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Exponential input-to-state stability of globally Lipschitz time-delay systems under sampled-data noisy output feedback and actuation disturbances

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ABSTRACT

In this paper we deal with the problem of global exponential practical stability preservation for globally Lipschitz time-delay systems, by Euler emulation of continuous-time dynamic output feedback controllers affected by measurement noises and actuation disturbances. Nonlinear time-delay systems not necessarily affine in the control input are studied. It is shown that, if the continuous-time closed-loop system at hand is globally exponentially stable and the maps describing the plant and the continuous-time dynamic output feedback controller are globally Lipschitz, then, under suitably fast sampling, the Euler emulation of the continuous-time controller at hand preserves the global exponential stability of the sampled-data closed-loop system (no matter whether periodic or aperiodic sampling is used). In the case of bounded measurement noises and bounded actuation disturbances affecting the control law, it is proved that, under suitable fast sampling, (global) exponential input-to-state stability with respect to both these external inputs is guaranteed. A generalisation of the Halanay's inequality is used as a tool in order to prove the results. The existence of a Lyapunov-Krasovskii functional for the continuous-time closed-loop system is sufficient to ensure the preservation of the global exponential practical stability. On the other hand, the explicit knowledge of a Lyapunov-Krasovskii functional allows us to compute an upper bound for the sampling period. An example is presented which validates the theoretical results.

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1. Introduction

Sampled-data stabilisation of linear, bilinear and nonlinear systems, even infinite dimensional ones, has been studied in the literature by many approaches. See, for instance, Briat (2013), Carnevale, Teel, and Nesic (2007), Clarke (2010), Clarke, Ledyaev, Sontag, and Subbotin (1997), Gomes da Silva, Queinnec, Seuret, and Tarbouriech (2016), Di Ferdinando and Pepe (2017), Fridman (2010, 2014), Fridman, Seuret, and Richard (2004), Hespanha (2005), Laila, Nesic, and Teel (2002), Monaco and Normand-Cyrot (2001, 2007), Nesic and Teel (2004), Omran, Hetel, Richard, and Lamnabhi-Lagarrigue (2014), Pepe (2016, 2017), Postoyan, Ahmed-Ali, and Lamnabhi-Lagarrigue (2009), Seuret and Briat (2015). The reader can refer to Hetel et al. (2017) for an interesting survey on the topic. The global (asymptotic, exponential) stability preservation under sampling, for delay-free systems has been extensively studied in Ahmed-Ali, Fridman, Giri, Burlion, and Lamnabhi-Lagarrigue (2016), Burlion, Ahmed-Ali, and Lamnabhi-Lagarrigue (2006), Carnevale et al. (2007), Herrmann, Spurgeon, and Edwards (1999), Hsu and Sastry (1987), Karafyllis and Jiang (2010), Laila, Nesic, and Astolfi (2006), Laila et al. (2002), Mazenc, Malisoff, and Dinh (2013).

On the other hand, as far as the sampled-data control problem of nonlinear time-delay systems is concerned, the results available in the literature are very few. Sufficient Lyapunov-like conditions for the global asymptotic stability preservation under sampling of control-affine time-varying systems with small delays in the input channel are given in Mazenc et al. (2013). Sampled-data stabilisation of finite-dimensional nonlinear systems, with large delays in the input/output channels, is extensively studied in Heemels, Teel, Wouw, and Nesic (2010), Karafyllis and Krstic (2012), Karafyllis, Malisoff, Mazenc, and Pepe (2015), Mattioni, Monaco, and Normand-Cyrot (2017a, 2017b). As far as nonlinear systems with state delays are concerned, in Pepe (2014, 2016, 2017), sufficient conditions, in terms of Lyapunov-Krasovskii functionals, are provided for the semi-global practical stability preservation under sampling, with arbitrarily small final target ball of the origin, in the case of static state feedback controllers. In Di Ferdinando and Pepe (2019), using the same approach of Pepe (2014, 2016, 2017), sufficient Lyapunov-like conditions are provided for the semi-global practical stability preservation by emulation of continuous-time dynamic output feedback controllers, for locally Lipschitz time-delay systems. Results, concerning the global exponential stability preservation under sampling, are provided in Pepe and Fridman (2017) for the class of nonlinear systems with state-delays, described by globally Lipschitz maps and admitting globally Lipschitz static state feedback exponential stabilisers. In Di Ferdinando, Pepe, and Fridman (in press), the input-to-state stability for globally Lipschitz time-delay systems under sampling, with respect to noisy output and actuation disturbances, is studied in the case of static

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state feedbacks. In Di Ferdinando et al. (in press), the implementation problems due to the non-availability in the buffer of some past values of the internal variables are not dealt with (see Remark 8 in Pepe, 2014). In general, some constraints on the sampling period are necessary. In this paper, the input-to-state stability for globally Lipschitz time-delay systems under sampling, with respect to noisy output and actuation disturbances, is studied in the case of dynamic output feedbacks. Here, the above problems, related to the non-availability in the buffer of some past values of the internal variables, are fully overcome.

It is well known that actuation disturbances and observation errors can deteriorate the performances of controllers (see, for instance, Malisoff & Sontag, 2004; Sontag, 1989), and the same, or even worse, kind of problems arises when the control law is applied by sampling and holding (see Ledyaev & Sontag, 1999; Sontag, 1999a, 1999b). In Karafyllis and Kravaris (2009) sufficient conditions ensuring input-to-state stability preservation, with respect to external disturbances, under suitably fast sampling, for nonlinear delay-free systems, are provided. In Pepe (2015), concerning the stabilisation in the sample-and-hold sense of nonlinear control-affine delay-free systems, an input-to-state stability redesign method (for static state feedback controllers) is exploited, in order to attenuate the effects of bounded actuation disturbances and of suitably bounded observation errors. The same approach has been used for nonlinear control-affine time-delay systems in Di Ferdinando and Pepe (2017). To our best knowledge, the problem related to the global practical exponential stability preservation by Euler emulation of continuous-time dynamic output feedback controllers, affected by bounded measurement noises and bounded actuation disturbances, has never been addressed in the literature concerning nonlinear, globally Lipschitz, time-delay systems. In this paper, we fill this gap.

In this paper, under the assumption that the continuous-time closed-loop system at hand is globally exponentially stable and the maps describing the plant and the continuous-time dynamic output feedback controller are globally Lipschitz, the following result is proved: there exists a suitably fast sampling period such that, emulation, by Euler approximation, of the continuous-time dynamic output feedback controller, affected by bounded measurement noises and bounded actuation disturbances, ensures the (global) exponential input-to-state stability of the sampled-data closed-loop system, with respect to both these external inputs (no matter whether periodic or aperiodic sampling is used). We highlight that, differently from the results provided in Pepe and Fridman (2017), here actuation disturbances, observation errors and dynamic output feedback controllers are addressed. We highlight also that, in Di Ferdinando and Pepe (2019), in order to preserve the semi-global practical stability of the sampled-data closed-loop system, the bounds of the actuation disturbances and of the observation errors must be sufficiently small. Moreover, in Di Ferdinando and Pepe (2019), only semi-global practical stability results are provided in the ideal case when actuation-measurement disturbances are zero. Instead, in the present contribution, the global practical (with offset depending on the disturbances and observation errors bounds) exponential stability preservation of the sampled-data closed-loop system is guaranteed for arbitrary actuation disturbances and arbitrary observation errors, as long as bounded, and the global exponential stability, in the case these disturbances and observation errors do not appear, is guaranteed. A linear ISS inequality is provided with respect to actuation disturbances and observation errors bounds. On the other hand, differently from Di Ferdinando and Pepe (2019) (concerning locally Lipschitz systems and semi-global practical stability preservation), the globally Lipschitz property, of the maps describing the plant and the continuous-time dynamic output feedback exponential stabiliser, is here assumed. Globally Lipschitz Time-Delay systems are very frequent in practice. In particular, such systems arise in the study of transport PDEs (e.g. purely convective/first-order hyperbolic PDE dynamics), which are very frequent in industrial practice (e.g. gas flow pipelines, gas-liquid flow in oil production pipes) as well as in theoretical studies (see, for instance, Cai, Liao, Zhang, & Zhang, 2016; Krstic, 2009a, 2009b; Krstic & Bekiaris-Liberis, 2013; Krstic & Karafyllis, 2019; Krstic & Smyshlyaev, 2008). Moreover, differently from Di Ferdinando and Pepe (2019), results concerning the study of other emulation schemes than the Euler ones are not provided. This last point is not a big limitation. Indeed, the Euler approximation is the simplest way to emulate continuous-time dynamic output feedback controllers because of its easy implementation and, for this reason, is the most used in practical engineering applications (see, for instance, Buccella, Cecati, Latafat, Pepe, & Razi, 2015; Di Ferdinando, Pepe, Palumbo, Panunzi, & De Gaetano, n.d.; Ikeda, 2017; Katayama, 2010; Katayama & Aoki, 2014).

Halanay's inequality (see Baker & Buckwar, 2005; Halanay, 1966; Hien, Phat, & Trinh, 2015) and converse Lyapunov theorems (see Karafyllis, Pepe, & Jiang, 2008; Krasovskii, 1963; Pepe & Karafyllis, 2013) are the main tools used to prove the results. As long as the continuous-time closed-loop system at hand is globally exponentially stable, since the existence of a Lyapunov-Krasovskii functional is ensured by the converse theorems, the proof of the existence of a suitably small sampling period does not require its explicit knowledge. If a Lyapunov-Krasovskii functional is explicitly known, then an explicit upper bound on the sampling period that preserves practical stability can be always provided. This upper bound may be conservative, but the results provided here are of the existence type and the provision of a non conservative sampling frequency is beyond the aims of this work. A numerical example is studied which validates the theoretical results.

Notation: $\mathbb{N}$ denotes the set of nonnegative integer numbers, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^\star$ denotes the extended real line $[-\infty, +\infty]$, $\mathbb{R}^+$ denotes the set of nonnegative reals $[0, +\infty)$. The symbol $|\cdot|$ stands for the Euclidean norm of a real vector, or the induced Euclidean norm of a matrix. For a given positive integer n, for a symmetric, positive definite matrix $P \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and the minimum eigenvalue of P, respectively. For a given positive integer n, $\underline{0}$ denotes a vector of all zero in $\mathbb{R}^n$. The essential supremum norm of an essentially bounded function is indicated with the symbol $\|\cdot\|_\infty$. For a positive integer n, for a positive real Δ (maximum involved time-delay): $\mathcal{C}^n$ denotes the space of the continuous functions mapping $[-\Delta, 0]$ into $\mathbb{R}^n$. For a positive real p, for $\phi \in \mathcal{C}^n$, $\mathcal{C}^n_p(\phi) = \{\psi \in \mathcal{C}^n : \|\psi - \phi\|_\infty \le p\}$.

The symbol $\mathcal{C}_p^n$ denotes $\mathcal{C}_p^n(0)$. For a continuous function $x : [-\Delta, c) \to \mathbb{R}^n$, with $0 < c \le +\infty$, for any real $t \in [0, c)$, x_t is the function in $\mathcal{C}^n$ defined as $x_t(\tau) = x(t + \tau)$, $\tau \in [-\Delta, 0]$. For positive integers n, m, for a map $f : \mathcal{C}^n \times \mathbb{R}^m \to \mathbb{R}^n$, and for a globally Lipschitz functional $V : \mathcal{C}^n \to \mathbb{R}^+$, $D^+V : \mathcal{C}^n \times \mathbb{R}^m \to \mathbb{R}^\star$ denotes the derivative in Driver's form of V, defined, for $\phi \in \mathcal{C}^n$, $u \in \mathbb{R}^m$, as follows (see Pepe, 2007) $D^+V(\phi, u) = \limsup_{h\to 0^+} \frac{V(\phi_{h,u})-V(\phi)}{h}$, where, for $0 \le h < \Delta$, $\phi_{h,u} \in \mathcal{C}^n$ is defined, for $s \in [-\Delta, 0]$, as

$$\phi_{h,u}(s) = \begin{cases} \phi(s+h), & s \in [-\Delta, -h), \\ \phi(0) + (s+h) f(\phi, u), & s \in [-h, 0]. \end{cases}$$

Throughout the paper, GES stands for *globally exponentially stable* or *global exponential stability*, RFDE stands for *retarded functional differential equation*.

2. Preliminaries and problem statement

Let us consider a nonlinear time-delay system (the plant), described by the following RFDE (see Hale & Lunel, 1993; Kolmanovskii & Myshkis, 1999)

$$\begin{aligned} \dot{x}(t) &= f(x_t, u(t)), \quad t \ge 0 \text{ a.e.}, \\ y(t) &= h(x_t), \\ x(\tau) &= x_0(\tau), \quad \tau \in [-\Delta, 0], \end{aligned} \tag{1}$$

where: $x(t) \in \mathbb{R}^n$; n is a positive integer; $x_0, x_t \in \mathcal{C}^n$; $\Delta > 0$ is the maximum involved time delay, which is assumed to be known; $u(t) \in \mathbb{R}^m$ is the input (the input signal is Lebesgue measurable and locally essentially bounded); m is a positive integer; $y(t) \in \mathbb{R}^q$ is the output; q is a positive integer; f is a map from $\mathcal{C}^n \times \mathbb{R}^m$ to $\mathbb{R}^n$; h is a map from $\mathcal{C}^n$ to $\mathbb{R}^q$. It is assumed that $f(0, 0) = h(0) = 0$ (regularity of the maps f and h will be established in forthcoming Assumption 2.1).

The following lemma is a particular case of Theorem 3.2 in Hien et al. (2015), as here needed for the forthcoming practical stability analysis. Theorem 3.2 in Hien et al. (2015) extends the Halanay's inequality (see Baker & Buckwar, 2005; Halanay, 1966) to a more general case.

Lemma 2.1 (see Theorem 3.2 in Hien et al., 2015): *Let a, b, γ, r be positive reals, $a > b$. Let $z : [-r, +\infty) \to \mathbb{R}^+$ be a continuous function satisfying the inequality $D^+z(t) \le -az(t) + b\sup_{\theta\in[-r,0]} z(t+\theta) + \gamma$, where $D^+z(t) = \limsup_{h\to 0^+} \frac{z(t+h)-z(t)}{h}$ is the upper right-hand Dini derivative of the function z. Let λ be the positive real solution of the scalar equation (with unknown variable $\bar{\lambda}$) $H(\bar{\lambda}) = \bar{\lambda} - 1 + \frac{b}{a} e^{\bar{\lambda}ar} = 0$. Let $\sigma = a - b$ and $N = e^{\lambda ar}$. Then the inequality $z(t) \le \sup_{\theta\in[-r,0]} z(\theta) N e^{-\lambda at} + \frac{\gamma}{\sigma}$ holds for any $t \ge 0$.*

Let us consider a dynamic output feedback controller for the nonlinear time-delay system (1), described by the following equations (see Ciccarella, Mora, & Germani, 1995; Germani, Manes, & Pepe, 2001, 2012)

$$\begin{aligned} \dot{\hat{x}}(t) &= \widehat{f}(\hat{x}_t, u(t), y(t)), \quad t \ge 0, \\ u(t) &= k(\hat{x}_t, y(t)), \\ \hat{x}(\tau) &= \hat{x}_0(\tau), \quad \tau \in [-\Delta, 0], \end{aligned} \tag{2}$$

where: $\hat{x}(t) \in \mathbb{R}^n$; $\hat{x}_0, \hat{x}_t \in \mathcal{C}^n$; Δ is the maximum involved time delay as in (1); $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^q$ are the input and the output as defined in (1), respectively; $\widehat{f}$ is a map from $\mathcal{C}^n \times \mathbb{R}^m \times \mathbb{R}^q$ to $\mathbb{R}^n$; k is a map from $\mathcal{C}^n \times \mathbb{R}^q$ to $\mathbb{R}^m$; it is assumed that $\widehat{f}(0, 0, 0) = k(0, 0) = 0$.

Assumption 2.1: *The maps f, h, $\widehat{f}$ and k in (1), (2) are globally Lipschitz. The continuous-time closed-loop system described by the RFDEs (see (1), (2))*

$$\begin{aligned} \dot{x}(t) &= f(x_t, k(\hat{x}_t, h(x_t))), \quad t \ge 0, \\ \dot{\hat{x}}(t) &= \widehat{f}(\hat{x}_t, k(\hat{x}_t, h(x_t)), h(x_t)), \\ x(\tau) &= x_0(\tau), \quad \hat{x}(\tau) = \hat{x}_0(\tau), \quad \tau \in [-\Delta, 0], \end{aligned} \tag{3}$$

is 0-GES.

We recall here the notion of partition of $[0, +\infty)$ (see Clarke et al., 1997; Pepe, 2017).

Definition 2.2: A partition $\pi = \{t_i, \ i = 0, 1, \ldots\}$ of $[0, +\infty)$ is a countable, strictly increasing sequence t_i, with $t_0 = 0$, such that $t_i \to +\infty$ as $i \to +\infty$. The diameter of π, denoted $diam(\pi)$, is defined as $\sup_{i\ge 0} t_{i+1} - t_i$. The dwell time of π, denoted $dwell(\pi)$, is defined as $\inf_{i\ge 0} t_{i+1} - t_i$. For any positive real $a \in (0, 1]$, $b > 0$, $\pi_{a,b}$ is any partition π with $ab \le dwell(\pi) \le diam(\pi) \le b$.

The real $a \in (0, 1]$, in Definition 2.2, is introduced in order to allow non-uniform sampling in sampled-data stabilisation. The problem that we want to address in this paper is the following one.

Problem 2.1: If Assumption 2.1 holds, prove the existence of a sufficiently small sampling period δ and of positive reals λ^*, r_1, r_2, r_3 (and eventually find them) such that: for any bounded sequence $d : \mathbb{N} \to \mathbb{R}^m$ (actuation disturbance), for any bounded sequence $e : \mathbb{N} \to \mathbb{R}^q$ (measurement noise), for any initial state x_0, $\hat{x}_0 \in \mathcal{C}^n$ and for any partition $\pi_{a,\delta}$ the solution of the sampled-data closed-loop system described by (1), with Euler emulated control input (see Di Ferdinando & Pepe, 2019)

$$u(t) = k(\hat{x}_{t_j}, h(x_{t_j}) + e_j) + d_j,$$

$$\hat{x}(t_{j+1}) = \hat{x}(t_j) + (t_{j+1} - t_j)\widehat{f}\left(\hat{x}_{t_j}, k(\hat{x}_{t_j}, h(x_{t_j}) + e_j), h(x_{t_j}) + e_j\right),$$

$$\hat{x}_{t_j}(\theta) = \begin{cases} \hat{x}_0(t_j + \theta), & t_j + \theta \le 0, \\ \hat{x}(t_k) + \dfrac{t_j + \theta - t_k}{t_{k+1} - t_k}(\hat{x}(t_{k+1}) - \hat{x}(t_k)), & t_j + \theta > 0, \\ k = \arg\max_{l\in N}\{t_l \in \pi_{a,\delta} : t_l \le t_j + \theta\}, & \end{cases}$$

$$\theta \in [-\Delta, 0], \quad t \in \left[t_j, t_{j+1}\right), \quad t_j \in \pi_{a,\delta}, \quad j = 0, 1, \dots, \tag{4}$$

exists for all $t \in \mathbb{R}^+$, and, furthermore, satisfies

$$\left\| \begin{bmatrix} x(t) \\ \hat{x}(t_j) \end{bmatrix} \right\| \le r_1 \left\| \begin{bmatrix} x_0 \\ \hat{x}_0 \end{bmatrix} \right\|_\infty \mathrm{e}^{-\lambda^* t} + r_2 \bar{d} + r_3 \bar{e}, \quad t \in [t_j, t_{j+1}],\ j = 0, 1, \dots, \tag{5}$$

where $\bar{d}$, $\bar{e}$ are the positive reals such that $|d_j| \le \bar{d}$, $|e_j| \le \bar{e}$, $j = 0, 1, 2, \dots$.

Remark 2.1: The continuous-time dynamic output feedback controller (2) is here discretised by the Euler method, and the control input is applied by means of a zero-order hold device (see (4)). The Euler approximation of (2) leads to a continuous, piece-wise linear, approximated solution $\hat{x}(t)$, and thus, at each t_j, $\hat{x}_{t_j}$ is described by (4). Notice that, in (4), when $t_j + \theta \le 0$, $\hat{x}_{t_j}(\theta)$ is a point-wise value of the controller initial state, and when $t_j + \theta > 0$, $\hat{x}_{t_j}(\theta)$ is a point-wise value of the linear interpolation between $\hat{x}(t_k)$ and $\hat{x}(t_{k+1})$, with $t_k \le t_j + \theta < t_{k+1}$.

Clearly, the dynamic output feedback sampled-data controller (4) does not have causality problems. On the other hand, possible time-delays, due to computation, are not taken into account here.

Firstly, in order to cope with Problem 2.1, by the same reasoning as in Laila et al. (2002) (see Remark 2.3 in Laila et al., 2002), taking into account (1) and (2), let us consider the open-loop system described by the following RFDEs

$$\begin{aligned} &\dot{x}(t) = f\left(x_t, \tilde{u}_1(t)\right), \\ &\dot{\hat{x}}(t) = \tilde{u}_2(t), \quad t \ge 0 \text{ a.e.}, \\ &y(t) = h(x_t), \\ &x(\tau) = x_0(\tau), \quad \hat{x}(\tau) = \hat{x}_0(\tau), \quad \tau \in [-\Delta, 0], \end{aligned} \tag{6}$$

where: x_0, $\hat{x}_0 \in \mathcal{C}^n$ are the initial states in (1) and (2); x_t, $\hat{x}_t \in \mathcal{C}^n$; $x(t), \hat{x}(t) \in \mathbb{R}^n$; f is the map in (1); $\tilde{u}_1(t) = u(t) \in \mathbb{R}^m$ is the input in (1); $\tilde{u}_2(t) \in \mathbb{R}^n$ is a new input (Lebesgue measurable and locally essentially bounded); $y(t) \in \mathbb{R}^q$ is the output in (1); h is the map in (1). Let (as long as the solution of (6) exists)

$$\begin{aligned} &\tilde{x}(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} \in \mathbb{R}^{2n}, \quad \tilde{x}_t = \begin{bmatrix} x_t \\ \hat{x}_t \end{bmatrix} \in \mathcal{C}^{2n}, \\ &\tilde{u}(t) = \begin{bmatrix} \tilde{u}_1(t) \\ \tilde{u}_2(t) \end{bmatrix} \in \mathbb{R}^{m+n}. \end{aligned} \tag{7}$$

By (7), system (6) can be rewritten as follows

$$\begin{aligned} &\dot{\tilde{x}}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} f\left(x_t, \tilde{u}_1(t)\right) \\ \tilde{u}_2(t) \end{bmatrix} = F\left(\tilde{x}_t, \tilde{u}(t)\right), \\ &\tilde{x}(\tau) = \tilde{x}_0(\tau) = \begin{bmatrix} x_0(\tau) \\ \hat{x}_0(\tau) \end{bmatrix}, \quad \tau \in [-\Delta, 0], \end{aligned} \tag{8}$$

where the map $F : \mathcal{C}^{2n} \times \mathbb{R}^{m+n} \to \mathbb{R}^{2n}$ is readily defined by (8). By Assumption 2.1, the map F is globally Lipschitz and $F(0, 0) = 0$. Taking into account the dynamic output feedback controller (2), let $K : \mathcal{C}^{2n} \times \mathbb{R}^q \to \mathbb{R}^{m+n}$ be the map defined, for all $\tilde{\phi} = \begin{bmatrix} \phi \\ \hat{\phi} \end{bmatrix} \in \mathcal{C}^{2n}$, $\phi, \hat{\phi} \in \mathcal{C}^n$ and for all $e \in \mathbb{R}^q$, as

$$\begin{bmatrix} k(\hat{\phi}, h(\phi) + e) \\ \widehat{f}(\hat{\phi}, k(\hat{\phi}, h(\phi) + e), h(\phi) + e) \end{bmatrix} = K(\tilde{\phi}, e), \tag{9}$$

where: $\widehat{f}$ and k are the maps in (2); h is the map in (1). By Assumption 2.1 the map K is globally Lipschitz and satisfies $K(0, 0) = 0$. Taking into account (6), (7), (8), (9), the continuous-time closed-loop system (3) can be rewritten as follows

$$\begin{aligned} &\dot{\tilde{x}}(t) = F(\tilde{x}_t, K(\tilde{x}_t, 0)), \\ &\tilde{x}(\tau) = \tilde{x}_0(\tau) = \begin{bmatrix} x_0(\tau) \\ \hat{x}_0(\tau) \end{bmatrix}, \quad \tau \in [-\Delta, 0]. \end{aligned} \tag{10}$$

From Assumption 2.1, it follows that the continuous-time closed-loop system described by (10) is 0-GES.

The following lemma, concerning a standard Lyapunov converse result (see Karafyllis et al., 2008; Krasovskii, 1963; Pepe & Karafyllis, 2013), holds for the system described by (10).

Lemma 2.3: *Let Assumption* 2.1 *hold. Then, there exist a globally Lipschitz functional $V : \mathcal{C}^{2n} \to \mathbb{R}^+$, with L_V as Lipschitz constant, and positive reals α_i, $i = 1, 2, 3$, such that the following inequalities hold for any $\phi \in \mathcal{C}^{2n}$*

$$\begin{aligned} &\alpha_1 \|\phi\|_\infty \le V(\phi) \le \alpha_2 \|\phi\|_\infty, \\ &D^+ V(\phi, K(\phi, 0)) \le -\alpha_3 \|\phi\|_\infty. \end{aligned} \tag{11}$$

3. Main results

The main result of the paper is given by next Theorem 3.1 which solves Problem 2.1. The proof follows the lines of the ones in Pepe and Fridman (2016, see Theorem 8), Pepe and Fridman (2017, see Theorem 5).

Theorem 3.1: *Let Assumption* 2.1 *hold. Let α_1, α_2, α_3 and L_V be the positive reals provided in Lemma* 2.3 *for the closed-loop system described by* (10) *(see also* (3)*). Let L_F, L_K be the Lipschitz constants related to the maps F and K, respectively (see* (8), (9)*). Let $a \in (0, 1]$. Let δ be a positive real satisfying*

$$\delta < \min\left\{\Delta, \frac{\alpha_1 \alpha_3}{\alpha_2 L_V L_F^2 L_K (1 + L_K)}\right\}. \tag{12}$$

Let λ be the positive real solution of the scalar equation

$$\bar{\lambda} - 1 + \frac{\alpha_2 L_V L_F^2 L_K (1 + L_K)}{\alpha_1 \alpha_3} \delta\, \mathrm{e}^{\frac{\alpha_3 \bar{\lambda} (\Delta + 2\delta)}{\alpha_2}} = 0. \tag{13}$$

Let l be the positive integer such that

$$la\delta \le \Delta < (l + 1)a\delta. \tag{14}$$

Let

$$\lambda^* = \frac{\alpha_3}{\alpha_2} \lambda. \tag{15}$$

Let r_1, r_2, r_3, be the following positive reals

$$r_1 = \frac{\alpha_2}{\alpha_1}(2 + L_F L_K \delta)^{l+1} \, \mathrm{e}^{(L_F + 2\lambda^*)(\Delta + 2\delta)},$$
$$r_2 = r_1 L_F \delta + \frac{\alpha_2 L_V L_F (1 + L_F L_K \delta)}{\alpha_1 \alpha_3 - \alpha_2 L_V L_F^2 L_K (1 + L_K) \delta}, \qquad (16)$$
$$r_3 = r_2 L_K.$$

Then, for any partition $\pi_{a,\delta}$ (see Definition 2.2), for any initial state $x_0, \hat{x}_0 \in \mathcal{C}^n$, for any sequence $e : \mathbb{N} \to \mathbb{R}^q$ (measurement noise) such that, for some positive real $\bar{e}$, $|e_j| \le \bar{e}$, $j = 0, 1, 2, \ldots$, for any sequence $d : \mathbb{N} \to \mathbb{R}^m$ (actuation disturbance) such that, for some positive real $\bar{d}$, $|d_j| \le \bar{d}$, $j = 0, 1, 2, \ldots$, the solution of the sampled-data closed-loop system described by (see (1), (4))

$$\dot{x}(t) = f(x_t, k(\hat{x}_{t_j}, h(x_{t_j}) + e_j) + d_j),$$
$$\hat{x}(t_{j+1}) = \hat{x}(t_j) + (t_{j+1} - t_j)\widehat{f}(\hat{x}_{t_j}, k(\hat{x}_{t_j}, h(x_{t_j}) + e_j), h(x_{t_j}) + e_j), \qquad (1)$$

$$\hat{x}_{t_j}(\theta) = \begin{cases} \hat{x}_0(t_j + \theta), & t_j + \theta \le 0, \\ \hat{x}(t_k) + \dfrac{t_j + \theta - t_k}{t_{k+1} - t_k}(\hat{x}(t_{k+1}) - \hat{x}(t_k)), & t_j + \theta > 0, \\ k = \arg\max\limits_{l \in N}\{t_l \in \pi_{a,\delta} : t_l \le t_j + \theta\}, & \end{cases}$$
$$\theta \in [-\Delta, 0], \quad t \in [t_j, t_{j+1}), \quad t_j \in \pi_{a,\delta}, \quad j = 0, 1, \ldots,$$
$$x(\tau) = x_0(\tau), \quad \hat{x}(\tau) = \hat{x}_0(\tau), \quad \tau \in [-\Delta, 0], \qquad (17)$$

exists for all $t \ge 0$ and, furthermore, satisfies (5).

Proof: In the following, the structure of the proofs used in Pepe and Fridman (2016, see Theorem 8), Pepe and Fridman (2017, see Theorem 5) is suitably adapted in order to cope with sampled-data dynamic output feedback controllers affected by actuation disturbances and measurement noises. Let us consider the system described by (8) with (as long as the related solution exists)

$$\tilde{u}(t) = \begin{bmatrix} k(\hat{x}_{t_j}, h(x_{t_j}) + e_j) \\ \widehat{f}(\hat{x}_{t_j}, k(\hat{x}_{t_j}, h(x_{t_j}) + e_j), h(x_{t_j}) + e_j) \end{bmatrix} + \begin{bmatrix} d_j \\ \underline{0} \end{bmatrix} = K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j, \qquad (18)$$
$$t_j \le t < t_{j+1}, \quad t_j \in \pi_{a,\delta}, \quad j = 0, 1, \ldots,$$

where the sequence $\tilde{d} : \mathbb{N} \to \mathbb{R}^{m+n}$ is readily defined by (18) with $d_j \in \mathbb{R}^m$ (the actuation disturbance) and $\underline{0} \in \mathbb{R}^n$. Notice that $|\tilde{d}_j| \le \bar{d}$, $j = 0, 1, 2, \ldots$. From the global Lipschitz property of the map F and the strictly increasing property of the partition $\pi_{a,\delta}$, it follows that the closed-loop system described by (8), (18) admits a unique locally absolutely continuous solution in $\mathbb{R}^+$ (see Hale & Lunel, 1993). Let $\tilde{x}_t \in \mathcal{C}^{2n}$ be the solution of the closed-loop system (8), (18). Let p be the positive integer such that, for the partition at hand, $t_p \le \Delta < t_{p+1}$. Notice that, by (14), $p \le l$. Firstly, let us consider the intervals $[t_j, t_{j+1}]$, $j = 0, 1, \ldots, p$. For $t \in [0, t_1]$, we have

$$\begin{aligned} \|\tilde{x}_t\|_\infty &\le \|\tilde{x}_0\|_\infty + \sup_{\theta \in [-\Delta, 0], \ t+\theta \ge 0} \left(\|\tilde{x}_0\|_\infty + \int_0^{t+\theta} |F(\tilde{x}_\tau, K(\tilde{x}_0, e_0) + \tilde{d}_0)| \, \mathrm{d}\tau \right) \\ &\le 2\|\tilde{x}_0\|_\infty + \sup_{\theta \in [-\Delta, 0], \ t+\theta \ge 0} \int_0^{t+\theta} L_F(\|\tilde{x}_\tau\|_\infty + L_K\|\tilde{x}_0\|_\infty + L_K|e_0| + |\tilde{d}_0|) \, \mathrm{d}\tau \\ &\le (2 + L_F L_K \delta)\|\tilde{x}_0\|_\infty + L_F L_K \delta \bar{e} + L_F \delta \bar{d} + \int_0^t L_F \|\tilde{x}_\tau\|_\infty \, \mathrm{d}\tau. \end{aligned} \qquad (19)$$

Let

$$c_1 = 2 + L_F L_K \delta, \quad c_2 = L_F L_K \delta \bar{e} + L_F \delta \bar{d}. \qquad (20)$$

By the Gronwall-Bellman Lemma (see Lemma A.1, pp. 651–652, in Khalil, 2000) and taking into account (20), it follows from (19) that, for $t \in [0, t_1]$, the following inequality holds $\|\tilde{x}_t\|_\infty \le c_1 \, \mathrm{e}^{L_F t}\|\tilde{x}_0\|_\infty + c_2 \, \mathrm{e}^{L_F t}$. Taking into account that, for any nonnegative integer y, the inequalities $\sum_{i=0}^{y} c_1^i \le c_1^{y+1}$, $\|\tilde{x}_t\|_\infty \le c_1 \, \mathrm{e}^{L_F(t - t_y)}\|\tilde{x}_{t_y}\|_\infty + c_2 \, \mathrm{e}^{L_F(t - t_y)}$, $t \in [t_y, t_{y+1}]$ hold, by an induction reasoning it follows that, for $t \in [t_j, t_{j+1}]$, $j = 0, 1, 2, \ldots, p$, the inequality holds

$$\|\tilde{x}_t\|_\infty \le c_1^{j+1} \, \mathrm{e}^{L_F t}\|\tilde{x}_0\|_\infty + c_1^{j+1} c_2 \, \mathrm{e}^{L_F t}. \qquad (21)$$

From (21), recalling that $p \le l$ and taking into account (20), for any $t \in [0, t_{p+1}]$ the following inequality/equality holds

$$\begin{aligned} \|\tilde{x}_t\|_\infty &\le c_1^{l+1} \, \mathrm{e}^{L_F(\Delta + 2\delta)}\|\tilde{x}_0\|_\infty + c_1^{l+1} c_2 \, \mathrm{e}^{L_F(\Delta + 2\delta)} \\ &= (2 + L_F L_K \delta)^{l+1} \, \mathrm{e}^{L_F(\Delta + 2\delta)}\|\tilde{x}_0\|_\infty \\ &\quad + (2 + L_F L_K \delta)^{l+1}(L_F L_K \delta \bar{e} + L_F \delta \bar{d}) \, \mathrm{e}^{L_F(\Delta + 2\delta)}. \end{aligned} \qquad (22)$$

Now, let $w : \mathbb{R}^+ \to \mathbb{R}^+$ be the continuous function defined, for $t \in \mathbb{R}^+$, as $w(t) = V(\tilde{x}_t)$, with V the functional provided in Lemma 2.3 (for the system described by (10)). The following equalities hold for $t_j \le t < t_{j+1}$, $j = p+1, p+2, \ldots$,

$$\begin{aligned} D^+ w(t) &= \limsup_{h \to 0^+} \frac{1}{h}(V(\tilde{x}_{t+h}) - V(\tilde{x}_t)) \\ &= \limsup_{h \to 0^+} \frac{1}{h}(V(\tilde{x}_{t+h}) - V((\tilde{x}_t)_{h, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j}) \\ &\quad + V((\tilde{x}_t)_{h, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j}) - V(\tilde{x}_t)), \end{aligned} \qquad (23)$$

where, for $t \ge 0$, and any $v \in \mathbb{R}^{m+n}$, $(\tilde{x}_t)_{h,v} \in \mathcal{C}^{2n}$ is defined (see the Notation section), for $h \in [0, \Delta)$, as

$$(\tilde{x}_t)_{h,v}(\theta) = \begin{cases} \tilde{x}_t(\theta + h), & \theta \in [-\Delta, -h), \\ \tilde{x}_t(0) + (\theta + h) F(\tilde{x}_t, v), & \theta \in [-h, 0]. \end{cases}$$

Now, for any positive real $h < \min\{t_{j+1} - t_j, \Delta\}$, the following equalities/inequalities hold (see Driver, 1962; Pepe, 2007)

$$\begin{aligned}
&\frac{1}{h}|V(\tilde{x}_{t+h}) - V((\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j})| \\
&\quad \le \frac{L_V}{h}\|\tilde{x}_{t+h} - (\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j}\|_\infty \\
&\quad = \frac{L_V}{h}\sup_{\theta\in[-\Delta,0]}|\tilde{x}_{t+h}(\theta) - (\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j}(\theta)| \\
&\quad = \frac{L_V}{h}\sup_{\theta\in[-h,0]}|\tilde{x}(t+h+\theta) - \tilde{x}(t) \\
&\quad - (\theta+h)F(\tilde{x}_t, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)| \\
&\quad \le \frac{L_V}{h}\sup_{\theta\in[-h,0]}\left|\tilde{x}(t) + \int_t^{t+h+\theta} F(\tilde{x}_\tau, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)\,\mathrm{d}\tau \right. \\
&\qquad \left. -\tilde{x}(t) - (\theta+h)F(\tilde{x}_t, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)\right| \\
&\quad = \frac{L_V}{h}\sup_{\theta\in(-h,0]}\left|(\theta+h)\left(\frac{1}{(\theta+h)}\int_t^{t+h+\theta}\right.\right. \\
&\quad F(\tilde{x}_\tau, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)\,\mathrm{d}\tau \\
&\qquad \left.\left. -F(\tilde{x}_t, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)\right)\right| \\
&\quad \le L_V\sup_{\theta\in(-h,0]}\left|\left(\frac{1}{(\theta+h)}\int_t^{t+h+\theta} F(\tilde{x}_\tau, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)\,\mathrm{d}\tau\right.\right. \\
&\quad \left.\left. -F(\tilde{x}_t, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)\right)\right|.
\end{aligned} \tag{24}$$

From (24), taking into account of the continuity of the map F and of the solution $\tilde{x}_\tau \in \mathcal{C}^{2n}$, $\tau \in \mathbb{R}^+$ (see Lemma 2.1, p. 40, in Hale & Lunel, 1993) the limit follows

$$\limsup_{h\to0^+}\frac{1}{h}(V(\tilde{x}_{t+h}) - V((\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j})) = 0. \tag{25}$$

From (23), (25) and by the use of Lemma 2.3 we obtain

$$\begin{aligned}
D^+w(t) &= \limsup_{h\to0^+}\frac{1}{h}(V((\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j}) - V(\tilde{x}_t)) \\
&= \limsup_{h\to0^+}\frac{1}{h}(V((\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j}) - V((\tilde{x}_t)_{h,K(\tilde{x}_t,0)}) \\
&\quad + V((\tilde{x}_t)_{h,K(\tilde{x}_t,0)}) - V(\tilde{x}_t)) \\
&\le \limsup_{h\to0^+}\frac{1}{h}(V((\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j}) - V((\tilde{x}_t)_{h,K(\tilde{x}_t,0)})) \\
&\quad + \limsup_{h\to0^+}\frac{1}{h}(V((\tilde{x}_t)_{h,K(\tilde{x}_t,0)}) - V(\tilde{x}_t)) \\
&\le -\alpha_3\|\tilde{x}_t\|_\infty + \limsup_{h\to0^+}\frac{1}{h}(V((\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j}) \\
&\quad - V((\tilde{x}_t)_{h,K(\tilde{x}_t,0)})).
\end{aligned} \tag{26}$$

Moreover, we have

$$\begin{aligned}
&\limsup_{h\to0^+}\frac{1}{h}|V((\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j}) - V((\tilde{x}_t)_{h,K(\tilde{x}_t,0)})| \\
&\quad \le \limsup_{h\to0^+}\frac{L_V}{h}\left(\sup_{\theta\in[-\Delta,0]}|(\tilde{x}_t)_{h,K(\tilde{x}_{t_j},e_j)+\tilde{d}_j}(\theta)\right. \\
&\quad \left. -(\tilde{x}_t)_{h,K(\tilde{x}_t,0)}(\theta)|\right) \\
&\quad = \limsup_{h\to0^+}\frac{L_V}{h}\left(\sup_{\theta\in[-h,0]}|\tilde{x}(t) + (\theta+h)F(\tilde{x}_t, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)\right. \\
&\qquad \left. -\tilde{x}(t) - (\theta+h)F(\tilde{x}_t, K(\tilde{x}_t, 0))|\right) \\
&\quad = \limsup_{h\to0^+}\frac{L_V}{h}\left(\sup_{\theta\in[-h,0]}(\theta+h)|F(\tilde{x}_t, K(\tilde{x}_{t_j}, e_j) + \tilde{d}_j)\right. \\
&\quad \left. -F(\tilde{x}_t, K(\tilde{x}_t, 0))|\right) \\
&\quad \le L_VL_F(L_K\|\tilde{x}_{t_j} - \tilde{x}_t\|_\infty + L_K\|e_j\|_\infty + |\tilde{d}_j|) \\
&\quad \le L_VL_F(L_K\|\tilde{x}_{t_j} - \tilde{x}_t\|_\infty + L_K\bar{e} + \bar{d}).
\end{aligned} \tag{27}$$

From (26), taking into account (27), the following inequality holds

$$D^+w(t) \le -\alpha_3\|\tilde{x}_t\|_\infty + L_VL_F(L_K\|\tilde{x}_{t_j} - \tilde{x}_t\|_\infty + L_K\bar{e} + \bar{d}). \tag{28}$$

Now, let us consider any $j \ge p+1$. Let $\hat{t}_1 = \max_{i\in\mathbb{N}}\{t_i \in \pi_{a,\delta} : t_i \le t_j - \Delta\}$ and let $\hat{t}_2, \hat{t}_3, \ldots, \hat{t}_s$, s positive integer, be the set of all sampling times, in increasing order, in the interval $(t_j - \Delta, t_j]$. Taking into account (14) and since $\hat{t}_s = t_j$ it must be $2 \le s \le l+2$. Then, for $t_j \le t < t_{j+1}$, we have

$$\begin{aligned}
\|\tilde{x}_{t_j} - \tilde{x}_t\|_\infty &= \sup_{\theta\in[-\Delta,0]}|\tilde{x}(t_j+\theta) - \tilde{x}(t+\theta)| \\
&\le \sup_{q=1,2,\ldots,s-1}\ \sup_{\theta\in[\hat{t}_q-t_j,\hat{t}_{q+1}-t_j]}|\tilde{x}(t_j+\theta) - \tilde{x}(t+\theta)| \\
&\le \sup_{q=1,2,\ldots,s-1}\ \sup_{\theta\in[\hat{t}_q-t_j,\hat{t}_{q+1}-t_j]}\left|\int_{t_j+\theta}^{t+\theta} L_F\right. \\
&\qquad \left.\left((1+L_K)\sup_{\beta\in[t_j+\theta,t+\theta]}\|\tilde{x}_\beta\|_\infty + L_K\bar{e} + \bar{d}\right)\mathrm{d}\tau\right| \\
&\le \sup_{q=1,2,\ldots,s-1}\ \sup_{\theta\in[\hat{t}_q-t_j,\hat{t}_{q+1}-t_j]} L_F(1+L_K)\delta \\
&\qquad \sup_{\beta\in[t_j+\theta,t+\theta]}\|\tilde{x}_\beta\|_\infty + L_FL_K\delta\bar{e} + L_F\delta\bar{d} \\
&\le L_F(1+L_K)\delta\sup_{\beta\in[\hat{t}_1,t]}\|\tilde{x}_\beta\|_\infty + L_FL_K\delta\bar{e} + L_F\delta\bar{d} \\
&\le \frac{L_F(1+L_K)\delta}{\alpha_1}\sup_{\beta\in[\hat{t}_1,t]}w(\beta) + L_FL_K\delta\bar{e} + L_F\delta\bar{d}.
\end{aligned} \tag{29}$$

From (28), taking into account (29) and setting $w(\theta) = w(0)$ for $\theta \in [-2\delta, 0]$, by the use of Lemma 2.3, the following inequality holds for all $t \geq t_{p+1}$

$$D^+ w(t) \leq -\frac{\alpha_3}{\alpha_2} w(t) + \frac{L_V L_F^2 L_K (1+L_K)\delta}{\alpha_1} \sup_{\theta \in [-\Delta-2\delta,0]} w(t+\theta) + (\delta L_V L_F^2 L_K^2 + L_V L_F L_K)\bar{e} + (\delta L_V L_F^2 L_K + L_V L_F)\bar{d}. \tag{30}$$

From (30), by the use of Lemma 2.1 in the interval $[t_{p+1}, +\infty)$ and taking into account (15), we obtain

$$w(t) \leq \sup_{\theta \in [-\Delta-2\delta,0]} w(t_{p+1}+\theta)\, \mathrm{e}^{\lambda^*(\Delta+2\delta)}\, \mathrm{e}^{-\lambda^*(t-t_{p+1})} + \frac{\alpha_1\alpha_2(L_V L_F L_K + L_V L_F^2 L_K^2 \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{e} + \frac{\alpha_1\alpha_2(L_V L_F + L_V L_F^2 L_K \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{d}. \tag{31}$$

From (31), by Lemma 2.3 and taking into account (22), for $t \in [t_{p+1}, +\infty)$, the following inequalities hold

$$\begin{aligned} \|\tilde{x}_t\|_\infty \leq{}& \frac{\alpha_2}{\alpha_1}(2+L_F L_K \delta)^{l+1}\, \mathrm{e}^{(L_F+\lambda^*)(\Delta+2\delta)} \|\tilde{x}_0\|_\infty\, \mathrm{e}^{-\lambda^*(t-t_{p+1})} \\ &+ \frac{\alpha_2}{\alpha_1}(2+L_F L_K \delta)^{l+1}\, \mathrm{e}^{(L_F+\lambda^*)(\Delta+2\delta)} L_F \delta \bar{d}\, \mathrm{e}^{-\lambda^*(t-t_{p+1})} \\ &+ \frac{\alpha_2}{\alpha_1}(2+L_F L_K \delta)^{l+1}\, \mathrm{e}^{(L_F+\lambda^*)(\Delta+2\delta)} \bar{e} L_F L_K \delta \\ &\mathrm{e}^{-\lambda^*(t-t_{p+1})} \\ &+ \frac{\alpha_2(L_V L_F + L_V L_F^2 L_K \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{d} \\ &+ \frac{\alpha_2(L_V L_F L_K + L_V L_F^2 L_K^2 \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{e}. \end{aligned} \tag{32}$$

From (32), taking into account that $t_{p+1} \leq \Delta + \delta$ and that, for $t \in [0, t_{p+1}]$, the right-hand side of the inequality in (32) is greater than the right-hand side of the equality in (22), we can conclude that, for $t \in [0, +\infty)$, the following inequality holds

$$\begin{aligned} \|\tilde{x}_t\|_\infty \leq{}& \frac{\alpha_2}{\alpha_1}(2+L_F L_K \delta)^{l+1}\, \mathrm{e}^{(L_F+\lambda^*)(\Delta+2\delta)} \|\tilde{x}_0\|_\infty\, \mathrm{e}^{-\lambda^*(t-\delta-\Delta)} \\ &+ \frac{\alpha_2}{\alpha_1}(2+L_F L_K \delta)^{l+1}\, \mathrm{e}^{(L_F+\lambda^*)(\Delta+2\delta)} L_F \delta \bar{d}\, \mathrm{e}^{-\lambda^*(t-\delta-\Delta)} \\ &+ \frac{\alpha_2}{\alpha_1}(2+L_F L_K \delta)^{l+1}\, \mathrm{e}^{(L_F+\lambda^*)(\Delta+2\delta)} \bar{e} L_F L_K \delta \\ &\mathrm{e}^{-\lambda^*(t-\delta-\Delta)} \\ &+ \frac{\alpha_2(L_V L_F L_K + L_V L_F^2 L_K^2 \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{e} \\ &+ \frac{\alpha_2(L_V L_F + L_V L_F^2 L_K \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{d}. \end{aligned} \tag{33}$$

From (33), taking into account (16), we obtain

$$\begin{aligned} \|\tilde{x}_t\|_\infty \leq{}& \frac{\alpha_2}{\alpha_1}(2+L_F L_K \delta)^{l+1}\, \mathrm{e}^{(L_F+2\lambda^*)(\Delta+2\delta)} \|\tilde{x}_0\|_\infty\, \mathrm{e}^{-\lambda^* t} \\ &+ \frac{\alpha_2}{\alpha_1}(2+L_F L_K \delta)^{l+1}\, \mathrm{e}^{(L_F+2\lambda^*)(\Delta+2\delta)} \\ &(L_F L_K \delta \bar{e} + L_F \delta \bar{d}) \\ &+ \frac{\alpha_2(L_V L_F L_K + L_V L_F^2 L_K^2 \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{e} \\ &+ \frac{\alpha_2(L_V L_F + L_V L_F^2 L_K \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{d} \\ ={}& r_1 \|\tilde{x}_0\|_\infty\, \mathrm{e}^{-\lambda^* t} + r_1(L_F L_K \delta \bar{e} + L_F \delta \bar{d}) \\ &+ \frac{\alpha_2(L_V L_F L_K + L_V L_F^2 L_K^2 \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{e} \\ &+ \frac{\alpha_2(L_V L_F + L_V L_F^2 L_K \delta)}{\alpha_1\alpha_3 - \alpha_2 L_V L_F^2 L_K\,(1+L_K)\,\delta}\bar{d} \\ ={}& r_1 \|\tilde{x}_0\|_\infty\, \mathrm{e}^{-\lambda^* t} + r_2 \bar{d} + r_3 \bar{e}. \end{aligned} \tag{34}$$

Now, from (8), (18), it follows that $\tilde{x}_t = \left[\begin{smallmatrix} x_t \\ \hat{x}_t \end{smallmatrix}\right]$ is the solution, for $t \in R^+$, of the closed-loop system described by the equations

$$\begin{aligned} &\dot{x}(t) = f(x_t, k(\hat{x}_{t_j}, h(x_{t_j}) + e_j) + d_j), \\ &\dot{\hat{x}}(t) = \widehat{f}(\hat{x}_{t_j}, k(\hat{x}_{t_j}, h(x_{t_j}) + e_j), h(x_{t_j}) + e_j), \\ &\hat{x}_{t_j}(\theta) = \begin{cases} \hat{x}_0(t_j+\theta), & t_j + \theta \leq 0, \\ \hat{x}(t_k) + \dfrac{t_j + \theta - t_k}{t_{k+1} - t_k}(\hat{x}(t_{k+1}) - \hat{x}(t_k)), & t_j + \theta > 0, \\ k = \arg\max\limits_{l \in N}\{t_l \in \pi_{a,\delta} : t_l \leq t_j + \theta\}, & \end{cases} \\ &\theta \in [-\Delta, 0], \quad t \in \left[t_j, t_{j+1}\right), \quad t_j \in \pi_{a,\delta}, \quad j = 0, 1, \ldots, \\ &x(\tau) = x_0(\tau), \quad \hat{x}(\tau) = \hat{x}_0(\tau), \quad \tau \in [-\Delta, 0]. \end{aligned} \tag{35}$$

From (35), it follows that $\left[\begin{smallmatrix} x_t \\ \hat{x}_{t_j} \end{smallmatrix}\right]$ is the solution, for $t \in R^+$, $t_j \in \pi_{a,\delta}$, of the system described by (17) (see also (1), (4)). From (34) it follows that (5) holds. The proof of the theorem is complete. ■

Remark 3.1: The results provided in Theorem 3.1 allow aperiodic sampling. Indeed, the global exponential practical stability of the sampled-data closed-loop system described by (1)–(4) is preserved as long as the length of the sampling intervals $[t_j, t_{j+1}]$ is less than the upper bound δ (see (12)) and no matter whether periodic or aperiodic sampling is used (see the real $a \in (0, 1]$ in Theorem 3.1 and Definition 2.2). The knowledge of the sampling times $t_j, j = 0, 1, \ldots,$ is not needed. On the other hand, for the computation of the estimated state $\hat{x}(t_{j+1}), j = 0, 1, \ldots,$ the knowledge of the time elapsed between the latest two sampling times (i.e. $t_{j+1} - t_j$) is needed. Otherwise, the Euler numerical method could not be correctly applied. Notice, in (17), that, for $j = 0, 1, \ldots,$ $\hat{x}_{t_{j+1}}$ can be computed by suitably shifting $\hat{x}_{t_j}$, and then making use of $\hat{x}(t_j), \hat{x}(t_{j+1})$. Again, just the latest sampling times difference $t_{j+1} - t_j$ is involved. Furthermore, even a blow up of the variable $x(t)$, $t_j < t < t_{j+1}$, does not prevent the possibility of computing as well $\hat{x}(t)$, $t_j < t \le t_{j+1}$.

Remark 3.2: Differently from Theorem 5 (Pepe & Fridman, 2017), in Theorem 3.1, actuation disturbances and observation errors as well as dynamic output feedback controllers are addressed (see (4)). The linear ISS inequality (5) guarantees global exponential stability in the case these disturbances and observation errors do not appear. In order to prove Theorem 3.1, more complex tools with respect to the ones used in Pepe and Fridman (2017), such as a recent extension of the Halanay's inequality, are required (see Lemma 2.1). The main difficulties in solving Problem 2.1 are in the concurrent presence of sampled-data dynamic output feedback, and of actuation disturbances and observation errors. These difficulties are overcome by exploiting the following tools: (1) the introduction of a fictitious input in order to deal with the continuous-time closed-loop system, with dynamic output feedback, as an extended continuous-time closed-loop system with static state feedback (see Di Ferdinando & Pepe, 2019; Laila et al., 2002); (2) the coincidence of the increment function (see Di Ferdinando & Pepe, 2019 and the references therein) of the Euler scheme with the function describing the dynamic output controller; (3) the global Lipschitz property of the increment function, inherited by the functions describing the dynamic output feedback; (4) the global Lipschitz property of the involved Lyapunov-Krasovskii functional for the continuous-time closed-loop system. The coincidence in item (2) is a key issue for proving global exponential stability (not of the practical type) as well as the ISS inequality (5). Differently from Di Ferdinando and Pepe (2019) (concerning locally Lipschitz systems), here: (1) global exponential stability is guaranteed in the ideal case when actuation-measurement disturbances are zero; (2) the globally Lipschitz property, of the maps describing the plant and the continuous-time dynamic output feedback exponential stabiliser, is here assumed; (3) the global practical exponential stability of the sampled-data closed-loop system is ensured with respect to any bounded actuation disturbances and to any bounded measurements noise; (4) results concerning the study of other emulation schemes than the Euler ones are not provided. In Di Ferdinando and Pepe (2017), actuation disturbances and observation errors are addressed by ISS redesign methodologies for locally Lipschitz control affine systems with sampled-data static state feedback, and semi-global practical stability is proved to hold for sufficiently high sampling frequency, under the assumption that the observation error affects marginally the ISS redesigned term in the control law. The following table provides a picture of the comparison addressed in this remark.

Next corollary readily follows from Theorem 3.1, and therefore the proof is omitted.

Corollary 3.2: *Let Assumption 2.1 hold. Let $a \in (0,1]$. Then, there exist positive reals δ, λ^*, r_1, r_2, r_3 such that, for any partition $\pi_{a,\delta}$ (see Definition 2.2), for any initial state x_0, $\hat{x}_0 \in \mathcal{C}^n$, for any sequence $e : \mathbb{N} \to \mathbb{R}^q$(measurement noise) such that, for some positive real $\bar{e}$, $|e_j| \le \bar{e}$, $j = 0,1,2,\ldots$, for any sequence $d : \mathbb{N} \to \mathbb{R}^m$ (actuation disturbance) such that, for some positive real $\bar{d}$, $|d_j| \le \bar{d}$, $j = 0,1,2,\ldots$, the solution of the sampled-data closed-loop system described by (17) (see also (1), (4)) exists for all $t \ge 0$ and, furthermore, satisfies (5).*

Remark 3.3: We highlight that, when $|d_j| = |e_j| = 0$, for any $j = 0,1,\ldots$, the condition (5) implies the global exponential stability property of the sampled-data closed-loop system described by (17) (see Problem 2.1, Theorem 3.1 and Corollary 3.2).

4. Example

Let us consider the nonlinear time-delay system described by the following RFDEs

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) - 0.1x_2(t), \\ \dot{x}_2(t) &= -x_2(t) + \tanh(x_1(t) + x_2(t) + x_2(t-\Delta) + u(t)), \\ y(t) &= \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ x_2(t-\Delta) \end{bmatrix}, \end{aligned} \tag{36}$$

where: $x_1(t)$, $x_2(t) \in \mathbb{R}$; $\Delta = 0.1$ is the involved time delay; $u(t) \in \mathbb{R}$ is the input signal; $y(t) \in \mathbb{R}^2$ is the output signal. In this case, the functions f and h in (1) are functions from $\mathcal{C}^2 \times \mathbb{R}$ to $\mathbb{R}^2$ and from $\mathcal{C}^2$ to $\mathbb{R}^2$, respectively. We consider here the dynamic output feedback controller described by the following RFDEs

$$\begin{aligned} \dot{\hat{x}}_1(t) &= -\hat{x}_1(t) - 0.1y_1(t) + H_1(y_1(t) - \hat{x}_2(t)), \\ \dot{\hat{x}}_2(t) &= -y_1(t) + \tanh(\hat{x}_1(t) + y_1(t) + y_2(t) + u(t)) \\ &\quad + H_2(y_1(t) - \hat{x}_2(t)), \\ u(t) &= -\hat{x}_1(t) - y_1(t) - y_2(t), \end{aligned} \tag{37}$$

where: $\hat{x}_1(t)$, $\hat{x}_2(t) \in \mathbb{R}$; Δ is the time delay in (36); H_1, H_2 are tuning parameters, that will be chosen later. In this case, the

	Class of Systems	Controller	Provided Stability property	Actuation disturbances and measurements noises
(Di Ferdinando & Pepe, 2019)	Locally Lipschitz	Discretized dynamic output feedback (covering Euler scheme)	Semi-global practical stability	Sufficiently small
(Di Ferdinando & Pepe, 2017)	Locally Lipschitz, control-affine	Static state-feedback with ISS redesign	Semi-global practical stability	Arbitrary bounded actuation disturbances, observation errors affecting marginally the ISS redesigned term
(Pepe & Fridman, 2017)	Globally Lipschitz	Static state-feedback	Global exponential stability	Not addressed
Theorem 3.1	Globally Lipschitz	Dynamic output feedback discretized by Euler scheme	Global exponential stability in absence of disturbances, ISS in presence of disturbances	Arbitrary bounded, with known bounds

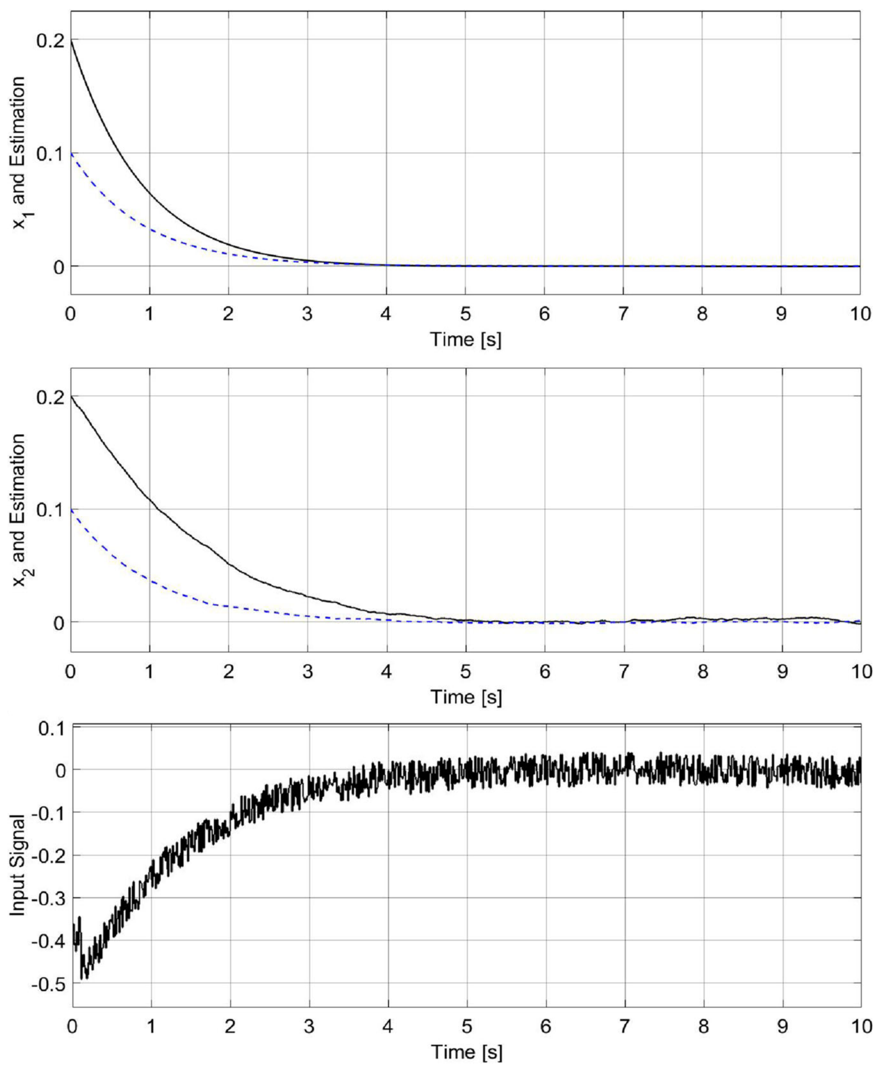

Figure 1. Emulation by Euler method with $\delta = 0.01$ [s]: in the first panel, $x_1(t)$ and $\hat{x}_1(t)$ (dashed line) are reported; in the second panel, $x_2(t)$ and $\hat{x}_2(t)$ (dashed line) are reported; the third panel reports the sampled-data control input signal $u(t)$.

functions $\widehat{f}$ and k in (2) are functions from $\mathcal{C}^2 \times \mathbb{R} \times \mathbb{R}^2$ to $\mathbb{R}^2$ and from $\mathcal{C}^2 \times \mathbb{R}^2$ to $\mathbb{R}$, respectively. Notice that, the maps f, h, $\widehat{f}$ and k, here involved, are globally Lipschitz. Let P and Q be two symmetric positive definite matrices defined as:

$$P = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}, \quad Q = 10^{-3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Let us consider the following Lyapunov-Krasovskii functional $V : \mathcal{C}^4 \to \mathbb{R}^+$, defined, $\forall \tilde{\phi} \in \mathcal{C}^4$, as $V(\tilde{\phi}) = V_1(\tilde{\phi}(0)) + V_2(\tilde{\phi})$, where: $V_1 : \mathbb{R}^4 \to \mathbb{R}^+$ is defined, for $\tilde{x} \in \mathbb{R}^4$, as $V_1(\tilde{x}) = \tilde{x}^T P \tilde{x}$; $V_2 : \mathcal{C}^4 \to \mathbb{R}^+$ is defined, for $\tilde{\phi} \in \mathcal{C}^4$, as $V_2(\tilde{\phi}) = \int_{-\Delta}^{0} \tilde{\phi}^T(\tau) Q \tilde{\phi}(\tau)\, d\tau$. Taking into account the functional V and the continuous-time closed-loop system described by (36), (37), rewritten as an augmented plant (see (6)–(10)), by choosing the tuning parameters in (37) as $H_1 = 0.1$, $H_2 = 1$, the following equality/inequalities hold:

$$\begin{aligned} D^+ V(\tilde{\phi}, K(\tilde{\phi}, 0)) &= -\phi_1^2(0) - 0.1\phi_1(0)\phi_2(0) - 0.1\phi_2^2(0) \\ &\quad + 0.1\phi_2(0)\tanh(\phi_1(0) - \hat{\phi}_1(0)) - \hat{\phi}_1^2(0) \\ &\quad - 0.1\hat{\phi}_1(0)\hat{\phi}_2(0) - \hat{\phi}_2^2(0) \\ &\quad + 10^{-3}(\phi_1^2(0) + \phi_2^2(0) + \hat{\phi}_1^2(0) + \hat{\phi}_2^2(0)) \\ &\quad - 10^{-3}(\phi_1^2(-\Delta) + \phi_2^2(-\Delta) + \hat{\phi}_1^2(-\Delta) + \hat{\phi}_2^2(-\Delta)) \\ &\le -\phi_1^2(0) + 0.1\left(2\phi_1^2(0) + \frac{\phi_2^2(0)}{8}\right) \\ &\quad - 0.1\phi_2^2(0) + 0.1\left(2\phi_1^2(0) + \frac{\phi_2^2(0)}{8} + 2\hat{\phi}_1^2(0) + \frac{\phi_2^2(0)}{8}\right) \\ &\quad - \hat{\phi}_1^2(0) - 0.1\left(\frac{\hat{\phi}_1^2(0)}{2} + \frac{\hat{\phi}_2^2(0)}{2}\right) \\ &\quad - \hat{\phi}_2^2(0) + 10^{-3}(\phi_1^2(0) + \phi_2^2(0) + \hat{\phi}_1^2(0) + \hat{\phi}_2^2(0)) \\ &\quad - 10^{-3}(\phi_1^2(-\Delta) + \phi_2^2(-\Delta) + \hat{\phi}_1^2(-\Delta) + \hat{\phi}_2^2(-\Delta)) \\ &\le -10^{-3}\|\tilde{\phi}\|_\infty. \end{aligned} \tag{38}$$

By Lemma 2.3, taking into account (38) and the functional V, given $\alpha_1 = \lambda_{\min}(P)$, $\alpha_2 = \lambda_{\max}(P) + \Delta\lambda_{\max}(Q)$, $\alpha_3 = 10^{-3}$, the continuous-time closed-loop system described by (36), (37) is 0-GES. Then, we can apply Corollary 3.2. In performed simulations, uniform sampling is used, and the sampling period has been chosen equal to 0.01 [s]. The actuator disturbance has been chosen as $d(j) = 0.005\sin(t_j)$, $j = 0, 1, \ldots$. As far as the observation error is concerned, we considered the sequence $e : \mathbb{N} \to \mathbb{R}^2$ defined, for any $j = 0, 1, \ldots$, as $e(j) = \begin{bmatrix} e_1(j) \\ e_2(j) \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$, with e_1 and e_2 taken from the interval $[-0.02, 0.02]$, by emulation of the uniform probability density function. The initial state has been chosen equal to $\begin{bmatrix} x_1(\tau) \\ x_2(\tau) \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}$, $\begin{bmatrix} \hat{x}_1(\tau) \\ \hat{x}_2(\tau) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $\tau \in [-\Delta, 0]$. The practical stability property of the sampled-data closed-loop system is evident by the plots in Figure 1.

5. Conclusions

In this paper, using the same reasonings as in Pepe and Fridman (2017), it has been shown that the global practical exponential stability property of globally Lipschitz fully nonlinear time-delay systems is preserved when the Euler emulation, under suitably fast sampling (aperiodic sampling is allowed), of globally Lipschitz dynamic output feedback controllers, affected by measurement noises and actuation disturbances, is used. The assumption needed to ensure the results is that the continuous-time closed-loop system is globally exponentially stable. The Halanay's inequality and the converse Lyapunov theorems have been used as tools to prove the results. Future investigations will concern the reduction of the conservativeness of the provided sampling period. The hybrid systems approach seems to be a very promising tool in such direction (see Carnevale et al., 2007 in the case of systems described by ordinary differential equations). The non-trivial analysis of sampled-data dynamic output feedback control, for nonlinear systems with state time-delays, in the case unknown time-delays affect also the input/output channels (see Mattioni et al., 2017a, 2017b), is left for future investigations.

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