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Review article

Survey on time-delay approach to networked control

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ABSTRACT

This paper provides a survey on time-delay approach to networked control systems (NCSs). The survey begins from a brief summary on fundamental network-induced issues in NCSs and the main approaches to the modelling of NCSs. In particular, a comprehensive introduction to time-delay approach to sampled-data and networked control is provided. Then, recent results on time-delay approach to event-triggered control are recalled. The survey highlights time-delay approach developed to modelling, analysis and synthesis of NCSs, under communication constraints, with a particular focus on Round-Robin, Try-once-discard and stochastic protocols. The time-delay approach allows communication delays to be larger than the sampling intervals in the presence of scheduling protocols. Moreover, some results on networked control of distributed parameter systems are surveyed. Finally, conclusions and some future research directions are briefly addressed.

Contents

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1. Introduction to NCSs

The point-to-point architecture is the traditional communication architecture for control systems, that is, sensors and/or actuators are connected to controllers via wires. Due to the expansion of physical setups and functionality, a traditional point-to-point architecture is no longer able to meet new requirements, such as modularity, integrated diagnostics, quick and easy maintenance, and low cost. Such requirements are particularly demanding in the control of complex control systems Chan and Özgüner (1995); Etkin and Reid (1996) and remote control systems (Baruch & Cox, 1996; Kondraske et al., 1993; Lee & Schneeman, 1999; Ray, 1988).

To satisfy these new requirements, common-bus network architectures have been introduced. The common-bus network architectures can improve the efficiency, flexibility and reliability of integrated applications, and reduce installation, reconfiguration and maintenance time and costs (Otanez, Moyne, & Tilbury, 2002). It gives rise to the so-called networked control systems (NCSs) Antsaklis and Baillieul (2007); Hespanha, Naghshtabrizi, and Xu (2007); Hristu-Varsakelis and Levine (2005); Walsh, Ye, and Bushnell (2002); Zhang, Branicky, and Phillips (2001).

In general, NCSs are a type of distributed control systems where sensors, actuators, and controllers are interconnected through a communication network. Sensors measure states of the plant and transmit these states over the communication network to controllers. The controllers receive these states, and calculate appropriate control actions and send them to actuators over the communication network. Actuators receive control actions and control the plant appropriately. Due to its low cost, flexibility, and less wiring, NCSs are rapidly increasing in industrial applications, including telecommunications, remote process control, altitude control of airplanes and so on (Baruch & Cox, 1996; Etkin & Reid, 1996; Kondraske et al., 1993; Lee & Schneeman, 1999; Ray, 1988).

In NCSs, the closed-loops are closed via communication networks. The insertion of the communication network in the feedback control loop makes the analysis and design of systems more complex than the traditional point-to-point architecture. The network can introduce unreliable and time-dependent levels of service in terms of, for example, delays, jitter, or losses. In general, network-induced imperfections can jeopardize the stability, safety, and performance of the units in a physical environment (Huang & Nguang, 2009).

Notations: The symbols $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{Z}^+$ and $\mathbb{N}$ denote the set of real numbers, non-negative real numbers, non-negative integers and positive integers, respectively. $\mathbb{R}^n$ denotes the n dimensional Euclidean space with vector norm $|\cdot|$, $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices, and the notation $P>0(\geq 0)$, for $P\in\mathbb{R}^{n\times n}$ means that P is symmetric and positive definite (semi-positive-definite). The symmetric term in a symmetric matrix is denoted by $*$. The superscript 'T' stands for matrix transposition. For $x:\mathbb{R}\to\mathbb{R}^n$, we denote $x_t(\theta)\stackrel{\Delta}{=}x(t+\theta), \theta\in[-h,0]$. The symbol $L_p(a, b)$, $p\in\mathbb{N}$, denotes the space of functions $\phi:(a,b)\to\mathbb{R}^n$ with the norm $\|\phi\|_{L_p}=\left[\int_a^b|\phi(s)|^p ds\right]^{\frac{1}{p}}$. The symbol $L_p(a, b)$, $p\in\mathbb{N}$, denotes the space of functions $\phi:(a,b)\to\mathbb{R}^n$ with the norm $\|\phi\|_{L_p}=\left[\int_a^b|\phi(s)|^p ds\right]^{\frac{1}{p}}$. The symbol $L_\infty(a, b)$ denotes the space of essentially bounded functions $\phi:(a,b)\to\mathbb{R}^n$ with the norm $\|\phi\|_{L_\infty}=\text{ess}\sup_{\theta\in(a,b)}|\phi(\theta)|$. The space of functions $\phi:[a,b]\to\mathbb{R}^n$, which are absolutely continuous on $[a, b]$, and have square integrable first order derivatives is denoted by $W[a, b]$ with the norm $\|\phi\|_W=\max_{\theta\in[a,b]}|\phi(\theta)|+\left[\int_a^b|\dot{\phi}(s)|^2 ds\right]^{\frac{1}{2}}$.

2. Fundamental issues in NCSs

The main network-induced imperfections and constraints can be categorized in the following five types (Heemels, Teel, van de Wouw, & Nesic, 2010a; Zhang, Han, & Yu, 2016b):

(i) Variable sampling/transmission intervals

Conventional computer-controlled systems theories assume equal-distance sampling of the plant outputs, which means the samples are taken periodically at the time instants kh, $k\in\mathbb{Z}^+$, where $h>0$ is the constant sampling period. This assumption leads to linear time-invariant (LTI) sampled-data systems and greatly simplifies the stability and performance analysis (Zhang, 2001).

However, the assumption of equal-distance sampling should not be imposed on the NCSs analysis. To transmit a continuous-time signal over a network, the signal must be sampled, encoded in a digital format, transmitted over the network, and finally the data must be decoded at the receiver side. This process is significantly different from the usual periodic sampling in digital control. A significant number of results have attempted to characterize maximum allowable transmission interval (MATI) for which stability can be guaranteed (Heemels et al., 2010a; Hetel, Kruszewski, Perruquetti, & Richard, 2011; Nesic & Liberzon, 2009).

Moreover, in contrast to periodic sampling control, event-based control aims at minimizing the bandwidth utilization while

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still guaranteeing the desired level of control performance. There are two main triggering strategies, one is event-triggering strategy (see e.g., (Borgers & Heemels, 2014a; Lunze & Lehmann, 2010; Tabuada, 2007)) and the other is self-triggering strategy (see e.g., (Heemels, Johansson, & Tabuada, 2012; Mazo, Anta, & Tabuada, 2010; Wang & Lemmon, 2009)). The difference between event-triggering strategy and self-triggering strategy is that the former is reactive, while the latter is proactive. In event-triggering strategy, a triggering condition based on current measurements is monitored and when violated, an event is triggered. In self-triggering strategy, the next updating time is pre-computed at a control updating time based on predictions using previously received data and knowledge on the plant dynamics (Heemels et al., 2012). Thus, growing attention is paid to event-based control for NCSs, e.g., see Peng and Han (2013); Peng, Ma, and Xie (2017); Peng and Yang (2013); Peng, Zhang, and Han (2019); Selivanov and Fridman (2016b); Wang and Lemmon (2011); Yue, Tian, and Han (2013) for event-triggered control over networks, and see Almeida, Silvestre, and Pascoal (2017); Anta and Tabuada (2010); Nowzari and Cortés (2012); Souza, Deaecto, Geromel, and Daafouz (2014); Zhang, Zhao, and Zheng (2015) for self-triggered control over networks.

(ii) Communication delay/network-induced delay

The network-induced delay, including sensor-to-controller delay and controller-to-actuator delay, that happens when data exchange among devices connected by the communication network, which will deteriorate the system performance as well as stability. This delay, depending on the network characteristics such as network load, topologies, routing schemes, can be constant, time-varying, or even random (Cloosterman et al., 2010; Feng, Lam, & Yang, 2015; Feng & Zheng, 2015; Fridman, 2014a; Heemels et al., 2010a; Liu & Fridman, 2012b; 2014; Ma, Lewis, & Song, 2016; Richard, 2003; Yang et al., 2017). In the literature, two ways of modeling network-induced uncertainties can be distinguished.

- The first approach bounds the network-induced delay and considers maximum allowable delay (MAD). The NCSs are modeled as discrete-time (uncertain) systems (Cloosterman et al., 2010; Zhang et al., 2001), time-delay systems (Gao & Chen, 2008; Gao, Chen, & Lam, 2008; Liu & Fridman, 2012a; 2012b; Sun, Liu, Wang, & Rees, 2010; Yue, Han, & Lam, 2005), hybrid systems (Heemels, Nesic, Teel, & van de Wouw, 2009; Heemels et al., 2010a) or switched systems (Sun et al., 2010; Zhang & Yu, 2009).
- The second one is stochastic modeling approach. By assuming that network-induced delay has a known probability distribution function (Hu & Zhu, 2003; Nilsson, Bernhardsson, & Wittenmark, 1998), or incorporating the network-induced delays as Markov process (Huang & Nguang, 2008; Shi & Yu, 2009; 2011; Zhang, Shi, Chen, & Huang, 2005), the resulting closed-loop is modeled as a stochastic system.

For some systems, the presence of communication delay may have the positive effect on system performance (Fridman & Shaikhet, 2016; 2017), e.g., see Liu and Fridman (2012b); Selivanov and Fridman (2018c) for sampled-data stabilization, and see Yu, Chen, Cao, and Ren (2013) for consensus of multi-agent systems.

(iii) Packet dropouts

Another significant difference between NCSs and standard digital control is the possibility that some packets not only suffer transmission delays but, even worse, may be lost while in transit through the network. Typically, packet dropouts result from transmission errors in physical network links (which is far more common in wireless than in wired networks) or from buffer overflows due to congestion. Thus, how much packet dropouts affect stability and performance of NCSs is an issue that must be considered (Hu & Yan, 2007; Xiong & Lam, 2007; 2009).

In general, in most of the literature two different strategies are considered for dealing with packet dropouts. The first one is zero-input, i.e., the actuator input to the plant is set to zero when the control packet from the controller to the actuator is lost (Imer, Yüksel, & Başar, 2006). The second one is hold-input, i.e., the latest control input stored in the actuator buffer is used when a packet is lost (Zhang et al., 2001). By studying linear quadratic performance of NCSs where control packets are subject to loss, in Schenato (2009) it was shown that none of these two control schemes can be claimed to be superior to the other.

Different control techniques have been developed for the modeling of NCSs with data packet dropouts. They can be roughly categorized into the following types based on the resulting closed-loop systems:

- Switching systems (Ishii, 2008; Zhang & Yu, 2008), asynchronous dynamical systems (Zhang et al., 2001), and jump linear systems with Markov chains (Schenato, 2009; Seiler & Sengupta, 2005). Note that packet dropouts defined in the aforementioned references have two cases, dropped or sent successfully, which are modeled as a Bernoulli or a two-state Markov chain process based on zero-input or hold-input.
- Another type of the resulting closed-loop system is in terms of time-delay systems (Gao et al., 2008; Yu, Wang, Chu, & Hao, 2005). By modeling dropouts as prolongations of the variable sampling intervals or communication delays, the NCSs with data packet dropouts are modeled as linear systems with time-varying input delays based on hold-input strategy. Then the delay-dependent approach can be applied to the resulting time-delay systems and the maximum allowable value of the successive packet dropouts can be determined by solving a set of linear matrix inequalities (LMIs).

(iv) Quantization

Due to the limited transmission capacity of the network, data transmitted in practical NCSs should be quantized before they are sent to the next network node (Fridman & Dambrine, 2009; Nesic & Liberzon, 2009; Xia, Fu, & Liu, 2011). A quantizer is a function that maps a real-valued function into a piecewise constant function taking on a finite set of values. At present, there exist two kinds of quantizers, which are uniform quantizers (Brockett & Liberzon, 2000) and logarithmic ones (Elia & Mitter, 2001).

- The uniform quantizer maps real-valued function to a finite number of quantization regions with rectilinear shape (Brockett & Liberzon, 2000) or arbitrary shape (Liberzon, 2003; Liberzon & Nesic, 2007; Liu, Fridman, & Johansson, 2015b; Nesic & Liberzon, 2009). The study of system affected by uniform quantizer is usually based on "zoom" strategy, which is composed of two stages, i.e., "zooming-out" and "zooming-in". In the first stage, the range of quantizer is increased to guarantee the states of system can be adequately measured. In the second stage, the quantization error is decreased to drive the states to the origin.
- When system is affected by logarithmic quantizer, in which the quantization levels are linear in logarithmic scale, the simple classical approach to analysis and mitigation of quantization effects is to treat the quantization error as uncertainty or nonlinearity and bound it using a sector bound (Fu & Xie, 2005; Gao et al., 2008; Yue, Peng, & Tang, 2006).

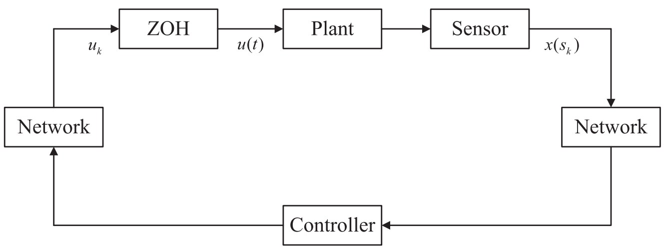

Fig. 1. Schematic diagram of a NCS.

(v) Communication scheduling

In NCSs, the performance of control loops not only depend on the design of the control algorithms, but also rely on the scheduling of the shared network resource. The communication constraints impose that, per transmission, only one node can access the network and send its information. Hence, many existing works are focused on that how often a plant should schedule to transmit the data and with what priority the packet should be sent out. There are three main classes of network protocols, namely:

- The class of static protocols, of which Round-Robin protocol is a special case (Donkers, Heemels, van de Wouw, & Hetel, 2011; Heemels et al., 2010a; Liu, Fridman, & Hetel, 2012; Nesic & Liberzon, 2009; Nesic & Teel, 2004). In the Round-Robin protocol, the node j is transmitted periodically with period l, where l is the total number of nodes. The transmission order is decided in advance and this order is repeated indefinitely.
- The class of dynamic protocols, which includes the well-known try-once-discard (TOD) protocol (Donkers et al., 2011; Heemels et al., 2010a; Liu, Fridman, & Hetel, 2015a; Nesic & Liberzon, 2009; Nesic & Teel, 2004). In the TOD protocol, the node that has the largest network-induced error, i.e., the largest difference between the latest transmitted values and the current values of the signals corresponding to the node, is granted access to the network.
  It was observed in some examples that the TOD protocol stabilized the system for larger *MATI* than the Round-Robin protocol whenever $l>1$ (Nesic & Teel, 2004; Walsh, Beldiman, & Bushnell, 2001), in some examples opposite conclusions were made Freirich and Fridman (2018); Liu et al. (2015a).
- The stochastic protocol, which was introduced in Donkers, Heemels, Bernardini, Bemporad, and Shneer (2012); Tabbara and Nesic (2008). The stochastic protocol determines the transmitted node through a Bernoulli or a two-state Markov chain process (Liu, Fridman, & Johansson, 2015c; Zou, Wang, & Gao, 2016a). The quadratic and stochastic protocols belong to dynamic protocols.

3. Three main approaches to NCSs

Consider a generic schematic diagram of NCSs as shown in Fig. 1. The LTI continuous-time plant is given by

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^{n_u}$ is the control input, $A\in\mathbb{R}^{n\times n}$ and $B\in\mathbb{R}^{n\times n_u}$ are system matrices with appropriate dimensions. Consider, for simplicity, the state-feedback case.

Let s_k denote the unbounded monotonously increasing sequence of sampling instants, i.e.,

$$0 = s_0 < s_1 < \cdots < s_k < \cdots, \quad k\in\mathbb{Z}^+, \ \lim_{k\to\infty} s_k = \infty$$

with the time-varying sampling intervals $h_k = s_{k+1} - s_k > 0$. There are two sources of delays from the network: sensor-to controller η_k^{sc} and controller-to-actuator η_k^{ca}. It is assumed that the sensor acts in a time-driven fashion (i.e., sampling occurs at the times s_k, $k\in\mathbb{Z}^+$) and that both the controller and the actuator act in an event-driven fashion (i.e., they respond instantaneously to newly arrived data). Under these assumptions, the two delays can be captured by a singe delay $\eta_k = \eta_k^{sc} + \eta_k^{ca}$. Assume that $\eta_k\in[\eta_m, MAD]$, where η_m and *MAD* denote the lower and upper delay bounds on the network-induced delays η_k, respectively. Denote by $t_k = s_k + \eta_k$ the updating instant time of the zero-order-hold (ZOH), finally the ZOH function transforms the discrete-time control input u_k to a continuous-time control input

$$u(t) = u_k = Kx(s_k), \ t\in[t_k, t_{k+1}), \ k\in\mathbb{Z}^+. \tag{2}$$

The resulting closed-loop system is (1), (2):

$$\dot{x}(t) = Ax(t) + BKx(s_k), \ t\in[t_k, t_{k+1}), \ k\in\mathbb{Z}^+. \tag{3}$$

In the literature, three main approaches have been used to the sampled-data control (see e.g., (Bamieh, Pearson, Francis, & Tannenbaum, 1991; Başar & Bernhard, 1995; Chen & Francis, 1995; Dullerud & Glover, 1993; Fridman, 2014a; Hetel et al., 2017; Mikheev, Sobolev, & Fridman, 1988; Sivashankar & Khargonekar, 1994)) and later to the NCSs.

3.1. Discrete-time modeling approach

For the sake of clarity, we focus here on the case of small network-induced delays, where these delays are smaller than the sampling intervals, i.e., $\eta_k = t_k - s_k < h_k$, $k\in\mathbb{Z}^+$. The most common discrete-time NCS model is explained in e.g., (Nilsson, 1998; Zhang et al., 2001). The discrete-time uncertain system can be obtained by describing the evolution of the states between s_k and $s_{k+1} = s_k + h_k$. Discretizing the linear plant (1) at the sampling time s_k, $k\in\mathbb{Z}^+$, we obtain (see e.g., (Cloosterman et al., 2010; Donkers et al., 2011))

$$\begin{aligned} x(s_{k+1}) \quad = \quad & e^{Ah_k}x(s_k) + \int_0^{h_k-\eta_k} e^{As}dsBu_k \\ & + \int_{h_k-\eta_k}^{h_k} e^{As}dsBu_{k-1}. \end{aligned}$$

Using now the state vector

$$\xi(s_k) = [x^T(s_k) \quad u_{k-1}^T]^T$$

that includes the current system state and the past system input, we obtain the following discrete model

$$\begin{aligned} \xi(s_{k+1}) \quad &= \quad \begin{bmatrix} e^{Ah_k} & \int_{h_k-\eta_k}^{h_k} e^{As}dsB \\ 0 & 0 \end{bmatrix}\xi(s_k) \\ & \quad + \begin{bmatrix} \int_0^{h_k-\eta_k} e^{As}dsB \\ I \end{bmatrix}u_k \\ &= \quad \tilde{A}_{h_k,\eta_k}\xi(s_k), \end{aligned} \tag{4}$$

where

$$\tilde{A}_{h_k,\eta_k} = \begin{bmatrix} e^{Ah_k} + \int_0^{h_k-\eta_k} e^{As}dsBK & \int_{h_k-\eta_k}^{h_k} e^{As}dsB \\ K & 0 \end{bmatrix}.$$

Hence, the stability analysis for the uncertain system (4) with the uncertainty $\eta_k\in[\eta_m, MAD]$, $h_k\in(0, MATI]$ is essentially a robust stability analysis problem. Then, the obstruction to apply existing robust stability analysis techniques directly is that the uncertainty appears in an exponential fashion in $\tilde{A}_{h_k,\eta_k}$ of (4). To make the formulation (4) suitable for robust stability analysis, over-approximation techniques are employed in the literature to embed the original model (as tight as possible) in a larger model that has nice structural properties suitable for the application of robust stability methods. Adopted over-approximation techniques are based on the real Jordan form (Cloosterman et al., 2010; Cloosterman, van de Wouw, Heemels, & Nijmeijer, 2007; 2009; van de

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Wouw, Naghshtabrizi, Cloosterman, & Hespanha, 2009), the Taylor series Hetel, Daafouz, and Iung (2006), gridding and norm-bounding Fujioka (2008); Skaf and Boyd (2009); Suh (2008) and the Cayley-Hamilton theorem Gielen et al. (2010). The over-approximation techniques typically result in discrete-time polytopic models Blanchini and Miani (2003); Heemels et al. (2010b); Lee (2006) with (or without) additive norm-bounded uncertainties. These models are amendable for robust stability assessment using LMIs. In Heemels et al. (2010b), a comparison was presented between the different over-approximation methods and the subsequent LMI-based stability analysis.

The most general and complete modeling based on discrete-time approach was provided in Cloosterman et al. (2010) (see also (Cloosterman et al., 2007; 2009)) that includes imperfection types (i), (ii), (iii) of network-induced uncertainties, i.e., time-varying sampling intervals, time-varying communication delays (both smaller and larger than the sampling interval) and explicit modeling of the dropouts.

In Donkers, Hetel, Heemels, van de Wouw, and Steinbuch (2009), the discrete-time modeling approach was further applied to the network-based stabilization including imperfection types (i), (iii), (v). In Donkers et al. (2011), small delays are taken into account. The work of Donkers et al. (2011) was extended in van Loon, Donkers, van de Wouw, and Heemels (2012) by including quantization, where quantization-induced disturbances were incorporated as one of the stability and performance limiting factors. A state-dependent sampling approach was introduced in Fiter, Hetel, Perruquetti, and Richard (2012) for the stabilization of sampled-data systems to reduce the number of computations.

For LTI systems, as was shown in e.g., Donkers et al. (2011), the discrete-time approach can lead to less conservative results in terms of the so-called *MATI* and the *MAD*. However, discrete-time methods become complicated for systems with uncertain coefficients. Moreover, it is tedious to include large delays (that are larger than the sampling intervals) in such models and the stability analysis methods may fail when the interval between two transmissions takes small values. In this case, one can resort to the delta operator approach proposed in Goodwin, Leal, Mayne, and Middleton (1986) and Middleton and Goodwin (1986).

3.2. Impulsive system approach

The second approach is based on the representation of the system in the form of hybrid/impulsive system Sivashankar and Khargonekar (1994). Impulsive dynamical systems exhibit continuous evolutions described by ordinary differential equations and instantaneous state jumps or impulses (see e.g., Goebel, Sanfelice, & Teel (2009)).

The idea of the impulsive system approach is to rewrite its closed-loop system (3) as the following delay impulsive system

$$\begin{aligned}\dot{\xi}(t) &= \begin{bmatrix} A & BK \\ 0 & 0 \end{bmatrix}\xi(t), \quad t \in [t_k, t_{k+1}),\\ \xi(t_k) &= \begin{bmatrix} x(t_k^-) \\ x(s_k) \end{bmatrix}, \quad k \in \mathbb{Z}^+,\end{aligned} \tag{5}$$

where $\xi(t) = [x^T(t) \;\; z^T(t)]^T$, $z(t) = x(s_k)$, $t \in [t_k, t_{k+1})$.

In Naghshtabrizi, Hespanha, and Teel (2008), the impulsive system approach was proposed to the analysis and control of sampled-data systems ($\eta_k \equiv 0$, i.e., $t_k = s_k$, $k \in \mathbb{Z}^+$) with variable sampling. A discontinuous Lyapunov functional, which is discontinuous at input update instants and is decreasing between discontinuities, is introduced. Later on in Naghshtabrizi, Hespanha, and Teel (2010) the impulsive system approach was further extended to the modeling and analysis of NCSs with variable sampling and communication delays. This discontinuous Lyapunov function method improved the existing Lyapunov-based results in e.g., Fridman (2005); Gao and Chen (2008); Gao et al. (2008); Yue et al. (2005). The main advantage of this modeling approach is the possibility to incorporate time-delays larger than the sampling interval without increasing model complexity, as is the case in the discrete-time modeling approach (Chen & Zheng, 2011; Cloosterman, van de Wouw, Heemels, & Nijmeijer, 2009).

Based on the impulsive system approach, the input-output stability properties of nonlinear NCSs have been studied in Nesic and Teel (2004) for NCSs with imperfection types (i), (iii), (v). In Nesic and Liberzon (2009), stabilization of nonlinear NCSs with dynamic quantization was studied. However, delays are not included in the analysis. In Heemels et al. (2010a), the imperfection types (i), (ii), (iii), (v) were considered and the methods for computing the *MATI* and *MAD* were provided, for which the stability of a nonlinear system is ensured. A unifying modeling framework was provided in Heemels et al. (2009) to incorporate all the five types of networked-induced effects. Note that some of the mentioned results that study varying transmission intervals and/or varying communication delays can be extended to include imperfection type (iii), i.e., data packet dropouts phenomena as well by modeling dropouts as prolongations of the *MATI* Heemels et al. (2009). In the above works, only small communication delays were considered.

3.3. Time-delay approach

Modelling of continuous-time systems with digital control in the form of continuous-time systems with delayed control input was introduced by Mikheev et al. (1988). The digital control law for sampled-data systems ($\eta_k \equiv 0$, i.e., $t_k = s_k$, $k \in \mathbb{Z}^+$) may be represented as delayed control as follows:

$$\begin{aligned}&u(t) = Kx(t_k) = Kx(t - (t - t_k)) = Kx(t - \tau(t)),\\ &t_k \le t < t_{k+1}, \quad \tau(t) = t - t_k,\end{aligned}$$

In this case, the closed-loop system becomes an infinite-dimensional delay differential equation

$$\dot{x}(t) = Ax(t) + A_1 x(t - \tau(t)), \;\; t_k \le t < t_{k+1}, \;\; k \in \mathbb{Z}^+, \tag{6}$$

where $A_1 = BK$, the time-varying delay $\tau(t) = t - t_k$ is piecewise linear with derivative $\dot{\tau}(t) = 1$ for $t \neq t_k$ (see Fig. 2). Moreover, $\tau(t) \le t_{k+1} - t_k = s_{k+1} - s_k = MATI$ for $t_k \le t < t_{k+1}$. The stability of (6) can be established using Lyapunov-Razumikin or Lyapunov-Krasovskii Theorems. The time-delay approach was applied to robust sampled-data stabilization via Lyapunov-Krasovskii technique Fridman, Seuret, and Richard (2004).

3.3.1. Delay-dependent analysis

The choice of Lyapunov-Krasovskii functionals (LKFs) (that we will call also Lyapunov functional) is crucial for deriving stability criteria Fridman (2014b). The first delay-dependent (both, Krasovskii and Razumikhin-based) conditions were derived by using the relation

$$x(t - \tau(t)) = x(t) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds \tag{7}$$

via different model transformations and by bounding the cross terms Kolmanovskii and Richard (1999); Li and de Souza (1997); Park (1999). The widely used first model transformation, where (7) is substituted into (6) with $\dot{x}(s)$ substituted by the right-hand side of (6), has the form

$$\dot{x}(t) = (A + A_1)x(t) - A_1 \int_{t-\tau(t)}^{t} [Ax(s) + A_1 x(s - \tau(s))]ds. \tag{8}$$

Note that this transformation is valid for $t - \tau(t) \ge t_0$. The latter system is not equivalent to the original one possessing some additional dynamics (Gu & Niculescu, 2001; Kharitonov & Melchor-Aguilar, 2000). The stability of the transformed system (8) guarantees the stability of the original one, but not vice versa.

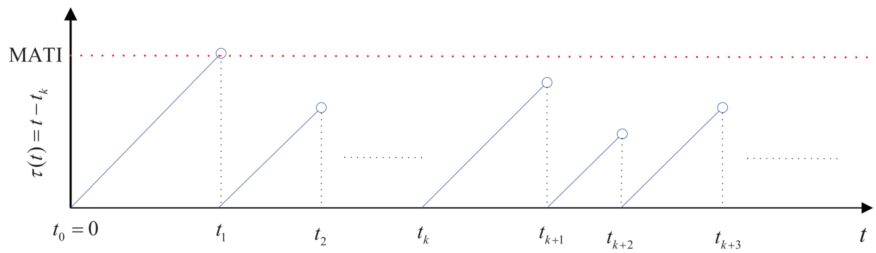

Fig. 2. Sampled-data systems: piecewise-continuous time-delay.

The first delay-dependent conditions treated only the slowly-varying delays with $\dot{\tau} \le d < 1$, whereas the fast-varying delay (without any constraints on the delay derivative) was analyzed via Lyapunov-Razumikhin functions.

For the first time, systems with fast-varying delays were analyzed by using Krasovskii method in Fridman and Shaked (2003), via the descriptor model transformation introduced in Fridman (2001):

$$\dot{x}(t) = y(t),$$
$$0 = -y(t) + (A + A_1)x(t) - A_1 \int_{t-\tau(t)}^{t} y(s)ds. \quad (9)$$

The descriptor system (9) is equivalent to (6) in the sense of stability. In the descriptor approach, $\dot{x}(t)$ is not substituted by the right-hand side of the differential equation. Instead, it is considered as an additional state variable of the resulting descriptor system (9). Therefore, the novelty of the descriptor approach is not in $V = x^T(t)Px(t) + \ldots$ $(P > 0)$, but in $\dot{V}$, where $\frac{d}{dt}\left[x^T(t)Px(t)\right]$ is found as

$$\frac{d}{dt}[x^T(t)Px(t)]$$
$$= 2x^T(t)P\dot{x}(t) + 2[x^T(t)P_2^T + \dot{x}^T(t)P_3^T]$$
$$\times[-\dot{x}(t) + (A + A_1)x(t) - A_1 \int_{t-\tau(t)}^{t} \dot{x}(s)ds], \quad (10)$$

and where $P_2 \in \mathbb{R}^{n\times n}$ and $P_3 \in \mathbb{R}^{n\times n}$ are "slack variables". This leads to $\dot{V} \le -\gamma(|x(t)|^2 + |\dot{x}(t)|^2)$, $\gamma > 0$.

The descriptor method brought free-weighting matrices P_2 and P_3 into the Lyapunov-based analysis. Later it was shown that Finsler's lemma leads to the same slack matrices (see Gouaisbaut and Peaucelle (2006) and Suplin, Fridman, and Shaked (2004)). The advantages of the descriptor method are:

- less conservative conditions (even without delay) for uncertain systems,
- "unifying" LMIs for the discrete-time and for the continuous-time systems, having almost the same form and the same advantages (Fridman & Shaked, 2006),
- simple conditions for neutral type systems can be derived (where the stability of the difference operator follows from LMIs) (Fridman, 2002),
- design is obtained for systems with state, input and output delays by choosing $P_3 = \varepsilon P_2$ with a tuning scalar parameter ε (Suplin, Fridman, & Shaked, 2007),
- simple delay-dependent conditions can be derived for diffusion partial differential equations (Fridman & Orlov, 2009).

Most of the recent Krasovskii-based results do not use model transformations and cross terms bounding. They are based on the application of Jensen's inequality (see e.g., Gu, Kharitonov, & Chen (2003)).

3.3.2. Simple delay-dependent conditions

The first Krasovskii-based LMI conditions for systems with fast-varying delays were derived in Fridman and Shaked (2003) via the descriptor method. We differentiate $x^T(t)Px(t)$ as in (10) along system (6) with $MATI \triangleq h$. To "compensate" $\int_{t-\tau(t)}^{t} \dot{x}(s)ds$, consider the double integral term (Fridman & Shaked, 2003):

$$V_R(\dot{x}_t) = \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta, \ R > 0.$$

The term V_R can be rewritten equivalently as

$$V_R(\dot{x}_t) = \int_{t-h}^{t} (h + s - t)\dot{x}^T(s)R\dot{x}(s)ds.$$

Differentiating $V_R(\dot{x}_t)$, we obtain

$$\begin{aligned}\frac{d}{dt}V_R(\dot{x}_t) &= -\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds + h\dot{x}^T(t)R\dot{x}(t) \\ &= -\int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds + h\dot{x}^T(t)R\dot{x}(t) \\ &\quad \underbrace{-\int_{t-h}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)ds}_{\text{will be ignored}}.\end{aligned}$$

We apply further Jensen's inequality

$$-\int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le -\frac{1}{h}\int_{t-\tau(t)}^{t} \dot{x}^T(s)dsR\int_{t-\tau(t)}^{t} \dot{x}(s)ds.$$

Then, for the Lyapunov functional

$$V(x(t), \dot{x}_t) = x^T(t)Px(t) + V_R(\dot{x}_t),$$

we find

$$\begin{aligned}\frac{d}{dt}V(x(t), \dot{x}_t) \le\ & 2x^T(t)P\dot{x}(t) + h\dot{x}^T(t)R\dot{x}(t) \\ & -\frac{1}{h}\int_{t-\tau(t)}^{t} \dot{x}^T(s)dsR\int_{t-\tau(t)}^{t} \dot{x}(s)ds \\ & +2[x^T(t)P_2^T + \dot{x}^T(t)P_3^T] \\ & \times[(A+A_1)x(t) - A_1\int_{t-\tau}^{t} \dot{x}(s)ds - \dot{x}(t)] \\ \le\ & \eta^T(t)\Psi\eta(t) \\ \le\ & -\varepsilon(|x(t)|^2 + |\dot{x}(t)|^2), \ \varepsilon > 0,\end{aligned}$$

where $\eta(t) = col\{x(t), \dot{x}(t), \frac{1}{h}\int_{t-\tau}^{t} \dot{x}(s)ds\}$, if

$$\Psi = \begin{bmatrix} \Phi & P - P_2^T + (A + A_1)^T P_3 & -hP_2^T A_1 \\ * & -P_3 - P_3^T + hR & -hP_3^T A_1 \\ * & * & -hR \end{bmatrix} < 0,$$

$$\Phi = P_2^T(A + A_1) + (A + A_1)^T P_2.$$

As it was understood later (Fridman & Orlov, 2009; Suplin, Fridman, & Shaked, 2006), the equivalent delay-dependent conditions can be derived without the descriptor method, where $\dot{x}$ is substituted by the right-hand side of (6) and the Schur complements is applied further.

Note that $\Psi < 0$ yields that the eigenvalues of hA_1 are inside of the unit circle. In the example $\dot{x}(t) = -x(t - \tau(t))$ with $A_1 = -1$, the simple delay-dependent conditions cannot guarantee the stability for $h \ge 1$, which is far from the analytical bound 1.5. This illustrates the conservatism of the simple conditions.

3.3.3. Improved delay-dependent conditions

The relation between $x(t-\tau(t))$ and $x(t-h)$ (and not only between $x(t-\tau(t))$ and $x(t)$) has been taken into account in He, Wang, Lin, and Wu (2007). The widely used by now LKF for delay-dependent stability is state-derivative dependent one of the form

$$\begin{aligned} V(t,x_t,\dot{x}_t) &= x^T(t)Px(t) + \int_{t-h}^{t} x^T(s)Sx(s)ds \\ &\quad + h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta \\ &\quad + \int_{t-\tau(t)}^{t} x^T(s)Qx(s)ds, \end{aligned} \tag{11}$$

where $P>0$, $R\geq 0$, $S\geq 0$, $Q\geq 0$. This functional with $Q=0$ leads to delay-dependent conditions for systems with fast-varying delays, whereas for $R=S=0$ it leads to delay-independent conditions (for systems with slowly-varying delays). The above V with $S=0$ was introduced in Fridman and Shaked (2003), whereas the S-dependent term was added in He et al. (2007).

Differentiating V given by (11), we find

$$\begin{aligned} \frac{d}{dt}V &\leq 2x^T(t)P\dot{x}(t) + h^2\dot{x}^T(t)R\dot{x}(t) \\ &\quad -h\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)ds + x^T(t)(S+Q)x(t) \\ &\quad -x^T(t-h)Sx(t-h) \\ &\quad -(1-d)x^T(t-\tau(t))Qx(t-\tau(t)) \end{aligned}$$

and employ the representation

$$\begin{aligned} -h\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)ds = -h\int_{t-h}^{t-\tau(t)}\dot{x}^T(s)R\dot{x}(s)ds \\ -h\int_{t-\tau(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds. \end{aligned} \tag{12}$$

Applying Jensen's inequality to both terms on the right-hand-side of (12), we arrive at

$$-h\int_{t-h}^{t}\dot{x}^T(s)R\dot{x}(s)ds \leq -\tfrac{h}{\tau(t)}e_1^TRe_1 - \tfrac{h}{h-\tau(t)}e_2^TRe_2, \tag{13}$$

where

$$e_1 = x(t)-x(t-\tau(t)),\ \ e_2 = x(t-\tau(t))-x(t-h).$$

Here, for $\tau=0$ and $\tau=h$, we mean the following limits:

$$\lim_{\tau(t)\to 0}\tfrac{h}{\tau(t)}e_1^TRe_1 = h\lim_{\tau(t)\to 0}\tau(t)\dot{x}^T(t)R\dot{x}(t) = 0$$

and

$$\lim_{\tau(t)\to h}\frac{h}{h-\tau(t)}e_2^TRe_2 = 0.$$

In He et al. (2007), the right-hand side of (13) was upper-bounded by $-e_1^TRe_1 - e_2^TRe_2$ that was conservative. The convex analysis of Park, Ko, and Jeong (2011) allowed to avoid the latter restrictive bounding. The novelty of this method consists in merging the non-convex terms into a single expression to derive an accurate convex inequality. It was notably shown in Liu and Seuret (2017) that the reciprocally convex combination lemma Park et al. (2011) leads to the same conservatism as the Moon et al.'s inequality Moon, Park, Kwon, and Lee (2001) when considering Jensen-based stability criteria, but with a lower computational burden. We reformulate the reciprocally convex combination lemma and the Moon et al.'s inequality in a more convenient form for the Lyapunov-based analysis:

Lemma 1. *(Reciprocally convex combination lemma) Let $R_1\in\mathbb{R}^{n_1\times n_1},\ldots,R_N\in\mathbb{R}^{n_N\times n_N}$ be positive matrices. For $e_1\in\mathbb{R}^{n_1},\ldots,e_N\in\mathbb{R}^{n_N}$, and for all $\alpha_i>0$ with $\sum_{i=1}^N\alpha_i=1$, define a reciprocally convex combination as a function of the form $\sum_{i=1}^N\frac{1}{\alpha_i}e_i^TR_ie_i$. Then, for all $S_{ij}\in\mathbb{R}^{n_i\times n_j}, j=2,\ldots,N,\ i=1,\ldots,j-1$, such that*

$$\begin{bmatrix} R_i & S_{ij} \\ * & R_j \end{bmatrix} \geq 0,$$

the following inequality holds:

$$\sum_{i=1}^{N}\tfrac{1}{\alpha_i}e_i^TR_ie_i \geq \zeta_N^T\Phi_N\zeta_N,$$

where

$$\zeta_N = \begin{bmatrix} e_1^T & e_2^T & \cdots & e_N^T \end{bmatrix}^T,$$

$$\Phi_N = \begin{bmatrix} R_1 & S_{12} & \cdots & S_{1N} \\ * & R_2 & \cdots & S_{2N} \\ & & \ddots & \vdots \\ * & * & & \\ * & * & \cdots & R_N \end{bmatrix}. \tag{14}$$

Lemma 2. *(Moon et al.'s inequality) Let $R_1\in\mathbb{R}^{n_1\times n_1},\ldots,\ R_N\in\mathbb{R}^{n_N\times n_N}$ be positive matrices. Then for all $\alpha_i>0$ with $\sum_{i=1}^N\alpha_i=1$ and for any matrices M_i in $\mathbb{R}^{Nn\times n}$, $i=1,\ldots,N$, the following inequality holds:*

$$\sum_{i=1}^{N}\tfrac{1}{\alpha_i}e_i^TR_ie_i \geq \zeta_N^T\Psi_N\zeta_N,$$

where ζ_N is given in (14) and

$$\begin{aligned} \Psi_N &= \Upsilon_N + \Upsilon_N^T - \sum_{i=1}^{N}\alpha_iM_iR_i^{-1}M_i^T, \\ \Upsilon_N &= M_1[I\ \ 0_{n\times(N-1)n}] + M_2[0\ \ I\ \ 0_{n\times(N-2)n}] + \cdots \\ &\quad + M_N[0_{n\times(N-1)n}\ \ I]. \end{aligned}$$

Furthermore, a relaxed reciprocally convex combination lemma was developed in Zhang, He, Jiang, Wu, and Zeng (2016a) without requiring any extra decision variable. This inequality was extended by the same authors in Zhang, He, Jiang, Wu, and Wang (2017). More insights on the relationship between some existing matrix inequalities was provided in a recent paper Seuret, Liu, and Gouaisbaut (2018) that revealed strong links between the existing inequalities of Moon et al. (2001); Park et al. (2011); Shao (2009); Zeng, He, Wu, and She (2015a); Zhang et al. (2016a).

3.3.4. Stability analysis of systems with interval or non-small delay

The time-delay approach became popular in NCSs, being applied to uncertain systems under uncertain sampling with the known upper bound on the sampling intervals (Gao & Chen, 2008; Gao et al., 2008; Yue et al., 2005). The time-delay approach was further extended to event-triggered networked control (Selivanov & Fridman, 2016b; Yue et al., 2013), distributed networked control of partial differential equations (Bar Am & Fridman, 2014; Fridman & Blighovsky, 2012; Kang & Fridman, 2018; Selivanov & Fridman, 2016a), etc.

By defining

$$\tau(t) = t - t_k + \eta_k,\ \ t_k\leq t<t_{k+1},\ \ k\in\mathbb{Z}^+,$$

the digital control law has the following form:

$$u(t) = Kx(t-\tau(t)),\ \ t_k\leq t<t_{k+1}. \tag{15}$$

Hence, the LTI system (1) under (15) is then modeled as a time-delay system with time-varying delay

$$\dot{x}(t) = Ax(t) + BKx(t-\tau(t)),\ \ t_k\leq t<t_{k+1},\ \ k\in\mathbb{Z}^+, \tag{16}$$

where the time-varying delay $\tau(t)$ is piecewise linear with derivative $\dot{\tau}(t)=1$ for $t\neq t_k$. Moreover, we have

$$\eta_m\leq\eta_k\leq\tau(t)<t_{k+1}-t_k+\eta_k\leq\tau_M,$$

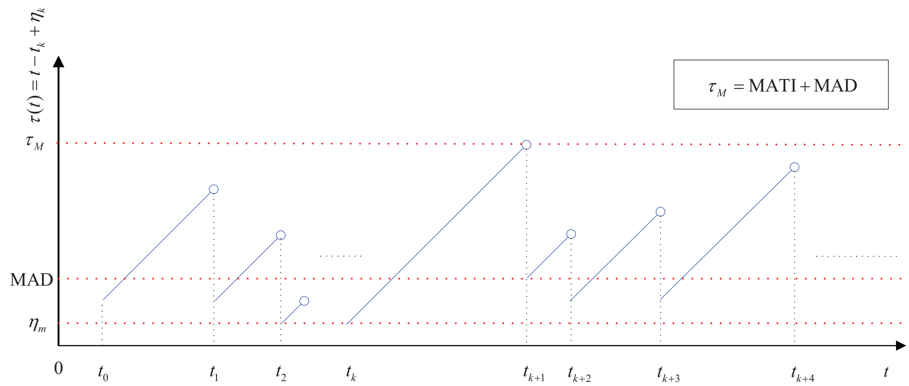

Fig. 3. NCSs: piecewise-continuous time-delay.

where η_m is a lower bound on the network-induced delay, τ_M denotes the maximum time span between the time $s_k = t_k - \eta_k$ at which the state is sampled and the time t_{k+1} at which next update arrives at the ZOH and $\tau_M = MATI + MAD$. See Fig. 3 for an example of $\tau(t)$.

This is one of applications that motivate the stability analysis of systems with interval (or non-small) delay $\tau(t) \in [h_0, h_1]$ with $h_0 > 0$ (see e.g., (Fridman, 2006b; He et al., 2007; Kharitonov & Niculescu, 2003)). Keeping in mind that (16) can be represented as

$$\dot{x}(t) = Ax(t) + A_1 x(t - h_0) - A_1 \int_{t-\tau(t)}^{t-h_0} \dot{x}(s)ds, \ \ h_0 \triangleq \eta_m,$$

the stability of (16) can be analyzed via Lyapunov functionals of the form Fridman (2006b):

$$V(t, x_t, \dot{x}_t) = V_n(x_t, \dot{x}_t) + V_1(t, x_t, \dot{x}_t),$$

where V_n is a "nominal" functional for the "nominal" system with constant delay

$$\dot{x}(t) = Ax(t) + A_1 x(t - h_0)$$

and where

$$V_1 = \int_{t-h_1}^{t-h_0} x^T(s) S_1 x(s) ds + \int_{t-\tau(t)}^{t-h_0} x^T(s) Q_1 x(s) ds$$
$$+ (h_1 - h_0) \int_{-h_1}^{-h_0} \int_{t+\theta}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta, \ \ h_1 \triangleq \tau_M$$

with $S_1 > 0$, $Q_1 > 0$, $R_1 > 0$.

In the case where the nominal system is stable for all constant delays from $[0, h_0]$, V_n can be chosen in the form of (11), where $h = h_0$ and $Q = 0$. Then the stability conditions in terms of LMIs can be derived by using standard arguments for the delay-dependent analysis, e.g., in Fridman (2014a); Park et al. (2011).

3.3.5. Time-dependent Lyapunov functionals for sampled-data systems

Note that the existing methods in the framework of time-delay approach are based on some Lyapunov-based analysis of systems with uncertain and bounded fast-varying delays. Therefore, these methods cannot guarantee the stability if the delay is not smaller than the analytical upper bound on the constant delay that preserves the stability. However, it is well-known that in many systems the upper bound on the sampling that preserves the stability may be higher than the one for the constant delay, see examples in Louisell (1999), as well as the following Example.

Example 1. Consider the following simple and much-studied problem (see e.g., (Papachristodoulou, Peet, & Niculescu, 2007) and the references therein):

$$\dot{x}(t) = -x(s_k), \ \ t_k \le t < t_{k+1}, \ \ k \in \mathbb{Z}^+.$$

It is well-known that the equation

$$\dot{x}(t) = -x(t - \tau) \tag{17}$$

with constant delay τ is asymptotically stable for $\tau \le \pi/2$ and unstable for $\tau > \pi/2$, whereas for the fast-varying delay it is stable for $\tau < 1.5$ and there exists a destabilizing delay with an upper bound greater than 1.5. This means that all the existing methods via time-independent Lyapunov functionals cannot guarantee the stability of system (17) for the sampling intervals that may be greater than $\pi/2$.

It is easy to check, that in the case of pure (uniform) sampling, the system remains stable for all constant samplings less than 2 and becomes unstable for samplings greater than 2.

Therefore, it is necessary to develop new Lyapunov functional-based techniques for sampled-data control to improve the results. Inspired by the construction of discontinuous Lyapunov functions in Naghshtabrizi et al. (2008) for the impulsive systems, time-dependent Lyapunov functionals were introduced in Fridman (2010) for the analysis of sampled-data systems in the framework of time-delay approach. The main idea is that for (6) the standard time-independent term

$$\int_{-h}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R \dot{x}(s) ds d\theta, \ \ R > 0$$

with $h \triangleq MATI$, can be advantageously replaced by the term

$$V_R^s = (h - t + t_k) \int_{t_k}^{t} \dot{x}^T(s) R \dot{x}(s) ds, \ \ t_k \le t < t_{k+1}, \tag{18}$$

which provides time-dependent LKFs $\bar{V}(t) = V(t, x_t, \dot{x}_t)$. The function $\bar{V}(t)$ may be discontinuous in time, but it is not allowed to grow in the jumps as shown in Fig. 4. The introduced time-dependent Lyapunov functionals lead to qualitatively new results for time-delay systems, allowing a superior performance under the sampling, than the one under the constant delay. The stability of system (6) is based on the following

Lemma 3. *Let there exist positive numbers α, β, δ and a functional $V: \mathbb{R}^+ \times W[-h, 0] \times L_2[-h, 0] \to \mathbb{R}^+$ such that*

$$\beta|\phi(0)|^2 \le V(t, \phi, \dot{\phi}) \le \delta \|\phi\|_W^2. \tag{19}$$

Let the function $\bar{V}(t) = V(t, x_t, \dot{x}_t)$ be continuous from the right for $x(t)$ satisfying (6), absolutely continuous for $t \ne t_k$ and satisfy

$$\lim_{t \to t_k^-} \bar{V}(t) \ge \bar{V}(t_k). \tag{20}$$

(i) *If along (6)*

$$\dot{\bar{V}}(t) \le -\tilde{\beta}|x(t)|^2$$

holds for $t \ne t_k$ and for some scalar $\tilde{\beta} > 0$, then (6) is asymptotically stable.

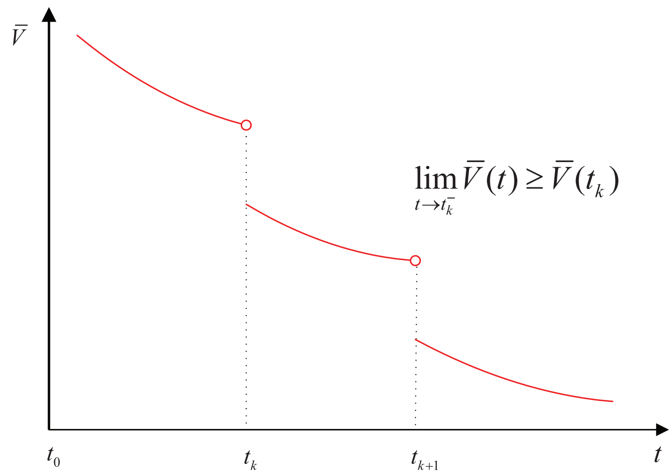

Fig. 4. Discontinuous in time Lyapunov functional.

(ii) *If along (6)*

$$\dot{\bar{V}}(t) + 2\alpha\bar{V}(t) \leq 0, \quad \textit{for } t \neq t_k,$$

then $\bar{V}(t) \leq e^{-2\alpha t}\bar{V}(0)$, *which implies that*

$$|x(t)|^2 \leq e^{-2\alpha t}\frac{\delta}{\beta}\|x_0\|_W^2,$$

and thus, system (6) is exponentially stable with the decay rate α.

Furthermore, a novel discontinuous in time Lyapunov functional was constructed in Liu and Fridman (2012b) based on the extension of the Wirtinger inequality (Hardy, Littlewood, & Pólya, 1934) to the vector case:

Lemma 4. *(Liu and Fridman, 2012b; Liu, Suplin, and Fridman, 2010) Let* $z : [a, b] \to \mathbb{R}^n$ *be an absolutely continuous function with* $\dot{z} \in L_2(a, b)$ *and with* $z(a) = 0$. *Then for any* $n \times n$*-matrix* $R > 0$, *the following inequality holds:*

$$\int_a^b z^T(\xi)Rz(\xi)d\xi \leq \frac{4(b-a)^2}{\pi^2}\int_a^b \dot{z}^T(\xi)R\dot{z}(\xi)d\xi. \tag{21}$$

An extended Wirtinger inequality of Lemma 4 for Lyapunov-based exponential stability analysis was presented in Selivanov and Fridman (2016c):

Lemma 5. *Selivanov and Fridman (2016c) Let* $\alpha \in \mathbb{R}$ *and* $z : [a, b] \to \mathbb{R}^n$ *be an absolutely continuous function with* $\dot{z} \in L_2(a, b)$ *such that* $z(a) = 0$ *or* $z(b) = 0$. *Then for any* $n \times n$*-matrix* $R > 0$, *the following inequality holds:*

$$\begin{aligned}&\int_a^b e^{2\alpha\xi}z^T(\xi)Rz(\xi)d\xi \\ &\quad \leq e^{2|\alpha|(b-a)}\frac{4(b-a)^2}{\pi^2}\int_a^b e^{2\alpha\xi}\dot{z}^T(\xi)R\dot{z}(\xi)d\xi.\end{aligned} \tag{22}$$

Based on Wirtinger inequality of Lemma 5, a discontinuous Lyapunov functional was constructed as follows:

$$\bar{V}(t) = x^T(t)Px(t) + V_R(t, x_t, \dot{x}_t), \quad P > 0 \tag{23}$$

with a novel Wirtinger-based discontinuous term

$$\begin{aligned}V_R &= h^2e^{2\alpha h}\int_{t_k}^t e^{2\alpha(s-t)}\dot{x}^T(s)R\dot{x}(s)ds \\ &\quad -\frac{\pi^2}{4}\int_{t_k}^t e^{2\alpha(s-t)}[x(s)-x(t_k)]^TR[x(s)-x(t_k)]ds, \\ &\quad R > 0, \ t \in [t_k, t_{k+1}).\end{aligned} \tag{24}$$

Since $[x(s) - x(t_k)]|_{s=t_k} = 0$, by Lemma 5 we have $V_R \geq 0$. Moreover, V_R vanishes at $t = t_k$. Hence, the condition $\lim_{t\to t_k^-}\bar{V}(t) \geq \bar{V}(t_k)$ holds. This new method leads to numerically simplified LMI condition for the stability analysis Liu and Fridman (2012b); Selivanov and Fridman (2016c), and it can be also applied to performance analysis such as exponential stability (Selivanov & Fridman, 2016c), input-to-state stability (Selivanov & Fridman, 2016c) and L_2-gain analysis (Liu et al., 2012).

Remark 1. The above discontinuous Lyapunov constructions and their extensions (Fridman, 2010; Seuret, 2012) give efficient tool for different control problems, see e.g., Liu and Fridman (2012a) for stabilization of NCSs with large network-induced delays, Freirich and Fridman (2016, 2018); Liu et al. (2012, 2015a); Liu et al. (2015b) for scheduling protocols, Zhang and Han (2015) for H_∞ filter of sampled-data systems, Hua, Ge, and Guan (2015); Lee, Park, Lee, and Kwon (2013); Lu, Shi, Su, Wu, and Lu (2018); Wu, Park, Su, and Chu (2012) for synchronisation of complex systems.

Remark 2. The discrete-time counterpart of the Wirtinger inequality (21) was presented in Seuret and Fridman (2018) for stability analysis of discrete-time sampled-data systems: for a sequence of $N+1$ real n-dimensional vectors $\xi_0, \xi_1, \ldots, \xi_N$ such that $\xi_0 = 0$, the following inequality holds:

$$\sum_{i=0}^{N-1}(\xi_{i+1} - \xi_i)^TR(\xi_{i+1} - \xi_i) \geq \varrho_N^2\sum_{i=0}^{N-1}\xi_i^TR\xi_i, \tag{25}$$

where $0 < R \in \mathbb{R}^{n\times n}$ and $\varrho_N = 2\sin\frac{\pi}{2(2N+1)}$.

Note that the discrete-time Wirtinger inequality (25) has been recently employed to the security analysis of discrete-time multi-sensor NCSs (Liu, Guo, Zhang, & Xia, 2019).

3.3.6. General and augmented Lyapunov functional method

A necessary condition for the application of the simple LKFs considered in the previous sections is the asymptotic stability of (6) or (16) with $\tau(t) = 0$. Consider e.g., the following system with a constant delay

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 0.1 \end{bmatrix}x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}x(t-h), \ x(t) \in \mathbb{R}^2.$$

This system is unstable for $h = 0$ and is asymptotically stable for the constant delay $h \in [0.10017, 1.7178]$ (Gu et al., 2003). For analysis of such systems (particularly, for using delay for stabilization) the simple Lyapunov functionals considered in the previous sections are not suitable. For stability conditions of system (6) or (16) with constant $\tau(t) \equiv h$ and with $A + A_1$ not necessary to be Hurwitz, one can use a general quadratic Lyapunov functional:

$$\begin{aligned}V(x_t) &= x^T(t)Px(t) + 2x^T(t)\int_{-h}^0 Q(\xi)x(t+\xi)d\xi \\ &\quad + \int_{-h}^0\int_{-h}^0 x^T(t+s)R(s,\xi)dsx(t+\xi)d\xi \\ &\quad + \int_{-h}^0 x^T(t+\xi)S(\xi)x(t+\xi)d\xi,\end{aligned} \tag{26}$$

where $0 < P \in \mathbb{R}^n$ and where $n \times n$ matrix functions $Q(\xi)$, $R(\xi, \eta) = R^T(\eta, \xi)$ and $S(\xi) = S^T(\xi)$ are absolutely continuous. For the sufficiency of (26), one have to formulate conditions for $V \geq \alpha_0|x(t)|^2$, $\alpha_0 > 0$ and $\dot{V} \leq -\alpha|x(t)|^2$, $\alpha > 0$.

LMI sufficient conditions via general Lyapunov functional of (26) and discretization were found in Gu (1997), where $Q(\xi), R(\xi, \eta) = R^T(\eta, \xi)$ and $S(\xi) = S^T(\xi) \in \mathbb{R}^{n\times n}$ are continuous and piecewise-linear matrix-functions. The resulting LMI stability

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conditions appeared to be very efficient, leading in some examples to results close to analytical ones. For the discretized Lyapunov functional method, see Section 5.7 of Gu et al. (2003). In Peet and Bliman (2011); Peet, Papachristodoulou, and Lall (2009), another method was proposed based on polynomial parameters, that successfully address the asymptotic stability of time-delay systems through the SoS framework.

Till Fridman (2006a) no design problems were solved by this method due to bilinear terms in the resulting matrix inequalities. The latter terms arise from the substitution of $\dot{x}(t)$ by the right-hand side of the differential equation in $\dot{V}$. The descriptor discretized method suggested in Fridman (2006a) avoids this substitution. The descriptor discretized method was applied to state-feedback design of H_∞ controllers for neutral type systems with discrete and distributed delays Fridman and Tsodik (2009) and to dynamic output-feedback H_∞ control of retarded systems with state, input and output delays Suplin, Fridman, and Shaked (2009). For differential-algebraic systems with delay, the corresponding general LKFs were studied in Gu (2010).

A more general quadratic Lyapunov functional has a form of

$$\begin{aligned} V(x_t) = {} & x^T(t)Px(t) + 2x^T(t)\int_{-h}^{0} Q(\xi)x(t+\xi)d\xi \\ & + \int_{-h}^{0}\int_{-h}^{0} x^T(t+s)R(s,\xi)ds\, x(t+\xi)d\xi \\ & + \int_{t-h}^{t} x^T(\xi)Sx(\xi)d\xi \\ & + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_0\dot{x}(s)dsd\theta, \end{aligned} \tag{27}$$

where $P>0$, $S>0$, $R_0>0$. Matrix-functions $Q(\xi)\in\mathbb{R}^{n\times n}$ and $R(\xi,\eta)=R^T(\eta,\xi)\in\mathbb{R}^{n\times n}$ are absolutely continuous. For the sufficiency of (27), one have to formulate conditions for $V\geq\beta|x(t)|^2$, $\beta>0$ and $\dot{V}\leq-\alpha|x(t)|^2$, $\alpha>0$.

Choosing in (27) $R=Q=0$ and replacing R_0 by hR, we arrive at the simple Lyapunov functional (11), where $Q=0$. Consider now (27) with constant $R\equiv Z$ and Q, and replace R_0 by hR. Then we arrive at the augmented Lyapunov functional of the form

$$\begin{aligned} V(x_t,\dot{x}_t) = {} & \begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} P & Q \\ * & Z \end{bmatrix} \begin{bmatrix} x(t) \\ \int_{t-h}^{t} x(s)ds \end{bmatrix} \\ & + \int_{t-h}^{t} x^T(s)Sx(s)ds \\ & + h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta, \end{aligned} \tag{28}$$

where

$$\begin{bmatrix} P & Q \\ * & Z \end{bmatrix} > 0,\ S>0,\ R>0.$$

Note that the term $Q\neq0$ in (28) allows to derive non-convex in h conditions that do not imply the stability of the original system with $h=0$. A remarkable result was obtained in Seuret and Gouaisbaut (2013) for systems with constant discrete and distributed delays and in Seuret, Gouaisbaut, and Fridman (2013) for systems with fast-varying discrete delays: LMI conditions that may guarantee the stability of systems which are unstable with the zero delay (i.e., in the case of "stabilizing delay") were derived by the Wirtinger-based integral inequality, which includes Jensen's inequality as a particular case, and by the augmented Lyapunov functional (28).

In recent years, several other attempts have been done concerning the extension of Jensen's inequality such as auxiliary-based Park, Lee, and Lee (2015), Bessel-Legendre inequality Seuret and Gouaisbaut (2015, 2018); Seuret, Gouaisbaut, and Ariba (2015) or polynomials-based inequality Lee, Lee, and Park (2017). By construction of more general augmented LKFs and the application of these developed integral inequalities, a series of less conservative stability conditions was achieved (Liu, Seuret, & Xia, 2017; Seuret & Gouaisbaut, 2015; 2018; Seuret et al., 2015; Zeng et al., 2015a; 2015b).

Remark 3. In the literature, there are many other methods that have been proposed to stability analysis and/or control synthesis of NCSs. Networked predictive method is effective for NCSs with communication delays and packet dropouts (Liu, Xia, Chen, Rees, & Hu, 2007; Wang, Liu, Wang, Rees, & Zhao, 2010; Xia et al., 2011; Xia, Liu, Fu, & Rees, 2009). A looped-functional approach is for robust analysis of sampled-data system either considering directly the sampled-data system formulation (Seuret, 2012) or the impulsive system formulation (Briat & Seuret, 2012; Seuret & Briat, 2015). Model predictive control plays an important role in dealing with constraints, such as actuator or physical limitations (Hashimoto, Adachi, & Dimarogonas, 2017a; 2017b; Liu, Ma, Xia, Sun, & Johansson, 2019). Sliding model control, as an effective robust control strategy has also been applied to NCSs (Han, Fridman, & Spurgeon, 2014; Shah & Mehta, 2018). With the development of cloud computing technologies, the cloud-based networked control has exhibited potential advantages (Adaldo, Liuzza, Dimarogonas, & Johansson, 2018; Xia, 2015).

4. Time-delay approach to event-triggered control

The event-triggering mechanism is used to reduce the amount of signals transmitted through a communication network. The basic idea is to send the signal only when its change is large enough. This idea has a long history (see, e.g., (Årzén, 1999; Dodds, 1981; Draper, Wrigley, & Hovorka, 1960; Gelig & Churilov, 1998; Kopetz, 1993; Tsypkin, 1958; 1984)) and lately became quite popular since it allows to reduce the workload in NCSs. However, the workload reduction has been analytically proved only for stochastic systems (Åström, 2008; Åström & Bernhardsson, 1999; 2002; Antunes & Heemels, 2014). For deterministic systems, the benefits of the event-triggering scheme are usually demonstrated by numerical simulations.

In this section, we describe the time-delay approach to the event-triggered control. We consider three types of the event-triggering mechanisms: continuous, periodic, and switching (with a dwell time). Many other types of the event-triggering mechanisms can be studied in a similar manner using the time-delay approach. Another approach is based on the impulsive systems (Heemels & Donkers, 2013; Postoyan, Tabuada, Nešić, & Anta, 2015; Seuret, Prieur, Tarbouriech, & Zaccarian, 2016). A survey on the event-triggered control can be found in Heemels et al. (2012).

4.1. Continuous event-triggering mechanism

Consider the system

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned} \qquad x\in\mathbb{R}^n,\ u\in\mathbb{R}^m,\ y\in\mathbb{R}^l \tag{29}$$

with the output $y(t)$ being transmitted through a communication channel at the sampling instants t_k, $k\in\mathbb{Z}^+$. We start by considering the following event-based sampling (similar to the one suggested in Tabuada (2007)): $t_0=0$,

$$t_{k+1} = \min\{t > t_k \mid |\sqrt{\Omega}(y(t)-y(t_k))|^2 \geq \sigma^2|\sqrt{\Omega}y(t)|^2\}, \tag{30}$$

where $k\in\mathbb{Z}^+$, $\sigma>0$ and $0<\Omega\in\mathbb{R}^{l\times l}$. According to (30), the output y is transmitted only when a certain "event" occurs. The "event" is that the relative change of y since the last transmission exceeded a predefined threshold $\sigma>0$. Since this condition is checked continuously, we call (30) the continuous event-triggering mechanism.

Let there exist a control gain $K \in \mathbb{R}^{m\times l}$ such that the output feedback $u(t) = -Ky(t)$ stabilizes the system (29). The event-triggered control is given by

$$u(t) = -Ky(t_k), \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{Z}^+ \tag{31}$$

with t_k defined by (30). To study the stability of (29)–(31), introduce the event-triggering error

$$e(t) = y(t_k) - y(t), \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{Z}^+.$$

Then the closed-loop system (29)–(31) can be presented as

$$\dot{x}(t) = A_{cl}x(t) - BKe(t), \quad A_{cl} = A - BKC. \tag{32}$$

Note that A_{cl} is Hurwitz since $u = -Ky$ is assumed to be stabilizing. Namely, there exists $P \in \mathbb{R}^{n\times n}$ such that

$$P > 0, \quad A_{cl}^T P + PA_{cl} < 0. \tag{33}$$

The event-triggering mechanism (30) guarantees that the event-triggering error is bounded:

$$e^T\Omega e \le \sigma^2 x^T C^T \Omega C x. \tag{34}$$

Then, for the Lyapunov function $V = x^T Px$, we have

$$\begin{aligned}\dot V &\overset{(32)}{=} 2x^TPA_{cl}x - 2x^TPBKe\\ &\overset{(34)}{\le} 2x^TPA_{cl}x - 2x^TPBKe - \sigma^{-1}e^T\Omega e + \sigma x^TC^T\Omega Cx\\ &= \begin{bmatrix} x\\ e\end{bmatrix}^T \Phi \begin{bmatrix} x\\ e\end{bmatrix},\end{aligned} \tag{35}$$

where

$$\Phi = \begin{bmatrix} A_{cl}^TP + PA_{cl} + \sigma C^T\Omega C & -PBK\\ * & -\sigma^{-1}\Omega\end{bmatrix}.$$

The addition of the quadratic form $-\sigma^{-1}e^T\Omega e + \sigma x^TC^T\Omega Cx \ge 0$ to the right-hand side of $\dot V$ is called the S-procedure (Lurie, 1957; Yakubovic, 1977). If $\Phi < 0$, then $\dot V < 0$ and (32) is stable. Since $\Omega > 0$, by Schur complements, the condition $\Phi < 0$ is equivalent to

$$A_{cl}^TP + PA_{cl} + \sigma[C^T\Omega C + (PBK)\Omega^{-1}(PBK)^T] < 0.$$

Due to (33), this inequality holds for a small enough $\sigma > 0$. Therefore, if $\operatorname{rank} C = n$ and the system (29) is stable under the feedback $u(t) = -Ky(t)$ (i.e., in the full state-feedback case), then it remains stable under the sampled-data control (31) with the event-based sampling (30) provided the threshold $\sigma > 0$ is small enough.

This result is very intuitive: if σ is small, then the event-triggering mechanism (30) gets more sensitive to the output change and transmits the signals more often, what makes the control (31) more similar to the stabilizing continuous-time controller.

The implicitly defined sampling instants (30) can be such that $\lim_{k\to\infty} t_k < \infty$. This is called the Zeno phenomenon. Such sampling cannot be implemented since it requires to transmit an infinite number of signals in finite time. To use (30), one must ensure that $\inf\{t_k - t_{k-1}\} > 0$. This can be guaranteed if $\operatorname{rank} C = n$ Tabuada (2007). However, the Zeno phenomenon can occur when $\operatorname{rank} C < n$ or in the presence of disturbances Borgers and Heemels (2014a). To avoid this, one may use the periodic event-triggering mechanism or introduce a dwell time.

4.2. Periodic event-triggering mechanism

The periodic event-triggering mechanism is given by

$$t_{k+1} = \min_i\{t_k + ih \mid |\sqrt{\Omega}(y(t_k+ih) - y(t_k))|^2 > \sigma^2|\sqrt{\Omega}y(t_k+ih)|^2\} \tag{36}$$

with $t_0 = 0$, $\sigma > 0$, and $0 < \Omega \in \mathbb{R}^{l\times l}$ Heemels and Donkers (2013); Heemels, Donkers, and Teel (2013); Peng and Yang (2013); Yue, Tian, and Han (2011); Zhang and Han (2014). Differently from (30), the event-triggering condition of (36) is checked periodically with the period $h > 0$. This approach guarantees that the inter-event time is at least h, which rules out the Zeno behavior.

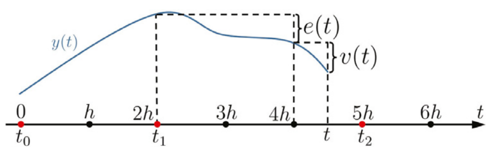

Fig. 5. The errors of the periodic event-triggering mechanism.

To study the stability of (29) under the feedback (31) with the event-based sampling (36), introduce the errors (Fig. 5)

$$\begin{aligned} e(t) &= y(t_k) - y(t_k + ih),\\ \nu(t) &= C\int_{t_k+ih}^{t} \dot x(s)\,ds, \qquad t \in [t_k + ih, t_k + (i+1)h).\end{aligned}$$

Then, $y(t_k) = Cx(t) - \nu(t) + e(t)$ and the closed-loop system can be presented as

$$\dot x(t) = A_{cl}x(t) + BK\nu(t) - BKe(t). \tag{37}$$

Due to (36), the error due to triggering satisfies

$$\begin{aligned} e^T(t)\Omega e(t) &\le \sigma^2 y^T(t_k+ih)\Omega y(t_k+ih)\\ &= \sigma^2(Cx(t) - \nu(t))^T\Omega(Cx(t) - \nu(t)).\end{aligned}$$

Therefore, it can be compensated using the S-procedure similarly to (35). The integral term ν, representing the error due to sampling, can be compensated using an appropriate LKF, e.g., $V = x^TPx + V_R^s$ with V_R^s defined by (18) or $V = x^TPx + V_R$ with V_R defined by (24) or $V = x^TPx + V_U + V_X$, where V_U and V_X are defined in (13) and (26) of Fridman (2010). This allows to derive LMIs guaranteeing the exponential stability of the system (29) under the periodic event-triggered control (31), (36) (Selivanov & Fridman, 2015, Proposition 1).

For $\sigma = 0$, the event-triggering mechanism (36) leads to the periodic sampling $t_k = kh$ and the conditions of (Selivanov & Fridman, 2015, Proposition 1) coincide with the stability conditions for periodic sampling from Fridman (2010).

4.3. Event-triggering mechanism with a dwell time

The event-triggering mechanism with a dwell time has the form

$$t_{k+1} = \min\{t \ge t_k + h \mid \eta \ge 0\}, \tag{38}$$

where $h > 0$ is a constant dwell time and η is the event-triggering condition. In Tallapragada and Chopra (2012a, 2012b, 2015), the value of h that preserves the stability was obtained by solving a scalar differential equation. For $\eta = |y(t) - y(t_k)| - C$ with a constant C, some qualitative results concerning practical stability have been obtained in Heemels, Sandee, and Van Den Bosch (2008). Here, we derive the stability conditions using the time-delay approach and switching as suggested in Selivanov and Fridman (2016b).

Let the sampling be given by

$$t_{k+1} = \min\{t \ge t_k + h \mid |\sqrt{\Omega}(y(t) - y(t_k))|^2 \ge \sigma^2|\sqrt{\Omega}y(t)|^2\} \tag{39}$$

with $t_0 = 0$, $\sigma > 0$, $0 < \Omega \in \mathbb{R}^{l\times l}$, and $h > 0$. Clearly, the inter-event time is not less than h, which rules out the Zeno behavior. The event-triggering mechanism (39) can be viewed as a switching between the periodic sampling $t_k = kh$ and the continuous event-triggering (30). Namely, the closed-loop system (29), (31), (39) can be presented as

$$\dot x(t) = A_{cl}x(t) + \chi(t)BK\nu(t) - (1 - \chi(t))BKe(t),$$

where $A_{cl} = A - BKC$,

$$\chi(t) = \begin{cases} 1, & t \in [t_k, t_k + h), \\ 0, & t \in [t_k + h, t_{k+1}), \end{cases}$$
$$e(t) = y(t_k) - y(t), \quad t \in [t_k + h, t_{k+1}),$$
$$\nu(t) = C \int_{t_k}^{t} \dot{x}(s)\, ds, \quad t \in [t_k + h, t_{k+1}).$$

Differently from (37), the errors e and ν operate on disjoint time intervals. This error separation makes it easier for the stable dynamics $\dot{x}(t) = A_{cl}x(t)$ to dominate the errors. Using the functional $V = x^T P x + \chi(t)(V_U + V_X)$, where V_U and V_X are defined in (13) and (26) of Fridman (2010), respectively, the LMI-based stability conditions can be derived for the system (29), (31), (39) (Selivanov & Fridman, 2015).

In this section, we showed that the time-delay approach can be used to study the event-triggered control. The general idea is to construct for the closed-loop system an appropriate LKF and apply the S-procedure to the quadratic form whose positiveness is guaranteed by the event-triggering condition. This idea can be extended to systems with time-varying communication delays Selivanov and Fridman (2016b), sampled-data predictors that compensate large constant input and output delays (Selivanov & Fridman, 2016c; 2016d), adaptive controllers (Selivanov, Fridman, & Fradkov, 2017), and event-triggered control of distributed parameter systems (Selivanov & Fridman, 2016a).

5. Time-delay approach to networked control under scheduling protocols

In many works on NCSs, it was usually assumed that all the nodes could simultaneously get access to the network to transmit their data. This assumption, however, is generally unrealistic for NCSs due to bandwidth limitations and interference channels. As such, the scheduling protocols are needed to orchestrate the transmission order of the nodes. The widely utilized protocols in the literature include the Round-Robin protocol, the TOD protocol and the stochastic protocol, see also Section 3. Three main approaches for the modelling and analysis of NCSs subject to scheduling protocols are based on discrete-time systems, impulsive/hybrid systems, and time-delay systems.

In the framework of discrete-time modelling approach, network-based stabilization of NCSs with TOD/Round-Robin protocol and without delay was considered in Donkers et al. (2009) (see also (Donkers et al., 2011) for delays less than the sampling interval). In van Loon et al. (2012), stability analysis of NCSs with Round-Robin protocol and uniform quantizers was studied. The stability of NCSs under a stochastic protocol, where the activated node is modeled by a Markov chain, was studied in Donkers et al. (2012) by applying the discrete-time modeling framework. In Donkers et al. (2012), data packet dropouts can be regarded as prolongations of the sampling interval for small delays.

In the framework of impulsive system approach, stabilization of nonlinear NCSs under TOD and Round-Robin protocols was studied in Nesic and Teel (2004), in which communication delays are not included in the analysis. In Heemels et al. (2010a), the authors provided methods for computing the MATI and MAD for which the stability of a nonlinear system is ensured. In Heemels et al. (2009) and Nesic and Liberzon (2009), stabilization of nonlinear NCSs including dynamic quantization was studied. In the same framework, a stochastic protocol was introduced in Tabbara and Nesic (2008) and analyzed for the input-output stability of NCSs in the presence of data packet dropouts or collisions. An independent and identically-distributed (iid) sequence of Bernoulli random variables is applied to describe the stochastic protocol. Communication delays, however, are not included in the analysis.

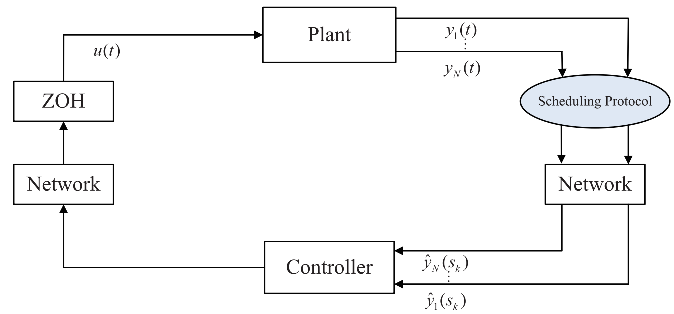

Fig. 6. System architecture with multiple sensors.

Till now only the time-delay system approach is applicable to non-small network-induced delays (that may be larger than sampling intervals) in the presence of scheduling protocols (Freirich & Fridman, 2016; 2018; Liu & Fridman, 2015; Liu et al., 2012; 2015a; Liu et al., 2015c; Liu, Fridman, Johansson, & Xia, 2016b; Liu, Seuret, Fridman, & Xia, 2018). In this section, we focus mainly on the time-delay approach to the modelling and analysis of NCSs under scheduling protocols. Consider the system architecture in Fig. 6 with plant

$$\dot{x}(t) = Ax(t) + Bu(t), \ t \geq 0, \tag{40}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ denote the state and the control input, respectively. The system matrices A and B can be uncertain with polytopic type uncertainties.

The system is equipped with N distributed sensors, a controller and an actuator, which are connected via the network. The measurements are given by $y_i(t) = C_i x(t) \in \mathbb{R}^{n_i}$, $i = 1, \ldots, N$, $\sum_{i=1}^{N} n_i = n_y$. Then we denote $C = [C_1^T \ \cdots \ C_N^T]^T, y(t) = [y_1^T(t) \ \cdots \ y_N^T(t)]^T \in \mathbb{R}^{n_y}$. Let s_k denote the unbounded and monotonously increasing sequence of sampling instants $0 = s_0 < s_1 < \cdots < s_k < \cdots, \quad k \in \mathbb{Z}^+, \lim_{k\to\infty} s_k = \infty$.

At each sampling instant s_k, one of the outputs $y_i(s_k) \in \mathbb{R}^{n_i}$ is transmitted via the sensor network. It is supposed that data loss does not occur and that the transmission of the information over the network experiences an uncertain, time-varying delay η_k. Then $t_k = s_k + \eta_k$ is the updating time instant of the ZOH device.

Assume that the maximum sampling interval and the maximum delay between the sampling instant s_k and its updating instant t_k are bounded by *MATI* and *MAD* respectively. The transmission delays are allowed to be non-small provided that the transmission order of data packets is maintained for reception (Naghshtabrizi et al., 2010). Assume that the network-induced delay η_k and the time span between the updating and the most recent sampling instants are bounded:

$$t_{k+1} - t_k + \eta_k \leq \tau_M, \ 0 \leq \eta_m \leq \eta_k \leq MAD, \ k \in \mathbb{Z}^+, \tag{41}$$

where τ_M denotes the maximum time span between the time $s_k = t_k - \eta_k$ at which the state is sampled and the time t_{k+1} at which the next update arrives at the destination. Here η_m and *MAD* are known bounds and $\tau_M = MATI + MAD$. Note that $MATI = \tau_M - MAD \leq \tau_M - \eta_m$, $\eta_m > \frac{\tau_M}{2}$, i.e., $\eta_m > \tau_M - \eta_m$ leads to MATI $\leq \tau_M - \eta_m < \eta_m \leq \eta_k$, which implies that the network delays are non-small.

Denote by

$$\hat{y}(s_k) = [\hat{y}_1^T(s_k) \ \cdots \ \hat{y}_N^T(s_k)]^T \in \mathbb{R}^{n_y} \tag{42}$$

the output information submitted to the scheduling protocol. At each sampling instant s_k, one of the system nodes $i \in \mathcal{I} = \{1, \ldots, N\}$ is active, that is only one of $\hat{y}_i(s_k)$ values is updated with the recent output $y_i(s_k)$. Let $i_k^* \in \mathcal{I}$ denote the active output node at the sampling instant s_k, which will be chosen due to scheduling protocols. Then

$$\hat{y}_i(s_k) = \begin{cases} y_i(s_k), & i = i_k^*, \\ \hat{y}_i(s_{k-1}), & i \neq i_k^*. \end{cases} \tag{43}$$

The choice of i_k^* will be determined by the scheduling protocol defined below.

It is supposed that the controller and the actuator are event-driven. The most recent output information on the controller side is denoted by $\hat{y}(s_k)$. Assume that there exists a matrix $K = [K_1 \ \cdots \ K_N]$, $K_i \in \mathbb{R}^{m\times n_i}$ such that $A + BKC$ is Hurwitz. Then, the static output feedback controller has a form

$$u(t) = K\hat{y}(s_k) = \sum_{i=1}^{N} K_i\hat{y}_i(s_k), \ \ t \in [t_k, t_{k+1}). \tag{44}$$

Therefore, due to (43), the controller for $t \in [t_k, t_{k+1})$ can be presented as

$$u(t) = K_{i_k^*} y_{i_k^*}(t_k - \eta_k) + \sum_{i=1, i\neq i_k^*}^{N} K_i\hat{y}_i(t_{k-1} - \eta_{k-1}), \tag{45}$$

where i_k^* is the index of the active node at s_k and η_k is communication delay.

5.1. Scheduling protocols

5.1.1. TOD protocol

In the TOD protocol, the output node $i \in \mathcal{I}$ with the greatest weighted error will have the highest priority to be access to the network.

Definition 1 (Weighted TOD protocol)**.** Let $Q_i > 0$, $i = 1, \ldots, N$, be some weighting matrices. At the sampling instant s_k, the weighted TOD protocol is a protocol for which the active output node with the index i_k^* is defined as any index that satisfies

$$\begin{aligned} &|\sqrt{Q_{i_k^*}} e_{i_k^*}(t)|^2 \geq |\sqrt{Q_i} e_i(t)|^2, \ \ t \in [t_k, t_{k+1}), \\ &\quad k \in \mathbb{Z}^+, \ \ i = 1, \ldots, N. \end{aligned} \tag{46}$$

A possible choice of i_k^* is given by

$$i_k^* = \min\{\arg\max_{i\in\mathcal{I}} |\sqrt{Q_i}\big(\hat{y}_i(s_{k-1}) - y_i(s_k)\big)|^2\},$$

i.e., if several errors are the same, we transmit the node i with a minimum index. The conditions for computing the weighting matrices $Q_1, \ldots, Q_N$ will be given in Lemma 6 below.

5.1.2. Periodic protocol

The active output node is chosen in a periodic order:

$$\begin{aligned} &i_k^* = i_{k+N}^*, \ \text{ for all } k \in \mathbb{Z}^+, \\ &i_j^* \neq i_l^*, \ \text{ for } \ 0 \leq j < l \leq N-1, \end{aligned} \tag{47}$$

where N is the period of the protocol. The well-known Round-Robin protocol belongs to this class of protocols.

5.1.3. Stochastic protocols

There are usually two classes of stochastic protocols, which are defined by iid and Markovian process, respectively.

Iid Protocol. The choice of i_k^* is assumed to be iid with the probabilities given by

$$\text{Prob}\{i_k^* = i\} = \beta_i, \ \ i \in \mathcal{I}, \tag{48}$$

where $\beta_i, i = 1, \ldots, N$ are non-negative scalars and $\sum_{i=1}^N \beta_i = 1$. Here β_j, $j = 1, \ldots, N$ are the probabilities of the measurement $y_j(s_k)$ to be transmitted at s_k.

Markovian Protocol. The protocol determines i_k^* through a Markov Chain. The conditional probability that node $j \in \mathcal{I}$ gets access to the network at time s_k, given the values of $i_{k-1}^* \in \mathcal{I}$, is defined by

$$\text{Prob}\{i_k^* = j | i_{k-1}^* = i\} = \pi_{ij}, \tag{49}$$

where $0 \leq \pi_{ij} \leq 1$ for all $i, j \in \mathcal{I}, \sum_{j=1}^N \pi_{ij} = 1$ for all $i \in \mathcal{I}$ and $i_0^* \in \mathcal{I}$ is assumed to be given. The transition probability matrix is denoted by $\Pi = \{\pi_{ij}\} \in \mathbb{R}^{N\times N}$.

Remark 4. The iid scheduling is a special case of the Markovian scheduling. For instance, assume that there are $N = 2$ sensor nodes, the Markovian scheduling with $\Pi = \begin{bmatrix} p & 1-p \\ p & 1-p \end{bmatrix}$, $0 \leq p \leq 1$, is an iid scheduling with $\beta_1 = p$, $\beta_2 = 1 - p$.

5.2. TOD protocol and an impulsive system model

Consider the error between the system output $y_i(s_k)$ and the last available information $\hat{y}_i(s_{k-1})$:

$$\begin{aligned} &e_i(t) = \hat{y}_i(s_{k-1}) - y_i(s_k), \ \ \hat{y}_i(s_{-1}) \triangleq 0, \ \ i = 1, \ldots, N, \\ &e(t) = col\{e_1(t), \cdots, e_N(t)\}, \ \ t \in [t_k, t_{k+1}), \ \ e(t) \in \mathbb{R}^{n_y}. \end{aligned} \tag{50}$$

Denote $\tau(t) = t - t_k + \eta_k$, $t \in [t_k, t_{k+1})$. From (41), it follows that $\eta_m \leq \tau(t) \leq \tau_M$. From (40), (45) and (50), we thus obtain the impulsive closed-loop model with the following dynamics:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1x(t - \tau(t)) + \sum_{i=1, i\neq i_k^*}^{N} B_ie_i(t), \\ \dot{e}(t) = 0, \ \ t \in [t_k, t_{k+1}), \end{cases} \tag{51}$$

and the delayed reset system

$$\begin{cases} x(t_{k+1}) = x(t_{k+1}^-), \\ e_i(t_{k+1}) = C_i[x(t_k - \eta_k) - x(t_{k+1} - \eta_{k+1})], \ \ i = i_k^*, \\ e_i(t_{k+1}) = e_i(t_k) + C_i[x(t_k - \eta_k) - x(t_{k+1} - \eta_{k+1})], \\ \qquad i \neq i_k^*, \ \ i \in \mathbb{N}, \end{cases} \tag{52}$$

where $A_1 = BKC$, $B_i = BK_i$, $i = 1, \ldots, N$.

Note that the differential equation for x given by (51) depends on $e_i(t) = e_i(t_k)$, $t \in [t_k, t_{k+1})$ with $i \neq i_k^*$ only. Consider the following Lyapunov functional:

$$\begin{aligned} V_e(t) &= V(t, x_t, \dot{x}_t) + \sum_{i=1}^{N} e_i^T(t)Q_ie_i(t), \\ V(t, x_t, \dot{x}_t) &= \tilde{V}(t, x_t, \dot{x}_t) + V_G, \\ V_G &= \sum_{i=1}^{N} (\tau_M - \eta_m) \int_{s_k}^{t} e^{2\alpha(s-t)} |\sqrt{G_i}C_i\dot{x}(s)|^2 ds, \\ \tilde{V}(t, x_t, \dot{x}_t) &= x^T(t)Px(t) + \int_{t-\eta_m}^{t} e^{2\alpha(s-t)} x^T(s)S_0x(s)ds \\ &\quad + \int_{t-\tau_M}^{t-\eta_m} e^{2\alpha(s-t)} x^T(s)S_1x(s)ds \\ &\quad + \eta_m \int_{-\eta_m}^{0} \int_{t+\theta}^{t} e^{2\alpha(s-t)} \dot{x}^T(s)R_0\dot{x}(s)dsd\theta \\ &\quad + (\tau_M - \eta_m) \int_{-\tau_M}^{-\eta_m} \int_{t+\theta}^{t} e^{2\alpha(s-t)} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta, \end{aligned} \tag{53}$$

where $P > 0$, $S_j > 0$, $R_j > 0$, $G_i > 0$, $Q_i > 0$, $\alpha > 0$, $j = 0, 1, i = 1, \ldots, N, t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}^+$, and where we define $x(t) = x_0$ for $t < 0$. The terms

$$e_i^T(t)Q_ie_i(t) \equiv e_i^T(t_k)Q_ie_i(t_k), \ \ t \in [t_k, t_{k+1})$$

are piecewise-constant, $\tilde{V}(t, x_t, \dot{x}_t)$ presents the standard Lyapunov functional for systems with interval delays $\tau(t) \in [\eta_m, \ \tau_M]$. The piecewise-continuous in time term V_G is inserted to cope with the delays in the reset conditions. It is continuous on $[t_k, t_{k+1})$ and does not grow in the jumps (when $t = t_{k+1}$). The function $V_e(t)$ is thus continuous and differentiable over $[t_k, t_{k+1})$. The following lemma gives sufficient conditions for the exponential stability of (46) and (51)–(52):

Lemma 6. *Liu et al. (2015a) Suppose that there exist positive constant α, $0 < Q_i \in \mathbb{R}^{n_i \times n_i}$, $0 < U_i \in \mathbb{R}^{n_i \times n_i}$, $0 < G_i \in \mathbb{R}^{n_i \times n_i}$, $i = 1, \dots, N$, and $V_e(t)$ of (53) such that along (51) the following inequality holds for $t \in [t_k, t_{k+1})$*

$$\dot{V}_e(t) + 2\alpha V_e(t) - \frac{1}{\tau_M - \eta_m} \sum_{i=1, i \neq i_k^*}^{N} |\sqrt{U_i} e_i(t)|^2 - 2\alpha |\sqrt{Q_{i_k^*}} e_{i_k^*}(t)|^2 \leq 0.$$

Assume additionally that

$$\begin{bmatrix} -\frac{1-2\alpha(\tau_M - \eta_m)}{N-1} Q_i + U_i & Q_i \\ * & Q_i - G_i e^{-2\alpha \tau_M} \end{bmatrix} < 0, \quad i = 1, \dots, N. \tag{54}$$

Then $V_e(t)$ does not grow in the jumps along (51)–(52) and (46):

$$V_e(t_{k+1}) - V_e(t_{k+1}^-) + \sum_{i=1, i \neq i_k^*}^{N} |\sqrt{U_i} e_i(t_k)|^2 + 2\alpha(\tau_M - \eta_m) |\sqrt{Q_{i_k^*}} e_{i_k^*}(t_k)|^2 \leq 0.$$

Moreover, the following bounds hold for the solutions of (51)–(52) and (46) initialized by $x_{t_0} \in W[-\tau_M, 0], e(t_0) \in \mathbb{R}^{n_y}$:

$$\begin{aligned} &V(t, x_t, \dot{x}_t) \leq e^{-2\alpha(t-t_0)} V_e(t_0), \ t \geq t_0, \\ &V_e(t_0) = V(t_0, x_{t_0}, \dot{x}_{t_0}) + \sum_{i=1}^{N} |\sqrt{Q_i} e_i(t_0)|^2, \end{aligned} \tag{55}$$

and

$$\sum_{i=1}^{N} |\sqrt{Q_i} e_i(t)|^2 \leq \tilde{c} e^{-2\alpha(t-t_0)} V_e(t_0), \tag{56}$$

where $\tilde{c} = e^{2\alpha(\tau_M - \eta_m)}$, implying exponential stability of (51)–(52) and (46).

The exponential stability of system (51)–(52) under (46) can be alternatively analyzed via the Lyapunov functional

$$\tilde{V}_e(t) = V_e(t) + V_W(t), \ t \in [t_k, t_{k+1}), \tag{57}$$

where $V_e(t)$ is given by (53) and

$$V_W(t) = 2\alpha(t_k - t) e_{i_k^*}^T(t) Q_{i_k^*} e_{i_k^*}(t) + \sum_{i=1, i \neq i_k^*}^{N} \frac{t_k - t}{t_{k+1} - t_k} e_i^T(t) U_i e_i(t).$$

The negative term $V_W(t)$ is a piecewise-continuous in time term that was employed in Freirich and Fridman (2016) to simplify the exponential stability analysis of the impulsive system. The following lemma gives sufficient conditions for the positivity of $\tilde{V}_e(t)$ and for the fact that it does not grow in the jumps t_k, and also for the exponential stability of (51)–(52) and (46):

Lemma 7. *Freirich and Fridman (2016) Given a tuning parameter $\alpha > 0$, let there exist matrices $0 < Q_i \in \mathbb{R}^{n_i \times n_i}$, $0 < U_i \in \mathbb{R}^{n_i \times n_i}$ and $0 < G_i \in \mathbb{R}^{n_i \times n_i}$, $i = 1, \dots, N$, that satisfy the LMIs (54). Then $\tilde{V}_e(t)$ of (57) is positive in the sense that*

$$\tilde{V}_e(t) \geq \beta \left(|x(t)|^2 + |e(t)|^2 \right), \ t \geq t_0,$$

for some $\beta > 0$. Moreover, $\tilde{V}_e(t)$ does not grow in the jumps along (51)–(52) and (46):

$$\tilde{V}_e(t_{k+1}) - \tilde{V}_e(t_{k+1}^-) \leq 0.$$

Furthermore, along (51) if the following inequality holds for $t \in [t_k, t_{k+1})$

$$\dot{\tilde{V}}_e(t) + 2\alpha \tilde{V}_e(t) \leq 0,$$

then the bounds (55) and (56) with $V_e(t_0)$ changed by $\tilde{V}_e(t_0)$ hold, for the solutions of (51), (52) and (46) initialized by $x_{t_0} \in W[-\tau_M, 0], e(t_0) \in \mathbb{R}^{n_y}$.

5.3. Round-Robin protocol

Under Round-Robin scheduling, the measurements are sent in a periodic manner one after another. In Section 5.3.1, we first show a simplified closed-loop system, which is modelled as one system with multiple independent delays. A more accurate model in the form of switched subsystems with ordered multiple delays is provided in Section 5.3.2. Section 5.3.3 presents an impulsive system model, which leads to a more complicated stability analysis.

5.3.1. Round-Robin protocol and a simplified system model with multiple independent delays

Under Round-Robin scheduling, the components of the most recent output on the controller side $\hat{y}(s_k)$ given by (42) can be presented as $\hat{y}_i(s_k) = y_i(s_{k-\Delta_k^i})$, $i = 1, \dots, N$ with some $\Delta_k^i \in \{0, \dots, N-1\}$. Following the time-delay approach to NCSs, denote $\tau_i(t) = t - s_{k-\Delta_k^i}$, $t \in [t_k, t_{k+1})$. We have

$$\begin{aligned} \eta_m \leq \tau_i(t) \leq t_{k+1} - s_{k-\Delta_k^i} &= s_{k+1} - s_{k-\Delta_k^i} + \eta_{k+1} \\ &\leq (\Delta_k^i + 1) \cdot MATI + MAD \\ &\leq N \cdot MATI + MAD \triangleq \tau_M^1. \end{aligned}$$

Therefore, for $t \geq t_{N-1}$ (when all the measurements are transmitted at least once) the static output-feedback (44) under Round-Robin protocol can be presented as

$$u(t) = \sum_{i=1}^{N} K_i y_i(t - \tau_i(t)), \ t \geq t_{N-1}. \tag{58}$$

The resulting closed-loop model is a system with multiple delays

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} A_i C_i x(t - \tau_i(t)), \ t \geq t_{N-1}, \tag{59}$$

where $A_i = BK_i$ and $\tau_i(t) \in [\eta_m, \tau_M^1,]$, $i = 1, \dots, N$.

The stability analysis of closed-loop model (59) can be provided by employing the Lyapunov functional proposed in Freirich and Fridman (2016).

5.3.2. Round-Robin protocol and a switched system model with multiple ordered delays

Consider Round-Robin scheduling for the choice of the active output node: $y_i(t) = C_i x(t)$, is transmitted only at the sampling instant $t = s_{N\ell+i-1}$, $\ell \in \mathbb{Z}^+$, $i = 1, \dots, N$. After each transmission and reception, the values in $y_i(t)$ are updated with the newly received values, while the values of $y_j(t)$ for $j \neq i$ remain the same, as no additional information is received. This leads to the constrained data exchange expressed as

$$\hat{y}_i(s_k) = \begin{cases} y_i(s_k) = C_i x(s_k), & k = N\ell + i - 1, \\ \hat{y}_i(s_{k-1}), & k \neq N\ell + i - 1, \end{cases} \ell \in \mathbb{Z}^+.$$

Following the assumptions given by (41) on the network-induced delay and sampling intervals, we have for $j = 1, \dots, N$

$$\begin{aligned} t_{k+1} - s_{k-N+j} &= s_{k+1} - s_{k-N+j} + \eta_{k+1} \\ &\leq (N - j + 1) MATI + MAD \triangleq \tau_M^j. \end{aligned}$$

The closed-loop system with Round-Robin protocol is modeled as a switched system:

$$\dot{x}(t) = Ax(t) + \sum_{j=1}^{N} A_{\theta(i,j)} x(t_{k-N+j} - \eta_{k-N+j}), \tag{60}$$

where $t \in [t_k, t_{k+1}),\ i = 1, \dots, N,\ A_{\theta(i,j)} = BK_{\theta(i,j)} C_{\theta(i,j)}$,

$$k = \begin{cases} N\ell + i - 1, & \text{for } i \in \mathcal{I}\backslash\{N\},\ \ell \in \mathbb{N} \\ N\ell - 1, & \text{for } i = N,\ \ell \in \mathbb{N} \end{cases}$$

$$\theta(i,j) = \begin{cases} i + j, & \text{if } i + j \le N, \\ i + j - N, & \text{if } i + j > N,\ j = 1, \dots, N. \end{cases}$$

We represent $t_{k-N+j} - \eta_{k-N+j} = t - \tau_j(t),\ j = 1, \dots, N$, where

$$\begin{aligned} &\tau_\vartheta(t) < \tau_{\vartheta-1}(t),\ \ \vartheta = 2, \dots, N, \\ &\tau_\vartheta(t) = t - t_{k-N+\vartheta} + \eta_{k-N+\vartheta}, \\ &\tau_{\vartheta-1}(t) = t - t_{k-N+\vartheta-1} + \eta_{k-N+\vartheta-1}, \\ &\tau_j(t) \in [\eta_m, \tau_M^j],\quad t \in [t_k, t_{k+1}),\ j = 1, \dots, N. \end{aligned}$$

Therefore, (60) can be considered as a system with N time-varying interval delays, where $\tau_\vartheta(t) < \tau_{\vartheta-1}(t),\ \vartheta = 2, \dots, N$. The stability analysis of the closed-loop system (60) can be based on the common (for both subsystems of the switched system) time-independent LKF for the exponential stability of systems with time-varying delay from the maximum delay interval $[\eta_m, \bar{\tau}_M^1]$ (see e.g., (Fridman, 2006b; He et al., 2007)). Moreover, system (60) can be analyzed by taking into account the order of the delays $\tau_\vartheta(t) < \tau_{\vartheta-1}(t),\ \vartheta = 2, \dots, N$, and applying convexity arguments of Park et al. (2011) via Lemma 1. In the special case when NCSs with $N = 2$ sensors and with constant measurement delay, one can adopt a switched time-dependent Lyapunov functional construction, which is based on the extension of Wirtinger inequality (Hardy et al., 1934). See more details in Liu et al. (2012).

Remark 5. The simplified closed-loop system model (59) consists of one system with independent delays from the maximum delay interval $[\eta_m, \tau_M^1]$. The closed-loop system (60) is presented in the form of switched N subsystems (instead of one system) with ordered multiple delays, which is a more accurate model of the closed-loop system under Round-Robin protocol.

5.3.3. Round-Robin protocol and an impulsive system model

Under Round-Robin protocol (47), the reset system (52) can be rewritten as

$$\begin{cases} x(t_{k+1}) = x(t_{k+1}^-), \\ e_{i^*_{k-j}}(t_{k+1}) = C_{i^*_{k-j}}[x(s_{k-j}) - x(s_{k+1})], \\ j = 0, \dots, N-1 \text{ if } k \ge N-1, \end{cases} \tag{61}$$

where the index $k - j$ corresponds to the last updated measurement in the node i^*_{k-j}.

The stability of system (47), (51), (61) can be analyzed by Lyapunov functional (33) of Liu et al. (2015a), leading to a more complicated analysis.

5.4. TOD/Round-Robin protocol: $N = 2$

For $N = 2$, less restrictive analysis of exponential stability of (51) and (52) under both TOD and Round-Robin protocols can be derived via a different from (53) Lyapunov functional:

$$\hat{V}_e(t) = V(t, x_t, \dot{x}_t) + \tfrac{t_{k+1}-t}{\tau_M - \eta_m}\{e_i^T(t) Q_i e_i(t)\}_{|i \ne i_k^*}, \tag{62}$$

where $Q_1 > 0$, $Q_2 > 0$, $\alpha > 0$, $t \in [t_k, t_{k+1})$, $k \in \mathbb{Z}^+, i_k^* \in \{1, 2\}$ and $V(t, x_t, \dot{x}_t)$ is given by (53) with $G_i = Q_i e^{2\alpha\tau_M}$. The term $\frac{t_{k+1}-t}{\tau_M-\eta_m}[e_i^T(t_k) Q_i e_i(t_k)]$ is inspired by the similar construction of Lyapunov functionals for the sampled-data systems (Fridman, 2010; Naghshtabrizi et al., 2008; Seuret, 2012). The following statement holds:

Lemma 8. Liu et al. (2015a) *Given $N = 2$, if there exist positive constant α and $\hat{V}_e(t)$ of (62) such that along (51), the following inequality holds*

$$\dot{\hat{V}}_e(t) + 2\alpha \hat{V}_e(t) \le 0,\quad t \in [t_k, t_{k+1}).$$

Then $\hat{V}_e(t)$ does not grow in the jumps along (51), (52), (46)) ((51), (52), (47)), where

$$\hat{V}_e(t_{k+1}) - \hat{V}_e(t_{k+1}^-) \le 0.$$

The bound (55) is valid for the solutions of (51), (52), (46)) ((51), (52), (47)) with the initial condition $x_{t_0} \in W[-\tau_M, 0]$, $e(t_0) \in \mathbb{R}^{n_y}$, implying exponential stability of (51), (52), (46)) ((51), (52), (47)) with respect to x.

Remark 6. Differently from Lemmas 6 and 7, Lemma 8 guarantees $\hat{V}_e(t_{k+1}) \le e^{-2\alpha(t_{k+1}-t_0)} \hat{V}_e(t_0)$, which does not give a bound on $e_{i_k^*}(t_k)$ since $\hat{V}_e(t)$ for $t \in [t_k, t_{k+1})$ does not depend on $e_{i_k^*}(t_k)$. That is why Lemma 8 guarantees exponential stability only with respect to x. However, it is explained that under Round-Robin protocol exponential stability with respect to x implies boundedness of e. See Remark 5.4 in Liu et al. (2015a).

5.5. Stochastic protocol and a stochastic impulsive system model

Following the modelling of NCSs under TOD protocol, the closed-loop system of NCSs under iid protocol (48) or under Markovian protocol (49) can be also formulated as an impulsive system model (51), (52) under (48) or under (49), respectively.

Following Yue, Tian, Zhang, and Peng (2009), we introduce the indicator functions

$$\pi_{\{i_k^* = i\}} = \begin{cases} 1, & i_k^* = i \\ 0, & i_k^* \ne i, \end{cases} i \in \mathcal{I},\ k \in \mathbb{Z}^+.$$

Thus, from (48) it follows that

$$\mathbb{E}\{\pi_{\{i_k^*=i\}}\} = \mathbb{E}\{[\pi_{\{i_k^*=i\}}]^2\} = \text{Prob}\{i_k^* = i\} = \beta_i,$$

$$\mathbb{E}\{[\pi_{\{i_k^*=i\}} - \beta_i][\pi_{\{i_k^*=j\}} - \beta_j]\} = \begin{cases} -\beta_i\beta_j, & i \ne j, \\ \beta_i(1-\beta_i), & i = j. \end{cases}$$

Therefore, the stochastic impulsive system model (51), (52) under (48) can be rewritten as

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1 x(t_k - \eta_k) + \sum_{i=1}^{N} (1 - \pi_{\{i_k^*=i\}}) B_i e_i(t), \\ \dot{e}(t) = 0,\ \ t \in [t_k, t_{k+1}) \end{cases} \tag{63}$$

with the delayed reset system

$$\begin{cases} x(t_{k+1}) = x(t_{k+1}^-), \\ e_i(t_{k+1}) = (1 - \pi_{\{i_k^*=i\}}) e_i(t_{k+1}^-) \\ \qquad + C_i[x(t_k - \eta_k) - x(t_{k+1} - \eta_{k+1})],\ \ i = 1, \dots, N. \end{cases} \tag{64}$$

Consider the LKF (53). The following lemma gives sufficient conditions for exponential stability of (63), (64) in the mean-square sense:

Lemma 9. Liu et al. (2015c) *If there exist positive constant α, $0 < Q_i \in \mathbb{R}^{n_i \times n_i}$, $0 < U_i \in \mathbb{R}^{n_i \times n_i}$, $0 < G_i \in \mathbb{R}^{n_i \times n_i}$, $i = 1, \dots, N$, and $V_e(t)$ of (53) such that along (63) for $t \in [t_k, t_{k+1})$*

$$\mathbb{E}\{\mathcal{L}V_e(t) + 2\alpha V_e(t) - \tfrac{1}{\tau_M - \eta_m} \sum_{i=1}^{N} e_i^T(t) U_i e_i(t)\} \le 0$$

with

$$\begin{bmatrix} -\beta_i Q_i + U_i & (1-\beta_i) Q_i \\ * & Q_i - G_i e^{-2\alpha\tau_M} \end{bmatrix} \le 0,\ \ i = 1, \dots, N.$$

Then $V_e(t)$ does not grow in the jumps along (63), (64)

$$\mathbb{E}\{V_e(t_{k+1}) - V_e(t_{k+1}^-) + \sum_{i=1}^{N} e_i^T(t_k)U_i e_i(t_k)\} \le 0.$$

Moreover, the following bounds hold for the solutions of (63), (64) with the initial condition $x_{t_0}, e(t_0)$:

$$\begin{aligned}&\mathbb{E}\{V(t, x_t, \dot{x}_t)\} \le e^{-2\alpha(t-t_0)}\mathbb{E}\{V_e(t_0)\}, \ t \ge t_0,\\ &V_e(t_0) = V(t_0, x_{t_0}, \dot{x}_{t_0}) + \sum_{i=1}^{N} e_i^T(t_0)Q_i e_i(t_0),\end{aligned} \tag{65}$$

and

$$\sum_{i=1}^{N} \mathbb{E}\{|\sqrt{Q_i}e_i(t)|^2\} \le \tilde{c}e^{-2\alpha(t-t_0)}\mathbb{E}\{V_e(t_0)\}, \tag{66}$$

where $\tilde{c} = e^{2\alpha(\tau_M - \eta_m)}$, implying exponential mean-square stability of (63), (64).

The exponential mean-square stability of stochastic Markovian jump impulsive system (51), (52) under (49) can be similarly analyzed. See more details in Liu et al. (2015c).

Remark 7. By the time-delay approach, the scheduling protocols were further considered in different control problems in the literature, e.g., in Ugrinovskii and Fridman (2014) for distributed estimation with H_∞ consensus, in Bar Am and Fridman (2014) to design network-based H_∞ filter for a parabolic system, in Zou, Wang, and Gao (2016b) for set-membership filtering problem of time-varying system, and in Wen, Wan, Cao, Huang, and Yu (2018) to achieve master-salve synchronization.

Remark 8. Decentralized networked control of large-scale interconnected systems with local independent networks was studied in the framework of impulsive systems (Borgers & Heemels, 2014b; Heemels, Borgers, van de Wouw, Nesic, & Teel, 2013), where variable sampling or/and small communication delays were taken into account. Distributed estimation in the presence of synchronous sampling of local networks and Round-Robin protocol was analyzed in Ugrinovskii and Fridman (2014) in the framework of time-delay approach. The time-delay approach was extended in Freirich and Fridman (2016) to decentralized NCSs with multiple local communication networks under TOD or Round-Robin protocol.

Remark 9. Note that the aforementioned works on scheduling protocols are concerned with continuous-time systems. The time-delay approach to the modelling and analysis of NCSs with discrete-time plant under scheduling protocols can be formulated by following the same lines of reasoning, see e.g., Liu and Fridman (2015) for discrete-time networked systems with actuator constraints and two sensor nodes under TOD or under Round-Robin protocol, Liu et al. (2016b) for quantized control with multiple sensor nodes under Round-Robin protocol, Liu et al. (2018) for discrete-time networked systems with multiple sensor nodes under dynamic scheduling protocols, Freirich and Fridman (2018) for decentralized networked control of large-scale discrete-time systems with local networks, where the multiple sensor nodes of each subsystem are orchestrated by TOD or Round-Robin protocol.

6. Networked control of parabolic PDEs

In this section, we show how the time-delay approach can be used for robust control of parabolic PDEs with different types of sampled in time measurements or/and actuations. Other methods for sampled-data control of PDEs include the discrete-time approach (Logemann, 2013; Logemann, Rebarber, & Townley, 2003; 2005; Tan, Trélat, Chitour, & Nešić, 2009) (for general LTI PDE systems) and the modal decomposition technique (Christofides, 2001; Dubljevic, El-Farra, Mhaskar, & Christofides, 2006; Ghantasala & El-Farra, 2012; Karafyllis & Krstic, 2018; Lao, Ellis, & Christofides, 2014).

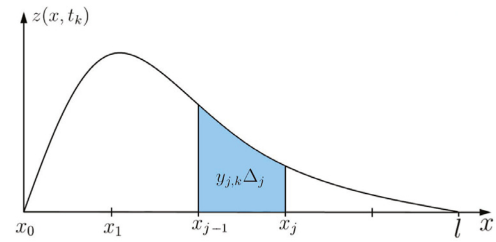

Fig. 7. Sampled in time averaged measurements.

In the setup, the control enters PDEs through shape functions $b_i \in L_2$ that was initiated in Fridman and Bar Am (2013); Fridman and Blighovsky (2012). For simplicity, we consider characteristic shape functions $b_i = \chi_i$. However, the results can be extended to arbitrary b_i such that $\|b_i - \chi_i\|_{L_2}$ is small enough (Selivanov & Fridman, 2017). Note that LKFs for heat equations with fast-varying delays were introduced in Fridman and Orlov (2009), where LMI stability conditions were derived via the descriptor method Fridman (2001) (see Section 4.7.3 of Fridman (2014a)). Sampled-data *point* control is a more challenging problem that is hard to solve using only LKFs. Robustness of state-feedback boundary control with respect to data sampling have been established in Karafyllis and Krstic (2018). The analysis is based on the Fourier series and the input-to-state stability ideas of Karafyllis and Krstic (2016).

6.1. Averaged measurements

Consider the reaction-diffusion PDE

$$\begin{aligned}&z_t(x,t) = z_{xx}(x,t) + az(x,t) + \sum_{j=1}^{N} \chi_j(x)u_j(t),\\ &z(0,t) = z(l,t) = 0\end{aligned} \tag{67}$$

with the state $z : [0, l] \times [0, \infty) \to \mathbb{R}$ and control inputs $u_j : (0, \infty) \to \mathbb{R}$, $j = 1, \dots, N$. The reaction coefficient a may depend on z, x, and t, but is supposed to be uniformly bounded: $|a| \le a_0$ and smooth with respect to all variables. Following (Fridman & Bar Am, 2013), we assume that there are $N+1$ points $0 = x_0 < x_1 < \dots < x_N = l$ dividing the space domain (0, l) into N subdomains (x_{j-1}, x_j). The control enters the system through the characteristic functions

$$\chi_j(x) = \begin{cases} 1, & x \in (x_{j-1}, x_j), \\ 0, & x \notin (x_{j-1}, x_j), \end{cases} \qquad j = 1, \dots, N.$$

The sampled in time averaged measurements (Fig. 7) are given by Bar Am and Fridman (2014); Foias, Mondaini, and Titi (2016); Fridman and Bar Am (2013)

$$y_{j,k} = \frac{1}{\Delta_j}\int_{x_{j-1}}^{x_j} z(\xi, t_k)\, d\xi, \tag{68}$$

where $\Delta_j = |x_j - x_{j-1}|$ are the subdomain sizes and the sampling instants t_k, $k = 0, 1, \dots$, satisfy

$$0 = t_0 < t_1 < t_2 < \cdots, \quad \lim_{k\to\infty} t_k = \infty, \quad t_{k+1} - t_k \le h.$$

Consider the sampled-data output-feedback control

$$u_j(t) = -Ky_{j,k}, \quad t \in [t_k, t_{k+1}). \tag{69}$$

The closed-loop system has the form

$$z_t(x,t) = z_{xx}(x,t) + az(x,t) - K\sum_{j=1}^{N} \chi_j(x) y_{j,k},\ t \in [t_k, t_{k+1}). \quad (70)$$

The existence of a unique classical solution to (70) can be established by applying (Pazy, 1983, Theorem 6.3.3) consecutively on each interval $[0, t_k], [t_k, t_{k+1}], \ldots,$ where the sampled-data terms can be treated as inhomogeneities.

If $-K\sum_{j=1}^{N} \chi_j(x) y_{j,k}$ is replaced by $-Kz(x,t)$, then the system is stable for a large enough K Fridman and Blighovsky (2012). The term $\sum_{j=1}^{N} \chi_j(x) y_{j,k}$ approximates the state $z(x, t)$:

$$\sum_{j=1}^{N} \frac{\chi_j(x)}{\Delta_j} \int_{x_{j-1}}^{x_j} z(\xi, t_k)\, d\xi = z(x,t) - \sigma(x,t) - \kappa(x,t),$$

where, for $x \in (0, l)$ and $t \in [t_k, t_{k+1})$,

$$\kappa(x,t) = \sum_{j=1}^{N} \frac{\chi_j(x)}{\Delta_j} \int_{x_{j-1}}^{x_j} [z(\xi,t) - z(\xi,t_k)] d\xi,$$
$$\sigma(x,t) = z(x,t) - \sum_{j=1}^{N} \frac{\chi_j(x)}{\Delta_j} \int_{x_{j-1}}^{x_j} z(\xi,t)\, d\xi.$$

The function σ represents the error that occurs when the state is approximated using the averaged measurements. It can be bounded using Pioncaré's inequality.

Lemma 10 (Poincaré's inequality Payne and Weinberger (1960)). *Let $f \in \mathcal{H}^1(0,l)$ be such that $\int_0^l f(x)\,dx = 0$. Then*

$$\|f\|_{L_2} \le \frac{l}{\pi}\|f'\|_{L_2}.$$

Since $\int_{x_{j-1}}^{x_j} \sigma = 0$ for $j = 1, \ldots, N$, Lemma 10 implies

$$\|\sigma\|_{L_2} \le \frac{\max \Delta_j}{\pi}\|\sigma_x\|_{L_2} = \frac{\max \Delta_j}{\pi}\|z_x\|_{L_2}.$$

Thus, the upper bound on σ can be made smaller by reducing the maximum subdomain size $\max \Delta_j$.

The error due to sampling κ can be compensated using an appropriate Lyapunov–Krasovskii term, e.g.,

$$\begin{aligned} V_W \ = \ & Wh^2 e^{2\alpha h} \int_{t_k}^{t} e^{-2\alpha(t-s)} \|z_s(\cdot,s)\|_{L_2}^2\, ds \\ & -W\frac{\pi^2}{4} \int_{t_k}^{t} e^{-2\alpha(t-s)} \|\kappa(\cdot,s)\|_{L_2}^2\, ds \end{aligned}$$

for $t \in [t_k, t_{k+1})$ with a scalar $W > 0$. To show that $V_W \ge 0$, note that, since $\kappa(\cdot, t_k) = 0$, Wirtinger inequality (Lemma 5) implies

$$\int_{t_k}^{t} e^{-2\alpha(t-s)} \|\kappa(\cdot,s)\|_{L_2}^2\, ds \le e^{2\alpha h}\frac{4h^2}{\pi^2} \int_{t_k}^{t} e^{-2\alpha(t-s)} \|\kappa_s(\cdot,s)\|_{L_2}^2\, ds.$$

Using Jensen's inequality, we have

$$\begin{aligned} & \int_{t_k}^{t} e^{-2\alpha(t-s)} \|\kappa_s(\cdot,s)\|_{L_2}^2\, ds \\ & = \int_{t_k}^{t} e^{-2\alpha(t-s)} \int_0^l \sum_{j=1}^{N} \frac{\chi(x)}{\Delta_j^2} \left[\int_{x_{j-1}}^{x_j} z_s(\xi,s)\, d\xi\right]^2 dx\, ds \\ & \le \int_{t_k}^{t} e^{-2\alpha(t-s)} \int_0^l \sum_{j=1}^{N} \frac{\chi(x)}{\Delta_j} \int_{x_{j-1}}^{x_j} z_s^2(\xi,s)\, d\xi\, dx\, ds \\ & = \int_{t_k}^{t} e^{-2\alpha(t-s)} \|z_s(\cdot,s)\|_{L_2}^2\, ds. \end{aligned}$$

Thus, $V_w \ge 0$. Calculating the derivative, we obtain

$$\dot V_W + 2\alpha V_W = Wh^2 e^{2\alpha h}\|z_t(\cdot,t)\|_{L_2}^2 - W\frac{\pi^2}{4}\|\kappa(\cdot,t)\|_{L_2}^2.$$

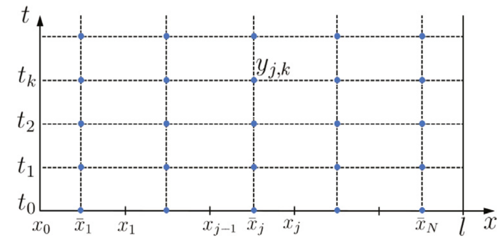

Fig. 8. Sampled in time point measurements.

While the negative term compensates the sampling error κ, the positive term can be made smaller by reducing the sampling period h.

The LMIs guaranteeing the exponential stability of (67)–(69) can be derived using $V = \|z(\cdot,t)\|_{L_2}^2 + p\|z_x(\cdot,t)\|_{L_2}^2 + V_W$. As for ODEs, for PDEs we derive $\dot V + 2\alpha V \le 0$ that, by the comparison principle, implies $V(t) \le e^{-2\alpha t}V(0)$, which guarantees the exponential stability. The ideas described in this subsection were originally presented in Fridman and Bar Am (2013), where the H_∞ control of semilinear 1D diffusion PDE was considered. The approach has been extended to multi-dimensional systems in the presence of Round-Robin protocol Bar Am and Fridman (2014) and to sampled-data relay control Selivanov and Fridman (2017).

6.2. Point measurements

As introduced in Fridman and Blighovsky (2012), consider the system (67) with the point measurements (Fig. 8)

$$y_{j,k} = z(\bar x_j, t_k), \quad \bar x_j = \frac{x_{j-1} + x_j}{2}, \quad j = 1, \ldots, N.$$

Let the control be given by (69), which leads to the closed-loop system (70). The existence of a unique classical solution to (70) with the point measurements can be established by applying (Pazy, 1983, Theorem 6.3.3) consecutively on each interval $[0, t_k], [t_k, t_{k+1}], \ldots,$ where the sampled-data terms can be treated as inhomogeneities.

The term $\sum_{j=1}^{N} \chi_j(x) y_{j,k}$ approximates the state $z(x, t)$:

$$\sum_{j=1}^{N} \chi_j(x) z(\bar x_j, t_k) = z(x,t) - \varsigma(x,t_k) - \varkappa(x,t),$$

where, for $x \in (0, l)$ and $t \in [t_k, t_{k+1})$,

$$\varsigma(x,t_k) = z(x,t_k) - \sum_{j=1}^{N} \chi_j(x) z(\bar x_j, t_k),$$
$$\varkappa(x,t) = z(x,t) - z(x,t_k).$$

The error due to sampling $\varkappa$ can be compensated using the Lyapunov–Krasovskii term similar to V_W from the previous subsection. Since $\varsigma(\cdot, t_k) = 0$, the error due to point measurements ς can be bounded using Wirtinger inequality:

$$\|\varsigma\|_{L_2} \le \frac{\max \Delta_j}{\pi}\|\varsigma_x(\cdot,t_k)\|_{L_2} = \frac{\max \Delta_j}{\pi}\|z_x(\cdot,t_k)\|_{L_2}. \quad (71)$$

To compensate the term $\frac{\max \Delta_j}{\pi}\|z_x(\cdot,t_k)\|_{L_2}$, Halanay's inequality was used in Fridman and Blighovsky (2012).

Lemma 11 (Halanay's inequality Fridman (2014a)). *Let $V : [-h, \infty) \to [0, \infty)$ be absolutely continuous and such that*

$$\dot V(t) + 2\delta_0 V(t) - 2\delta_1 \sup_{-h \le \theta \le 0} V(t+\theta) \le 0, \quad t \ge 0$$

for $0 < \delta_1 < \delta_0$. Then

$$V(t) \le e^{-2\alpha t} \sup_{-h \le \theta \le 0} V(\theta), \quad t \ge 0,$$

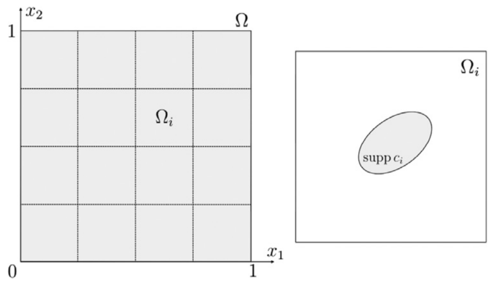

Fig. 9. Subdomains Ω_i and the subset $\operatorname{supp} c_i \subset \overline{\Omega}_i$.

where α *is the solution of* $\alpha = \delta_0 - \delta_1 e^{2\alpha h}$.

If $V(t) = \|z(\cdot,t)\|^2_{L_2} + p\|z_x(\cdot,t)\|^2_{L_2} + V_W(t)$, where V_W is the term compensating the sampling error $\varkappa$, then the term

$$-2\delta_1 \sup_{-h\le\theta\le 0} V(t+\theta) \le -2\delta_1 V(t_k)$$

contains $-2\delta_1 p\|z_x(\cdot,t_k)\|^2_{L_2}$, which can dominate the right-hand side of (71). The LMI-based stability conditions summing up the above ideas can be found in Fridman and Blighovsky (2012), where a nonlinear system was considered.

The ideas of Sections 6.1 and 6.2 have been developed for the event-triggered control (Selivanov & Fridman, 2016a). They can also be used to construct sampled-data observers compensating constant input delays in point actuators (both in-domain and boundary) (Selivanov & Fridman, 2018a). The approach can be used for sampled-data stabilization of the Kuramoto–Sivashinsky equation (Kang & Fridman, 2018).

There is an important difference between the averaged and point measurements. First, since the averaged measurements were analyzed using the direct Lyapunov–Krasovskii approach, these results can be easily extended to H_∞ filtering and control (Fridman & Bar Am, 2013; Fridman & Blighovsky, 2012). Halanay's inequality does not allow for such extension in the case of point measurements. Moreover, while averaged measurements can be easily treated in the case of multi-dimensional domains (Bar Am & Fridman, 2014), point measurements cannot since extension of (71) for higher dimensions contains higher-order derivatives in the right-hand side (cf. (81) below). However, this issue has been overcome in Selivanov and Fridman (2018b) by considering pointlike measurements for 2D heat equations presented in the next subsection.

6.3. Pointlike measurements for 2D domains

Consider the reaction-diffusion system

$$\begin{aligned} &z_t(x,t) = \Delta_D z(x,t) + az(x,t), \quad x\in\Omega,\ t>0,\\ &\quad z|_{\partial\Omega} = 0, \quad z|_{t=0} = z_0 \end{aligned} \tag{72}$$

defined on $\Omega = (0,1)\times(0,1) \subset \mathbb{R}^2$ with the state $z: \overline{\Omega}\times[0,\infty)\to\mathbb{R}$, reaction coefficient a, and diffusion term

$$\Delta_D z = \operatorname{div}(D\nabla z), \quad 0 < D = \begin{bmatrix} d_1 & d_2\\ d_2 & d_3\end{bmatrix} \in \mathbb{R}^{2\times 2}. \tag{73}$$

Let Ω be divided into N square subdomains Ω_i (Fig. 9) with a sensor placed in each Ω_i providing the measurements

$$\begin{aligned} &y_{i,k} = \int_{\Omega_i} c_i(\xi) z(\xi,t_k)\,d\xi,\\ &0 \le c_i \in L_\infty(\Omega_i), \quad \int_{\Omega_i} c_i = 1, \quad i = 1,\dots,N, \end{aligned} \tag{74}$$

where the sampling instants t_k satisfy

$$0 = t_0 < t_1 \cdots, \quad \lim_{k\to\infty} t_k = \infty, \quad t_{k+1} - t_k \le h.$$

For example,

$$c_i(\xi) = \begin{cases} \frac{1}{\varepsilon^2}, & |\xi - x_c^i|_\infty < \frac{\varepsilon}{2},\\ 0, & |\xi - x_c^i|_\infty \ge \frac{\varepsilon}{2} \end{cases} \tag{75}$$

with a small $\varepsilon \in (0, 1/\sqrt{N}]$ model point measurements at $x_c^i \in \Omega_i$. The case of $\varepsilon = 1/\sqrt{N}$ was considered in Bar Am and Fridman (2014).

Consider the sampled-data observer

$$\begin{aligned} &\hat z_t(x,t) = \Delta_D \hat z(x,t) + a\hat z(x,t) + L\sum_{i=1}^N \chi_i(x)\times\\ &\quad \left[y_{i,k} - \textstyle\int_{\Omega_i} c_i(\xi)\hat z(\xi,t_k)\,d\xi\right], \quad t\in[t_k,t_{k+1}),\ k\in\mathbb{N},\\ &\hat z|_{\partial\Omega} = 0, \quad \hat z|_{t=0} = 0 \end{aligned} \tag{76}$$

with the injection gain L and characteristic functions

$$\chi_i(x) = \begin{cases} 1, & x\in\Omega_i,\\ 0, & x\notin\Omega_i,\end{cases} \quad i = 1,\dots,N. \tag{77}$$

The estimation error $\bar z(x,t) = z(x,t) - \hat z(x,t)$ satisfies

$$\begin{aligned} &\bar z_t = \Delta_D \bar z + a\bar z - L\sum_{i=1}^N \chi_i(x)\int_{\Omega_i} c_i(\xi)\bar z(\xi,t_k)\,d\xi,\\ &\qquad\qquad t\in[t_k,t_{k+1}),\\ &\bar z|_{\partial\Omega} = 0, \quad \bar z|_{t=0} = z_0, \end{aligned} \tag{78}$$

The last term approximates the stabilizing feedback $-L\bar z$:

$$-L\sum_{i=1}^N \chi_i(x)\int_{\Omega_i} c_i(\xi)\bar z(\xi,t_k)\,d\xi = -L\bar z + \sigma + \kappa,$$

where

$$\begin{aligned} &\sigma(x,t) = L\bar z(x,t) - L\sum_{i=1}^N \chi_i(x)\int_{\Omega_i} c_i(\xi)\bar z(\xi,t)\,d\xi,\\ &\kappa(x,t) = L\sum_{i=1}^N \chi_i(x)\int_{\Omega_i} c_i(\xi)[\bar z(\xi,t) - \bar z(\xi,t_k)]d\xi. \end{aligned} \tag{79}$$

Using these notations, the system (78) can be presented as

$$\begin{aligned} &\bar z_t = \Delta_D \bar z + (a-L)\bar z + \sigma + \kappa, \quad x\in\Omega,\ t>0,\\ &\bar z|_{\partial\Omega} = 0, \qquad \bar z|_{t=0} = z_0. \end{aligned} \tag{80}$$

If $\sigma\equiv 0$ and $\kappa\equiv 0$, then the system (80) is stable for a large enough injection gain L. If $\Omega = (0,1)$, the error $\sigma\not\equiv 0$ can be bounded using Wirtinger's inequality, which was used in Fridman and Blighovsky (2012) to prove the stability of (80) for large L and N. The following inequality can be used to bound the error σ in the case of $\Omega = (0,1)^2$. This lemma refines (Jones & Titi, 1993, Lemma 4.1).

Lemma 12 (Selivanov & Fridman (2018b)). *Let* $f \in H^2((0,l)^2;\mathbb{R})$, $f(0,0) = 0$. *Then*

$$\begin{aligned} \|f\|^2_{L_2} \le \frac{1}{\alpha_1}\left(\frac{2l}{\pi}\right)^2 \left\|\frac{\partial f}{\partial x_1}\right\|^2_{L_2} + \frac{1}{\alpha_2}\left(\frac{2l}{\pi}\right)^2 \left\|\frac{\partial f}{\partial x_2}\right\|^2_{L_2}\\ + \frac{1}{\alpha_3}\left(\frac{2l}{\pi}\right)^4 \left\|\frac{\partial^2 f}{\partial x_1\partial x_2}\right\|^2_{L_2} \end{aligned} \tag{81}$$

for any positive $\alpha_1, \alpha_2, \alpha_3$ *such that* $\alpha_1+\alpha_2+\alpha_3 = 1$.

Corollary 1. *Let* $f \in H^2((0,l)^2;\mathbb{R})$, $f(0,0) = 0$, $\eta > 0$. *Then*

$$\begin{aligned} \eta\|f\|^2_{L_2} \le \lambda_1\left(\frac{2l}{\pi}\right)^2 \left\|\frac{\partial f}{\partial x_1}\right\|^2_{L_2} + \lambda_2\left(\frac{2l}{\pi}\right)^2 \left\|\frac{\partial f}{\partial x_2}\right\|^2_{L_2}\\ + \lambda_3\left(\frac{2l}{\pi}\right)^4 \left\|\frac{\partial^2 f}{\partial x_1\partial x_2}\right\|^2_{L_2} \end{aligned} \tag{82}$$

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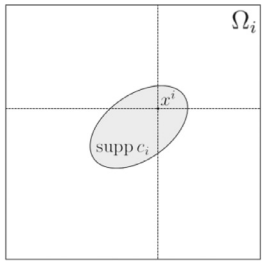

Fig. 10. Four rectangles cornered at $x^i \in \operatorname{supp} c_i$.

for any λ_1, λ_2, λ_3 *satisfying*

$$diag\{\lambda_1, \lambda_2, \lambda_3\} \geq \eta \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}. \tag{83}$$

By the mean value theorem (this idea comes from Wang & Wu (2014)),

$$\int_{\Omega_i} c_i(\xi)\bar{z}(\xi, t)\, d\xi = \bar{z}(x^i(t), t),$$

where $x^i(t)$ belongs to the convex hull of the support of c_i for $t \geq 0$ and $i = 1, \dots, N$. Each rectangle cornered at $x^i \in \operatorname{supp} c_i$ and lying in Ω_i (see Fig. 10) has sides smaller than

$$l = \max_{i=1,\dots,N} \max_{\substack{\omega \in \partial\Omega_i \\ d \in \operatorname{supp} c_i}} |\omega - d|_\infty. \tag{84}$$

Applying Corollary 1 to σ defined in (79) on each of such rectangles and summing over them, we obtain

$$0 \leq -\eta \frac{\|\sigma\|^2_{L_2}}{L^2} + \lambda_1 \left(\frac{2l}{\pi}\right)^2 \|\bar{z}_{x_1}\|^2_{L_2} + \lambda_2 \left(\frac{2l}{\pi}\right)^2 \|\bar{z}_{x_2}\|^2_{L_2} + \lambda_3 \left(\frac{2l}{\pi}\right)^4 \|\bar{z}_{x_1x_2}\|^2_{L_2} \tag{85}$$

with $\eta > 0$, λ_1, λ_2, λ_3 satisfying (83). The positive terms in (85) can be made arbitrarily small by reducing l, i.e., by increasing the number of sensors N. In order to compensate the second order derivative, one has to consider $V = \|\bar{z}\|^2_{L_2} + p\|\bar{z}_x\|^2_{L_2}$.

The error due to sampling κ is compensated by the Wirtinger-based term

$$V_\kappa = p_\kappa e^{2\alpha h} \int_{t_k}^{t} e^{-2\alpha(t-s)} \|\kappa_s(\cdot, s)\|^2_{L_2} ds - \nu \int_{t_k}^{t} e^{-2\alpha(t-s)} \|\kappa(\cdot, s)\|^2_{L_2} ds,$$

which gives

$$\dot{V}_\kappa + 2\alpha V_\kappa = \max_i \|c_i\|_{L_\infty} h^2 e^{2\alpha h} \tilde{p}_\kappa \int_\Omega \bar{z}_t^2(\xi, t)\, d\xi - \nu\|\kappa(\cdot, t)\|^2_{L_2}.$$

Note that the coefficient $\max_i \|c_i\|_{L_\infty} h^2$ makes this approach inapplicable to $c_i = \delta$. Moreover, if ε from (75) gets smaller (i.e., $\max_i \|c_i\|_{L_\infty}$ gets bigger), then the maximum allowable sampling h decreases. The LMI-based convergence conditions combining the above ideas can be found in Selivanov and Fridman (2018b).

If $\Omega = (0, 1)^3$, an upper bound for σ similar to (85) can be derived. This bound will involve the 3rd order space derivative, which we do not know how to compensate. Thus, it is not clear how to extend the proposed method to 3D domains.

7. Conclusions and suggestions for further research

This paper has presented fundamental network-induced issues in NCSs and the main approaches to the modelling of NCSs: discrete-time modeling approach, impulsive system approach and time-delay approach. Recent results on time-delay approach to event-triggered control, modelling and analysis of NCSs under scheduling protocols, and networked control of distributed parameter systems have been surveyed, respectively. This survey has mostly focused on linear systems, where constructive LMI conditions are available. Note that many of the presented ideas can be extended to nonlinear systems (see e.g., Kang & Fridman (2018)).

Although a number of remarkable results on networked control under communication constraints have been achieved in the literature, the following problems are of interest to future research:

- The time-delay approach has been utilized to the modelling and analysis of NCSs under scheduling protocols for the sensor nodes. How to include the consideration of scheduling protocols for the actuator nodes is worthy of further study.
- In Roesch, Roth, and Niculescu (2005), it is revealed that in general the network-induced communication delay is modelled as a gamma distribution model. By direct Lyapunov approach, stability analysis of linear continuous-time systems with gamma-distributed delays Liu, Fridman, Johansson, and Xia (2016a); Solomon and Fridman (2013) and linear discrete-time systems with poisson-distributed delays Liu, Johansson, Fridman, and Xia (2015d) has been studied. In network environments, how to include gamma-distributed or poisson-distributed delays in the model formulation of NCSs under scheduling protocols is significant in theory and in practice.
- Due to the openness of the network, NCSs are more vulnerable to malicious threats such as data tampering, eavesdropping and interception. The issues of network security have attracted ever-increasing attention in recent years, see e.g., Amin, Schwartz, and Sastry (2013); Dolk, Tesi, De Persis, and Heemels (2017); Hendrickx, Johansson, Jungers, Sandberg, and Sou (2014); Mitchell and Chen (2014); Sandberg, Amin, and Johansson (2015); Zhang, Liu, Xia, and Ma (2019) and the references therein. It would be interesting to extend the current frameworks on scheduling protocols to security control of NCSs.
- We have demonstrated that the time-delay approach can be used to study the networked control of PDEs with actuators modeled by shape functions. The analysis of sampled-data control with point actuators is a more challenging problem. The solution may be to combine the time-delay approach with the Fourier series.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.arcontrol.2019.06.005

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