bar{Y}^{\mathrm{T}} + \bar{Y}) + h_1\tilde{R}_{13} \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & *
\end{bmatrix}
$$

$$
\begin{bmatrix}
h_1SC_2 & h_1SC_2 & XB_1 & SD_{21} & C_1^{\mathrm{T}} \\
0 & 0 & \bar{Y}B_1 & 0 & C_1^{\mathrm{T}} - \tilde{Z}^{\mathrm{T}} \\
\varepsilon h_1SC_2 & \varepsilon h_1SC_2 & \varepsilon XB_1 & \varepsilon SD_{21} & 0 \\
0 & 0 & \varepsilon\bar{Y}B_1 & 0 & 0 \\
-h_1\bar{R}_{11} & -h_1\tilde{R}_{12} & 0 & 0 & 0 \\
* & -h_1\tilde{R}_{13} & 0 & 0 & 0 \\
* & * & -\gamma^2 I & 0 & 0 \\
* & * & * & -\gamma^2 I & 0 \\
* & * & * & * & -I
\end{bmatrix} < 0, \tag{34a}
$$

where:

$$\tilde{A}_0 = XA_0 + SC_2 + MA_{f0}N^{\mathrm{T}}\bar{Y}^{\mathrm{T}}, \tag{34b}$$

$$\tilde{Z} = C_f N^\mathrm{T} \bar{Y}^\mathrm{T}, \tag{34c}$$

$$S = M B_f, \tag{34d}$$

$$\tilde{T} = X + M N^\mathrm{T} \bar{Y}^\mathrm{T}. \tag{34e}$$

If a solution to the latter LMI exists then the transfer function matrix of the filter is given by:

$$T(s) = \tilde{Z} Y^\mathrm{T} N^{-\mathrm{T}} (sI - M^{-1}(\tilde{A}_0 - X A_0 - SC_2) Y^\mathrm{T} N^{-\mathrm{T}})^{-1} M^{-1} S = \tilde{Z}((\tilde{T} - X)s - (\tilde{A}_0 - X A_0 - SC_2))^{-1} S. \tag{35}$$

Remark 7. The above results for the two types of filter have been derived for the nominal case where the matrices of the state-space model of the process to be controlled or estimated are all known. However, the LMI in Corollary 1 is affine in these matrices. Thus, a robust type 2 filter can be obtained that satisfies the prescribed performance level, determined by γ, for systems with matrix parameters that lie in a given polytope if (34) is satisfied at all the vertices of this uncertainty polytope.

The same holds true for the filter of type 1. If we repeat the procedure that led to (34), this time with a nonzero A_{1f}, we obtain the LMI (34), with the exception that now SC_2 is replaced in the (1,6) and the (3,6) blocks by $SC_2 + \hat{A}_1$ where $\hat{A}_1 = M A_{f1} N^\mathrm{T} \bar{Y}^\mathrm{T}$. Also the resulting inequality is affine in the matrices of the state-space model of the process and thus a robust solution can be obtained also for the filter of type 1.

Remark 8. The filters that are derived above are of full order. Filters of reduced order can be obtained by imposing structural constrains on the decision variables in, say, (34). These constrains should lead to uncontrollable filters with the required number of controllable modes.

Example 2 (*filtering*). (i) *Estimation for nominal systems*: We consider here the process of Sagfors and Toivonen (1997). Given the system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -16 & -4.8 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 16 \end{bmatrix} w(t), \tag{36a}$$

$$z(t) = [1\ 0] x(t), \tag{36b}$$

$$y_k = [1\ 0] x(\sigma_k) + 0.1 v_k, \quad k = 0, 1, \ldots \tag{36c}$$

with $h_1 = \pi/4$. We seek an estimator of type 1 in the form of (9) that obtains a continuous estimate of $z(t)$ with an estimation error level of γ. We apply Theorem 2 and find a minimum achievable $\gamma = 1.0418$ for $\varepsilon = 0.01$. For $h_1 = \pi/25$ we get a better estimation. For $\varepsilon = 0.18$ we obtain $\gamma = 0.4647$.

The corresponding results that are obtained for the filter of type 2 (applying the LMI of Corollary 1) are $\gamma = 1.043$ (for $\varepsilon = 0.01$) and $\gamma = 0.4876$ (for $\varepsilon = 0.14$), respectively. The slight improvement that is achieved by the filter of type 1 is due to the additional degree of freedom A_{f1} that was introduced in (10).

(ii) *Robust estimation*:

We consider the system of (36a), (36c) where the (2, 2) element in the dynamic matrix A is uncertain and instead of being -4.8 in the latter solution it is now known to reside in the interval $[-3.80\ -5.8]$. The uncertainty polytope is in the present case an interval with two vertices. Applying the arguments of Corollary 1 we solve the LMI of (34a) at the two vertices and obtain for $h_1 = \pi/4$ a minimum value of $\gamma = 1.239$ for ε that tends to zero. For the smaller delay of $h_1 = \pi/25$ the corresponding result is $\gamma = 0.5794$ achieved for $\varepsilon = 0.13$.

7. Conclusions

A comprehensive H_∞ control and filtering design approach is presented for linear systems with sampled measurements. The sampling rate may be unknown but bounded by a known value h and it may vary in time. A new performance index is introduced which takes into account sampling rates and corresponds to the energy of the measurement noise. On the basis of h, design schemes are proposed for deriving various types of controllers. These stem from a new bounded real lemma that is developed in this paper to accommodate for delays in the disturbances and in the objective.

A state-feedback controller is first derived which in comparison to previous methods allow for weighting of the control effort in the performance index. When access (a delayed one) to the system states is unavailable, three types of controllers are derived which, under their special control set-up, achieve a minimum bound on the disturbance rejection level. These controllers cover the

three cases where the controller applies either a continuous-time control signal, or a sample and hold control (synchronized with the sampling of the measurement), or an unsynchronized sampled and hold control input. All the three types of the controllers are characterized by the fact that their (continuous-time) dynamics is affected not only by the current value of the controller state but also by its constant value at the last sampling instant. This additional degree of freedom should help in reducing the achieved performance level. Additional degrees of freedom may be introduced by considering also the value of the controller state in previous sampling instants. This should further reduce the value of the performance index but will significantly complicate the derivation of the corresponding controllers.

The theory developed is also applied to the case of H_∞ filtering that is based on uncertain time-varying sampling of the noisy measurements. A need for such a type of filter is encountered in many areas of modern communication and network control. A filtering scheme is introduced which produces a continuous-time estimate of the system state vector. Also this filter possesses the non conventional structure that includes the additional term of the sampled value of the estimate in the continuous time description of the estimator dynamics. The estimate is obtained by solving a linear matrix inequality that is affine in the parameters of the process to be estimated. It can be therefore used to obtain a robust estimation scheme for systems with polytopic type uncertainty in their parameters.

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Vladimir Suplin received the M.Sc. degree in 2000 from Tel-Aviv University, Israel. He is presently a Ph.D. student at Tel-Aviv University, Israel. His areas of current research interest include robust and sampled-data control, time-delayed dynamical systems, and networks.

Emilia Fridman received the M.Sc. degree from Kuibyshev State University, USSR, in 1981 and the Ph.D. degree from Voroneg State University, USSR, in 1986, all in mathematics. From 1986 to 1992 she was an Assistant and Associate Professor in the Department of Mathematics at Kuibyshev Institute of Railway Engineers, USSR. Since 1993 she has been at Tel Aviv University, where she is currently Professor of Electrical Engineering-Systems.

Her research interests include time-delay systems, H_∞ control, singular perturbations, nonlinear control and asymptotic methods. She has published over 60 articles in international scientific journals.

Uri Shaked received the B.Sc and the M.Sc degrees in physics from the Hebrew University, Jerusalem, Israel, in 1964 and 1968, respectively, and the Ph.D degree in applied mathematics from the Weizmann Institute, Rehovot, Israel in 1975.

From 1974 to 1976, he was a Senior Visiting Fellow at the Control and Management Science Division, Faculty of Engineering, Cambridge University, Cambridge, U.K. In 1976 he joined the Faculty of Engineering at Tel Aviv University, Israel, where, from 1985 to 1989, he was the Chairman of the Department of Electrical Engineering-Systems. From 1993 to 1998 he was the Dean of the Faculty of Engineering at Tel Aviv University. He has been the incumbent of Celia and Marcos Maus Chair of Computer Systems Engineering at Tel Aviv University since 1989. From 1983 to 1984, 1989 to 1990, and 1998 to 1999, he spent his sabbatical year in the Electrical Engineering Departments at the University of California, Berkeley, Yale University, New haven, CT, and Imperial College, London, U.K., respectively.

In 1993 he was elected for the grade of fellow of the IEEE, and in 2000 he was nominated as a Nanyang Professor at Nanyang University Singapore.

Prof. Shaked has published more than 180 papers in international scientific journals, a monograph on systems with multiplicative noise, four text books, and more than 110 reviewed papers in international conferences.

His research interests include linear optimal control and filtering, robust control and estimation, H-optimal control, and control and estimation of systems with time-delay.