A new Lyapunov technique for robust control of systems with uncertain non-small delays

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Stability, L_2-gain analysis and state-feedback H_∞ control of linear systems with uncertain time-varying delays are considered in the case, where the nominal values of delays are constant and *non-zero*. A new construction of Lyapunov–Krasovskii functionals (LKFs) is introduced: to a nominal LKF, which is appropriate to the system with nominal delays, terms are added that correspond to the perturbed system and that vanish when the delay perturbations approach 0. In the present paper we apply the nominal Lyapunov–Krasovskii functional (LKF) which is based on the descriptor model transformation. Sufficient conditions are given in terms of linear matrix inequalities. Numerical examples illustrate the efficiency of the method.

Keywords: uncertain delay; Lyapunov–Krasovskii method; stability; LMI; H_∞ control.

1. Introduction

During the last decade, a considerable amount of attention has been paid to stability and control of linear systems with uncertain constant or time-varying delays lying in the given segment $[0, \mu]$ (see, e.g. Cao *et al.*, 1998; Fridman & Shaked, 2003; Kolmanovskii & Myshkis, 1999; Kolmanovskii & Richard, 1999; Li & de Souza, 1997; Mahmoud, 2000; Niculescu, 2001; Richard, 2003 and the references therein). This type of delays may be considered as uncertain delays with *zero nominal values* and perturbations from $[0, \mu]$. Throughout the paper, such a delay will be called the uncertain *small* delay (note that the delay perturbations may not be necessarily small). For linear systems with uncertain small delays, the so-called delay-dependent sufficient conditions in terms of linear matrix inequalities (LMIs) have been derived by using Lyapunov–Krasovskii or Lyapunov–Razumikhin approaches (Razumikhin approach usually leads to more conservative results and it is not applicable to H_∞ control). Delay-dependent conditions via LKFs are based on different model transformations. The most recent one, the *descriptor* representation introduced in Fridman (2001), leads to less conservative results and is applicable, unlike the other LKF methods, to the case of fast-varying uncertain *small* delays. By the descriptor approach, the derivative of the LKF along the trajectories of the system depends on both the state vector and *the state derivative*. The latter allows the treatment of the delay uncertainty in a less conservative way.

The case of uncertain *non-small* time-varying delays, where the *nominal delay values are non-zero* and constant, appears in different applications: in engineering and biological systems (see Part 1, Chapter 2 of Kolmanovskii & Myshkis, 1999), in networked control with constant transport delay and time-varying data packets dropout (Yu *et al.*, 2004). This case is mathematically more complicated than the case of small delay, since it may include the systems which are not asymptotically stable without delays. Only a few papers have been published on this topic. The *stability* of linear *retarded-type* systems

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with *one* time-varying *non-small* delay has been studied by Kharitonov & Niculescu (2003). Sufficient stability conditions in their study have been derived via a modification of *complete* LKF. The *complete* LKF for robust stability was introduced in Kharitonov & Zhabko (2003) and it corresponds to necessary and sufficient stability conditions of the nominal system. The modified *complete* LKF of Kharitonov & Niculescu (2003) does not depend explicitly on the delay perturbation and, as a result, the conditions obtained are conservative.

H_∞ *control* of systems with non-small delays as well as the *stability* for neutral-type systems with *multiple non-small* delays have not yet been studied. In the present paper we consider a linear neutral system with multiple (for simplicity two) uncertain state delays $h_i + \eta_i(t), i = 1, 2$, where the constant nominal values $h_i > 0$ are non-zero and the time-varying perturbations η_i satisfy the bounding condition $|\eta_i(t)| \leqslant \mu_i, \forall t \geqslant 0$, with given bounds $\mu_i > 0$. We introduce a new construction of the LKF: to a nominal LKF, which is appropriate to the nominal system (with nominal delays), the terms are added which correspond to the perturbed system and which vanish when the delay perturbations approach 0. The derivative of the nominal LKF along the trajectories of the nominal system depends on the state and the state derivative which allows a less conservative treatment of the delay perturbations.

In the present paper we combine a nominal LKF which is based on the descriptor model transformation (Fridman & Shaked, 2002) with appropriate additional terms. Sufficient conditions for stability, L_2-gain analysis and state-feedback H_∞ control are given in terms of LMIs. As a by-product, new effective criteria for small fast-varying delays are obtained. For the case of analysis, we have to assume that for the nominal values of the delays the sufficient stability conditions of Fridman & Shaked (2002) are feasible. If this is not the case, the *complete* nominal LKF may be applied. The latter will not be considered in the present paper.

Notations. Throughout the paper the superscript 'T' stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices with the Euclidean norm $\|\cdot\|$ and the notation $P > 0$, for $P \in \mathbb{R}^{n \times n}$, means that P is symmetric and positive definite. Symmetric terms in symmetric matrices are denoted by $*$, i.e.

$$\begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix} = \begin{bmatrix} A & B \\ * & C \end{bmatrix}.$$

The space of vector functions that are square integrable over $[0, \infty)$ is denoted by $\mathcal{L}_2$.

2. Problem formulation

We consider the following linear system with uncertain time-varying delays $\tau_i(t), i = 1, 2$, and $g(t)$:

$$\dot{x}(t) - F\dot{x}(t - g(t)) = \sum_{i=0}^{2} A_i x(t - \tau_i(t)) + B_2 u(t) + B_1 w(t), \tag{2.1a}$$

$$x(t) = \phi(t), \quad t \in [-h, 0], \tag{2.1b}$$

$$z(t) = Cx(t) + Du(t), \tag{2.1c}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^q$ is the disturbance input, $z(t) \in \mathbb{R}^l$ is the objective vector, $\tau_0 \equiv 0$, A_i, B_1, B_2, C and D are constant matrices, ϕ is a continuously differentiable initial function and h is an upper-bound on the time delays $\tau_i, i = 1, 2$, and g. For simplicity, we took only two delays τ_1, τ_2 and one delay g. The results of this paper can be easily generalized to the case of multiple delays $\tau_1, \ldots, \tau_m, g_1, \ldots, g_k$.

The uncertain delays $\tau_i(t)$ are supposed to have the following form:

$$\tau_i(t) = h_i + \eta_i(t), \quad i = 1, 2, \tag{2.2}$$

where $h_i > 0$ is a nominal constant value and η_i is a time-varying perturbation. We will consider two cases of delay perturbation:

Case 1 $\eta_i(t)$ are *sign-varying* (take both positive and negative values) piecewise-continuous functions satisfying

$$|\eta_i(t)| \leqslant \mu_i \leqslant h_i, \quad i = 1, 2, \tag{2.3}$$

with known upper-bounds μ_i, i.e. $\tau_i(t) \in [h_i - \mu_i, h_i + \mu_i]$.

Case 2 $\eta_i(t)$ are *non-negative* functions and $\eta_i(t) \leqslant \mu_i$, and thus $\tau_i(t) \in [h_i, h_i + \mu_i]$.

Our results for the case $h_1 = h_2 = 0$ will coincide with the corresponding results of Fridman & Shaked (2002). As in the case of *small* uncertain delays (with $h_1 = h_2 = 0$), we here consider two different subcases for time-varying delay perturbations:

Case 2A $\eta_i(t)$ are differentiable functions, satisfying for all $t \geqslant 0$

$$0 \leqslant \eta_i(t) \leqslant \mu_i, \quad \dot{\eta}_i(t) \leqslant d_i < 1, \quad i = 1, 2, \tag{2.4}$$

where μ_i and d_i are constant upper-bounds.

Case 2B $\eta_i(t)$ are piecewise-continuous functions, satisfying for all $t \geqslant 0$,

$$0 \leqslant \eta_i(t) \leqslant \mu_i, \quad i = 1, 2.$$

We do not consider the case of non-positive η_i because criteria for this case are feasible for the smaller intervals of the values of $\tau_i, i = 1, 2$, and thus are more conservative than for the Case 2.

Equation (2.1a) is a general neutral-type system. We assume that $g(t)$ is a differentiable function satisfying $\dot{g}(t) \leqslant d_0 < 1$, for all $t \geqslant 0$, where d_0 is a known upper-bound. Our results will be independent of g and dependent on d_0. For example, $g(t) = \tau_1(t)$ (usually such models appear in the applications) and one can apply the results of Case 2A with $d_0 = d_1$.

We assume that $\|F\| < 1$. The latter will guarantee the applicability of Lyapunov-Krasovskii method for stability of neutral type systems with time-varying delays (Kolmanovskii & Myshkis, 1999). Note that in the case of constant delay g we do not need any assumptions on F since our LMI conditions will guarantee the stability of the difference equation (see, e.g. Fridman & Shaked, 2003).

We will start with the stability analysis of (2.1a), where $B_1 = B_2 = 0$. Further, we will proceed with H_∞ control problem. For a pre-chosen $\gamma > 0$, we consider the following performance index:

$$J = \int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t)]\mathrm{d}t. \tag{2.5}$$

We seek a state-feedback control law

$$u(t) = Kx(t) \tag{2.6}$$

that will internally stabilize (2.1aa) and will lead to $J < 0$, for all $x(t)$ satisfying (2.1a), with the initial value $\phi = 0$ and for all $0 \neq w(t) \in \mathcal{L}_2$.

3. The stability issue

In this section we consider $B_1 = B_2 = 0$. We represent (2.1a) in the from

$$\dot{x}(t) = f(t, x_t, \dot{x}_t)$$

with $f(t, x_t, \dot{x}_t) = F\dot{x}(t - g(t)) + \sum_{i=0}^{2} A_i x(t - \tau_i(t))$. The assumption $\|F\| < 1$ implies that f satisfies Lipshitz condition in $\dot{x}_t$ with a constant less than 1. Hence, by Theorem 1.6 (p. 337 of Kolmanovskii & Myshkis, 1999) the existence of $V > 0$ such that $\dot{V} < 0$ guarantees asymptotic stability of (2.1a). Similar to delay-dependent methods and following Kharitonov & Niculescu (2003), we represent the system in the form

$$\dot{x}(t) - F\dot{x}(t - g(t)) = \sum_{i=0}^{2} A_i x(t - h_i) + \sum_{i=1}^{2} A_i [x(t - h_i - \eta_i(t)) - x(t - h_i)], \qquad (3.1)$$

or equivalently

$$\dot{x}(t) - F\dot{x}(t - g(t)) = \sum_{i=0}^{2} A_i x(t - h_i) - \sum_{i=1}^{2} A_i \int_{t-h_i-\eta_i(t)}^{t-h_i} \dot{x}(s)\mathrm{d}s, \qquad (3.2)$$

where $h_0 = 0$ since $\tau_0 \equiv 0$.

We suggest the following form of LKF:

$$V = V_{\mathrm{n}} + V_{\mathrm{a}}, \qquad (3.3)$$

where V_{n} is a nominal LKF corresponding to the nominal system (3.1) with $\eta_1 = \eta_2 = 0$, and V_{a} is an additional term.

For the nominal system in the present paper we choose LKF which corresponds to the descriptor model transformation. The latter transformation of (3.2) has the form

$$E\dot{\bar{x}}(t) = \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} = \begin{bmatrix} y(t) \\ -y(t) + \sum_{i=0}^{2} A_i x(t - h_i) + F y(t - g(t)) \end{bmatrix}$$
$$- \sum_{i=1}^{2} \begin{bmatrix} 0 \\ A_i \end{bmatrix} \int_{t-h_i-\eta_i}^{t-h_i} y(s)\mathrm{d}s, \qquad (3.4)$$

with $\bar{x}(t) = \mathrm{col}\{x(t), y(t)\}$, $E = \mathrm{diag}\{I, 0\}$. The nominal LKF is given by (see, e.g. Fridman & Shaked, 2002):

$$V_{\mathrm{n}} = \bar{x}^{\mathrm{T}}(t) E P \bar{x}(t) + \sum_{i=1}^{2} \int_{-h_i}^{0} \int_{t+\theta}^{t} y^{\mathrm{T}}(s) R_i y(s)\mathrm{d}s\,\mathrm{d}\theta$$
$$+ \sum_{i=1}^{2} \int_{t-h_i}^{t} x^{\mathrm{T}}(s) S_i x(s)\mathrm{d}s + \int_{t-g(t)}^{t} y^{\mathrm{T}}(s) U y(s)\mathrm{d}s, \qquad R_i > 0, S_i > 0, U > 0, \qquad (3.5)$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \qquad P_1 > 0. \qquad (3.6\mathrm{a,b})$$

The nominal system is asymptotically stable if there exist $n \times n$ matrices $0 < P_1$, P_2, P_3, S_i, U, Y_{i1}, Y_{i2}, Z_{i1}, Z_{i2}, Z_{i3}, R_i such that the following LMIs are feasible

$$
\Gamma_{\mathrm{n}} = \begin{bmatrix} \Psi_{\mathrm{n}} & P^{\mathrm{T}} \begin{bmatrix} 0 \\ A_1 \end{bmatrix} - Y_1^{\mathrm{T}} & P^{\mathrm{T}} \begin{bmatrix} 0 \\ A_2 \end{bmatrix} - Y_2^{\mathrm{T}} & P^{\mathrm{T}} \begin{bmatrix} 0 \\ F \end{bmatrix} \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & -(1-d_0)U \end{bmatrix} < 0, \qquad (3.7a,b)
$$

$$
\begin{bmatrix} R_i & Y_i \\ * & Z_i \end{bmatrix} \geqslant 0, \quad i = 1, 2,
$$

where

$$
Y_i = [Y_{i1} \quad Y_{i2}], \quad Z_i = \begin{bmatrix} Z_{i1} & Z_{i2} \\ * & Z_{i3} \end{bmatrix}, \quad i = 1, 2,
$$

$$
\Psi_{\mathrm{n}} = P^{\mathrm{T}} \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_0 & -I \end{bmatrix}^{\mathrm{T}} P + \sum_{i=1}^{2} h_i Z_i
$$

$$
+ \begin{bmatrix} \sum_{i=1}^{2} S_i & 0 \\ 0 & \sum_{i=1}^{2} h_i R_i + U \end{bmatrix} + \sum_{i=1}^{2} \begin{bmatrix} Y_i \\ 0 \end{bmatrix} + \sum_{i=1}^{2} \begin{bmatrix} Y_i \\ 0 \end{bmatrix}^{\mathrm{T}}. \qquad (3.8a\text{–}c)
$$

By the descriptor approach, the derivative of the nominal LKF depends on both $x(t)$ and $\dot{x}(t)$. Therefore, the additional terms V_{a} may be chosen in the following forms:

Case 1

$$
V_{\mathrm{a}} = \sum_{i=1}^{2} \int_{-\mu_i}^{\mu_i} \int_{t+\theta-h_i}^{t} \dot{x}^{\mathrm{T}}(s) R_{i\mathrm{a}} \dot{x}(s) \, \mathrm{d}s \mathrm{d}\theta, \quad R_{i\mathrm{a}} > 0. \qquad (3.9)
$$

Case 2A

$$
V_{\mathrm{a}} = \sum_{i=1}^{2} \int_{-\mu_i}^{0} \int_{t+\theta-h_i}^{t} \dot{x}^{\mathrm{T}}(s) R_{i\mathrm{a}} \dot{x}(s) \, \mathrm{d}s \mathrm{d}\theta
$$

$$
+ \sum_{i=1}^{2} \int_{t-\tau_i(t)}^{t} x^{\mathrm{T}}(s) S_{i\mathrm{a}} x(s) \mathrm{d}s, \quad R_{i\mathrm{a}} > 0, \; S_{i\mathrm{a}} > 0. \qquad (3.10)
$$

Case 2B V_{a} has the form (3.10) with $S_{i\mathrm{a}} = 0$.

Note that for $\mu_i \to 0$ (and $S_{i\mathrm{a}} \to 0$ in Case 2A) we have $V_{\mathrm{a}} \to 0$ in all the cases under consideration and thus $V \to V_{\mathrm{n}}$. The latter will guarantee that if the conditions for the stability of the nominal system are feasible, then the stability conditions for the perturbed system will be feasible for small enough μ_i.

We obtain:

THEOREM 1

(i) Under Case 1, the system (2.1a) with $B_1 = B_2 = 0$ is asymptotically stable if there exist $n \times n$ matrices $0 < P_1, P_2, P_3, S_i, U, Y_{i1}, Y_{i2}, Z_{i1}, Z_{i2}, Z_{i3}, R_i$ and $R_{ia} > 0, i = 1, 2$, that satisfy (3.7b) and the following LMIs:

$$\Gamma_1 = \begin{bmatrix} \Psi \ P^T\begin{bmatrix}0\\A_1\end{bmatrix} - Y_1^T \ P^T\begin{bmatrix}0\\A_2\end{bmatrix} - Y_2^T & P^T\begin{bmatrix}0\\F\end{bmatrix} & \mu_1 P^T\begin{bmatrix}0\\A_1\end{bmatrix} & \mu_2 P^T\begin{bmatrix}0\\A_2\end{bmatrix} \\ * & -S_1 & 0 & 0 & 0 & 0 \\ * & * & -S_2 & 0 & 0 & 0 \\ * & * & * & -(1-d_0)U & 0 & 0 \\ * & * & * & * & -\mu_1 R_{1a} & 0 \\ * & * & * & * & * & -\mu_2 R_{2a} \end{bmatrix} < 0,$$

(3.11)

where Y_i, Z_i and Ψ_n are given by (3.8) and

$$\Psi = \Psi_1 = \Psi_n + 2\sum_{i=1}^{2}\begin{bmatrix} 0 & 0 \\ 0 & \mu_i R_{ia} \end{bmatrix}.$$

(3.12)

(ii) Under Case 2A, the system (2.1a) with $B_1 = B_2 = 0$ is asymptotically stable if there exist $n \times n$ matrices $0 < P_1, P_2, P_3, S_i, U, S_{ia}, Y_{i1}, Y_{i2}, Y_{ia}, Z_{i1}, Z_{i2}, Z_{i3}, Z_{i1a}, Z_{i2a}, Z_{i3a}, R_i$ and $R_{ia} > 0, i = 1, 2$, that satisfy (3.7b) and the following LMIs:

$$\Gamma_{2A} = \begin{bmatrix} \Psi_{2A} \ P^T\begin{bmatrix}0\\A_1\end{bmatrix} - Y_{1a}^T \ P^T\begin{bmatrix}0\\A_2\end{bmatrix} - Y_{2a}^T & P^T\begin{bmatrix}0\\F\end{bmatrix} & Y_{1a}^T - Y_1^T \ Y_{2a}^T - Y_2^T \\ * & -(1-d_1)S_{1a} & 0 & 0 & 0 & 0 \\ * & * & -(1-d_2)S_{2a} & 0 & 0 & 0 \\ * & * & * & -(1-d_0)U & 0 & 0 \\ * & * & * & * & -S_1 & 0 \\ * & * & * & * & * & -S_2 \end{bmatrix} < 0,$$

(3.13a,b)

$$\text{and} \quad \begin{bmatrix} R_{ia} & Y_{ia} \\ * & Z_{ia} \end{bmatrix} \geqslant 0, \quad i = 1, 2,$$

where Y_i, Z_i and Ψ_n are given by (3.8) and

$$Y_i = [Y_{i1a} \ \ Y_{i2a}], \quad Z_{ia} = \begin{bmatrix} Z_{i1a} & Z_{i2a} \\ * & Z_{i3a} \end{bmatrix}, \quad i = 1, 2,$$

$$\Psi_{2A} = \Psi_n + \sum_{i=1}^{2}\mu_i Z_{ia} + \begin{bmatrix} \sum_{i=1}^{2} S_{ia} & 0 \\ 0 & \sum_{i=1}^{2}\mu_i R_{ia} \end{bmatrix}.$$

(iii) Under Case 2B, the system (2.1aa) with $B_1 = B_2 = 0$ is asymptotically stable if there exist $n \times n$ matrices $0 < P_1, P_2, P_3, S_i, U, Y_{i1}, Y_{i2}, Z_{i1}, Z_{i2}, Z_{i3}, R_i$ and $R_{ia} > 0, i = 1, 2$, that

satisfy (3.11), (3.7b), where Y_i, Z_i and Ψ_n are given by (3.8) and

$$\Psi = \Psi_{2B} = \Psi_n + \sum_{i=1}^{2} \begin{bmatrix} 0 & 0 \\ 0 & \mu_i R_{ia} \end{bmatrix}.$$

Proof. Derivative of V_n in t along the trajectories of the nominal system satisfies the following inequality (see Fridman & Shaked, 2002):

$$\dot{V}_n \leqslant \xi^T(t) \Gamma_n \xi(t), \tag{3.14}$$

where Γ_n is given by (3.7a) and

$$\xi(t) = \mathrm{col}\{x(t), y(t), x(t - h_1), x(t - h_2), y(t - g(t))\}, \tag{3.15}$$

provided (3.7b) is satisfied. Note that

$$\bar{x}^T(t) E P \bar{x}(t) = x^T(t) P_1 x(t)$$

and, hence, differentiating this term in t along the trajectories of the perturbed system (3.4) gives:

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\bar{x}^T(t) E P \bar{x}(t)\} = 2x^T(t) P_1 \dot{x}(t) = 2\bar{x}^T(t) P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix}$$

$$= 2\bar{x}^T(t) P^T \begin{bmatrix} y(t) \\ -y(t) + \sum_{i=0}^{2} A_i x(t - h_i) + F y(t - g(t)) \end{bmatrix} + \sum_{i=1}^{2} \Delta_i(t), \quad (3.16)$$

where

$$\Delta_i(t) = -2\bar{x}^T(t) P^T \int_{t-h_i-\eta_i(t)}^{t-h_i} \begin{bmatrix} 0 \\ A_i \end{bmatrix} y(s)\mathrm{d}s,$$

while differentiating the same term along the trajectories of the nominal system gives (3.16) with $\Delta_i(t) = 0$.

Therefore, $\dot{V}_n$ along the trajectories of the perturbed system satisfies the following inequality:

$$\dot{V}_n \leqslant \xi^T(t) \Gamma_n \xi(t) + \sum_{i=1}^{2} \Delta_i(t). \tag{3.17}$$

We will bound $\Delta_i(t)$ differently for each case.
Case 1

$$\Delta_i(t) \leqslant \left| \int_{t-h_i-\eta_i}^{t-h_i} \bar{x}^T(t) P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_{ia}^{-1} [0 \quad A_i^T] P \bar{x}(t)\mathrm{d}s \right| + \left| \int_{t-h_i-\eta_i}^{t-h_i} y^T(s) R_{ia} y(s)\mathrm{d}s \right|$$

$$\leqslant \mu_i \bar{x}^T(t) P^T \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_{ia}^{-1} [0 \quad A_i^T] P \bar{x}(t) + \int_{t-h_i-\mu_i}^{t-h_i+\mu_i} y^T(s) R_{ia} y(s)\mathrm{d}s. \tag{3.18}$$

For V_a of (3.9) we have

$$\dot{V}_a = 2y^T(t) \left[\sum_{i=1}^{2} \mu_i R_{ia} \right] y(t) - \sum_{i=1}^{2} \int_{t-h_i-\mu_i}^{t-h_i+\mu_i} y^T(s) R_{ia} y(s)\mathrm{d}s. \tag{3.19}$$

Hence, from (3.17)–(3.19) we find

$$\dot{V} = \dot{V}_{\mathrm{n}} + \dot{V}_{\mathrm{a}} \leqslant \xi^{\mathrm{T}}(t)\Gamma_{\mathrm{n}}\xi(t) + \sum_{i=1}^{2} \mu_i \bar{x}^{\mathrm{T}}(t) P^{\mathrm{T}} \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_{ia}^{-1}[0 \quad A_i^{\mathrm{T}}]$$

$$\times P\bar{x}(t) + 2y^{\mathrm{T}}(t) \left[\sum_{i=1}^{2} \mu_i R_{ia} \right] y(t) \tag{3.20}$$

and thus by Schur complements formula (3.11) implies $\dot{V} < 0$ and asymptotic stability of (2.1aa).
Case 2B Proof is similar to Case 1 with the following bounding of $\Delta_i(t)$:

$$\Delta_i(t) \leqslant \mu_i \bar{x}^{\mathrm{T}}(t) P^{\mathrm{T}} \begin{bmatrix} 0 \\ A_i \end{bmatrix} R_{ia}^{-1}[0 \quad A_i^{\mathrm{T}}] P\bar{x}(t) + \int_{t-h_i-\mu_i}^{t-h_i} y^{\mathrm{T}}(s) R_{ia} y(s) \mathrm{d}s.$$

Case 2A When $\eta_i(t) \geqslant 0$ we can apply less conservative bounding introduced in Moon *et al.* (2001). For any $a \in \mathbb{R}^n$, $b \in \mathbb{R}^{2n}$, $\mathcal{N} \in \mathbb{R}^{2n \times n}$, $R \in \mathbb{R}^{n \times n}$, $Y \in \mathbb{R}^{n \times 2n}$ and $Z \in \mathbb{R}^{2n \times 2n}$, the following holds

$$-2b^{\mathrm{T}}\mathcal{N}a \leqslant \begin{bmatrix} a \\ b \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R & Y - \mathcal{N}^{\mathrm{T}} \\ Y^{\mathrm{T}} - \mathcal{N} & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \quad \text{where} \quad \begin{bmatrix} R & Y \\ Y^{\mathrm{T}} & Z \end{bmatrix} \geqslant 0. \tag{3.21}$$

We apply the latter on the expression we have obtained above for Δ_i. From (3.21), taking $\mathcal{N} = \mathcal{N}_i = P^{\mathrm{T}}[\begin{smallmatrix} 0 \\ A_i \end{smallmatrix}]$, $R = R_{ia}$, $Z = Z_{ia}$, $Y = Y_{ia}$, $a = y(s)$ and $b = \bar{x}(t)$, we obtain, for $i = 1, 2$, that

$$\Delta_i(t) \leqslant \int_{t-h_i-\eta_i}^{t-h_i} [y^{\mathrm{T}}(s) \quad \bar{x}^{\mathrm{T}}(t)] \begin{bmatrix} R_{ia} & Y_{ia} - [0 \quad A_i^{\mathrm{T}}]P \\ Y_{ia}^{\mathrm{T}} - P^{\mathrm{T}}\begin{bmatrix} 0 \\ A_i \end{bmatrix} & Z_{ia} \end{bmatrix} \begin{bmatrix} y(s) \\ \bar{x}(t) \end{bmatrix} \mathrm{d}s$$

$$= \int_{t-h_i-\eta_i}^{t-h_i} y^{\mathrm{T}}(s) R_{ia} y(s) \mathrm{d}s + 2 \int_{t-h_i-\eta_i}^{t-h_i} y^{\mathrm{T}}(s)(Y_{ia} - [0 \quad A_i^{\mathrm{T}}]P)\bar{x}(t)\mathrm{d}s$$

$$+ \int_{t-h_i-\eta_i}^{t-h_i} \bar{x}(t)^{\mathrm{T}} Z_{ia} \bar{x}(t)\mathrm{d}s$$

$$= \int_{t-h_i-\eta_i}^{t-h_i} y^{\mathrm{T}}(s) R_i y(s) \mathrm{d}s + 2 \int_{t-h_i-\eta_i}^{t-h_i} \dot{x}^{\mathrm{T}}(s)(Y_{ia} - [0 \quad A_i^{\mathrm{T}}]P)\bar{x}(t)\mathrm{d}s$$

$$+ \eta_i \bar{x}(t)^{\mathrm{T}} Z_{ia} \bar{x}(t)$$

$$\leqslant \int_{t-h_i-\mu_i}^{t-h_i} y^{\mathrm{T}}(s) R_{ia} y(s) \mathrm{d}s + 2x^{\mathrm{T}}(t - h_i)(Y_{ia} - [0 \quad A_i^{\mathrm{T}}]P)\bar{x}(t)$$

$$- 2x^{\mathrm{T}}(t - h_i - \eta_i)(Y_{ia} - [0 \quad A_i^{\mathrm{T}}]P)\bar{x}(t) + \mu_i \bar{x}(t)^{\mathrm{T}} Z_i \bar{x}(t).$$

Substituting the latter into (3.17) and applying (3.3), (3.10) we find that $\dot{V} \leqslant \bar{\xi}^{\mathrm{T}}(t)\Gamma_{2A}\bar{\xi}(t)$, where

$$\bar{\xi}(t) = \mathrm{col}\{\bar{x}(t), x(t - h_1 - \eta_1), x(t - h_2 - \eta_2), y(t - g(t)), x(t - h_1), x(t - h_2)\}$$

and thus the LMIs of (ii) guarantee $\dot{V} < 0$ and asymptotic stability of (2.1a).   □

REMARK 1 Similar to the case of *small* uncertain delays, in Case 2A choosing $\mu_i \to 0$ and $S_{ia} \to 0$ we obtain conditions of Case 2B.

In Case 2 for $h_i \to 0$ choosing $S_i \to 0$ and $Y_{ia} \to Y_i$ we obtain that the criteria of Cases 2A and 2B coincide with those of Fridman & Shaked (2002) for *small* time-varying delays.

REMARK 2 For feasibility of the LMIs in Theorem 1, the LMIs $\Gamma_n < 0$ and (3.7b) should necessarily be feasible for the nominal system, which guarantee that the original system (2.1a) with constant delays τ_i is asymptotically stable $\forall \tau_i \in [0, h_i], i = 1, 2$.

If the *nominal* LMIs (3.7a,b) are feasible, then, evidently, the *perturbed* LMIs of Theorem 1 in the Cases 1 and 2B have solutions for small enough values of μ_i. In the Case 2A, the same is true for $S_{ia} = \rho I$, where ρ is small enough .

REMARK 3 The only difference between LMIs in Cases 1 and 2B is that Γ_1 has an additional positive term $\sum_{i=1}^{2} \mu_i R_{ia}$ and thus LMIs for Case 1 are more restrictive. On the other hand, in Case 1 the lengths of the delay intervals are $2\mu_i$, while in Case 2 it is μ_i. In most of the examples, criterion of Case 1 leads to the larger delay intervals, but in some examples (see Example 2 below) criterion of Case 2B gives less conservative results.

REMARK 4 It follows from (3.13a) that the diagonal elements $-S_{ia}(1 - d_i), i = 1, 2$, are negative, and thus $S_{ia} > 0$, since by assumption $d_i < 1$.

REMARK 5 If we apply the criterion of Case 1 of Theorem 1 for $h_i = \mu_i, i = 1, 2$, we obtain a new criterion for stability in the case of *small* fast-varying $\tau_i(t) \in [0, 2\mu_i]$. In all the examples that we have considered (see, e.g. Examples 1 and 2 below) the resulting criterion is less conservative than those of Fridman & Shaked (2002) for the case of fast-varying delays.

Theorem 1 can be readily used to verify the stability of (2.1a) over the uncertainty polytope

$$\bar{\Omega} = \sum_{j=1}^{N} f_j \bar{\Omega}_j, \quad \text{for some } 0 \leqslant f_j \leqslant 1, \sum_{j=1}^{N} f_j = 1,$$

where the N vertices of the polytope are described by

$$\bar{\Omega}_j = [A_0^{(j)} \quad A_1^{(j)} \quad A_2^{(j)}],$$

by solving the LMI simultaneously for all the N vertices, applying the same matrices P_2 and P_3 and solving for the N vertices only.

EXAMPLE 1 Consider the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x(t - \tau_1(t)) + \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} x(t - \tau_2(t)). \tag{3.22}$$

The stability of (3.22) has been analysed by Kharitonov & Niculescu (2003) in the case of single delay, where $\tau_1 \equiv 0, \tau_2 = 1 + \eta_2(t)$ and $\eta_2(t)$ is a differentiable sign-varying function satisfying $|\eta_2| \leqslant \mu_2, \dot{\eta}_2 \leqslant d_2 < 1$. The following values of μ_2 and d_2 for the asymptotic stability of (3.22) have been found: $d_2 < 0.8$ and $\mu_2 < \frac{1}{25600} < 0.00004$, i.e. $\tau_2(t) \in (0.99996, 1.00004)$.

Applying Case 2A of Theorem 1 for $\tau_1 \equiv 0, h_2 = 0.8$ and $d_2 \leqslant 0.8$ we obtain the maximum value of $\mu_2 = 0.4$, and thus for essentially larger interval $\tau_2 \in [0.8, 1.2]$, (3.22) is asymptotically stable.

Applying Case 1 of Theorem 1 for $h_2 = 1$, where $\eta_2(t)$ may be fast varying, we obtain even a larger interval $\tau_2(t) \in [0.73, 1.27]$ with $\mu_2 = 0.27$. The reason for relative conservativeness of the results for Case 2A in this example is that d_2 is large enough (close to 1) and for $d_2 \to 1$ the conditions of Case 2A are close to conditions of Case 2B. The latter conditions may be more conservative than those of Case 1 (see Remark 3). Choosing e.g. $d_2 = 0.2$ for Case 2A we find for $h_2 = 0.6$ the maximum $\mu_2 = 2.6$ and the resulting interval $[0.6, 3.2]$ becomes essentially larger than that by Case 1 of Theorem 1.

Applying Case 1 of Theorem 1 for $\tau_1 \equiv 0, h_2 = \mu_2 = 0.33$, we find the following stability interval for *small* fast-varying delay: $\tau_2(t) \in [0, 0.66]$. This interval is larger than the interval $\tau_2(t) \in [0, 0.56]$ obtained by Shaked & Fridman (2002). In this example, Case 2B of Theorem 1 leads to more conservative results than Case 1.

Consider next the case of two delays: $\tau_1 = 0.1 + \eta_1(t)$, $\tau_2 = 1.1 + \eta_2(t)$. By Case 1 of Theorem 1 we find that the system is asymptotically stable for $|\eta_1| \leqslant 0.015$, $|\eta_2| \leqslant 0.1$, i.e. for all $\tau_1(t) \in [0.085, 0.115]$, $\tau_2(t) \in [1, 1.2]$. Simulation results for (3.22) with the initial condition $x(0) = [1 \quad 5]^{\mathrm{T}}, x(t) = 0, t < 0$, and with $\tau_1 = 0.11, \tau_2 = 1 + 0.2|\sin 5t|$ (see Fig. 1 for plots of $x_1(t)$ and $x_2(t)$) show that the system is stable, while for a greater value of the delay $\tau_1 = 0.23$ and τ_2 as above, (3.22) becomes unstable (see Fig. 2).

EXAMPLE 2 Consider the scalar system

$$\dot{x}(t) = -x(t - \tau). \tag{3.23}$$

It is well known that for all constant $\tau \in [0, \pi/2)$ this system is asymptotically stable. Consider $\tau = h + \eta(t)$. By Kharitonov & Niculescu (2003), (3.23) is asymptotically stable for $h = 1, \dot{\eta} \leqslant 0.9$ and $|\eta(t)| \leqslant 0.0002$.

Applying Case 1 of Theorem 1 (with $h = 1, \mu = 0.18$), we obtain that for all piecewise-continuous (including fast-varying) delays $\tau(t) \in [0.82, 1.18]$ the system is asymptotically stable. Application of Case 2B of Theorem 1 (with $h = 0.82, \mu = 0.38$) leads to a slightly wider interval $\tau(t) \in [0.82, 1.2]$.
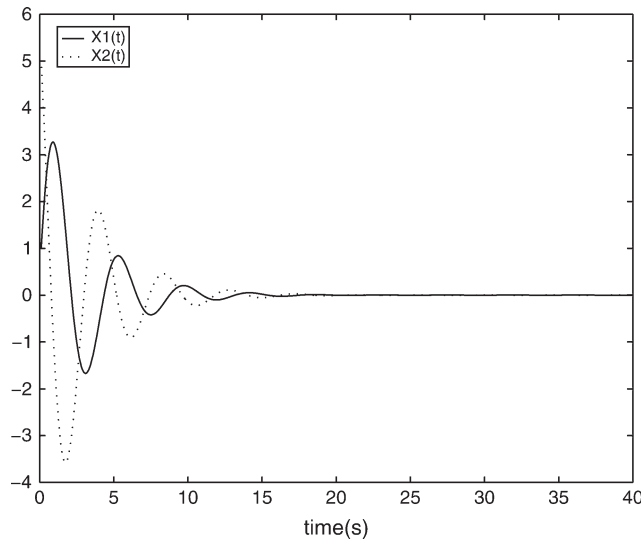

FIG. 1. Solution of (3.22) for $\tau_1 = 0.11, \tau_2 = 1 + 0.2|\sin 5t|$.
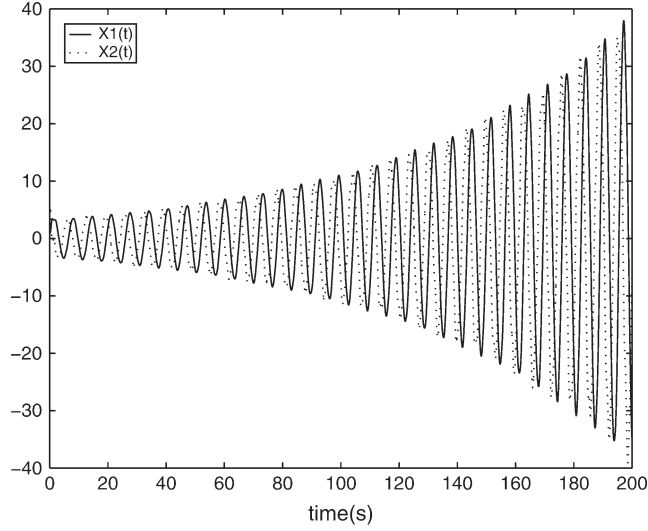
FIG. 2. Solution of (3.22) for $\tau_1 = 0.23$, $\tau_2 = 1 + 0.2|\sin 5t|$.

Also in this example, the method of Shaked & Fridman (2002) leads to a more conservative result of fast-varying $\tau(t) \in [0, 0.99]$ than that of $\tau(t) \in [0, 1.11]$ (for $h = \mu = 0.505$) obtained by Case 1 of Theorem 1.

Note that for constant τ by descriptor approach (Fridman & Shaked, 2002) the system is stable for all $\tau \in [0, 1.41]$. Hence, for $h \in (1.41, \pi/2)$ the method of the present paper is not applicable. Another (*complete*) nominal LKF should be used.

4. H_∞ control

4.1 *Bounded real lemma (BRL)*

In this section for simplicity we will consider the *mixed* case of delays: τ_1 of Case 1 and τ_2 of Case 2A. Consider (2.1a), where $B_2 = 0$, $D = 0$ and $\phi = 0$. We are looking for conditions which guarantee that $J < 0$, for all $w(t) \in \mathcal{L}_2$. Using the arguments of Theorem 1 and finding the conditions that $\dot{V} + z^T z - \gamma^2 w^T w < 0$, we obtain similarly to Fridman & Shaked (2003) the following BRL:

LEMMA 1 For a prescribed $\gamma > 0$, consider (2.1a) with $B_2 = 0$, $D = 0$, $\phi = 0$ and delays given by (2.2), where $\eta_1(t)$ is a piecewise-continuous function satisfying $|\eta_1(t)| \leqslant \mu_1 \leqslant h_1$ and $\eta_2(t)$ is a differentiable function satisfying (2.4), where $i = 2$. The cost function (2.5) achieves $J < 0$ for all non-zero $w \in \mathcal{L}_2$ if there exist $n \times n$ matrices $0 < P_1, P_2, P_3, S_i, U, S_{1a}, Y_{i1}, Y_{i2}, Y_{21a}, Y_{22a}, Z_{21a}, Z_{22a}, Z_{23a}, Z_{i1}, Z_{i2}, Z_{i3}, R_i$ and $R_{ia} > 0$, $i = 1, 2$, that satisfy (3.7b), (3.13b) with $i = 2$ and the following LMI:

$$\begin{bmatrix} \Gamma_{12} & P^T \begin{bmatrix} 0 \\ B_1 \end{bmatrix} & \begin{bmatrix} C^T \\ 0 \end{bmatrix} \\ & 0 & 0 \\ * & -\gamma^2 I_q & 0 \\ * & * & -I_l \end{bmatrix} < 0, \tag{4.1}$$

E. FRIDMAN

where

$$
\Gamma_{12} =
\begin{bmatrix}
\Psi_{12} & P^{\mathrm{T}}\begin{bmatrix}0\\A_1\end{bmatrix} - Y_1^{\mathrm{T}} & P^{\mathrm{T}}\begin{bmatrix}0\\A_2\end{bmatrix} - Y_{2a}^{\mathrm{T}} & P^{\mathrm{T}}\begin{bmatrix}0\\F\end{bmatrix} & \mu_1 P^{\mathrm{T}}\begin{bmatrix}0\\A_1\end{bmatrix} & Y_{2a}^{\mathrm{T}} - Y_2^{\mathrm{T}} \\
* & -S_1 & 0 & 0 & 0 & 0 \\
* & * & -(1-d_2)S_{2a} & 0 & 0 & 0 \\
* & * & * & -(1-d_0)U & 0 & 0 \\
* & * & * & * & -\mu_1 R_{1a} & 0 \\
* & * & * & * & * & -S_2
\end{bmatrix},
\quad (4.2)
$$

$$
Y_{2a} = [Y_{21a} \quad Y_{22a}], \quad Z_{2a} = \begin{bmatrix} Z_{21a} & Z_{22a} \\ * & Z_{23a} \end{bmatrix},
$$

$$
\Psi_{12} = \Psi_n + \mu_2 Z_{2a} + \begin{bmatrix} S_{2a} & 0 \\ 0 & 2\mu_1 R_{1a} + \mu_2 R_{2a} \end{bmatrix},
$$

(4.3)

and where Y_i, Z_i and Ψ_n are given by (3.8).

4.2   *State-feedback H_∞ control*

Consider (2.1a) with $B_2 \neq 0$, $D \neq 0$. We apply the BRL of Lemma 1 to the closed-loop system (2.1a), where $u(t) = Kx(t)$. Following Suplin *et al.* (2004) we choose $P_3 = \varepsilon P_2$, $\varepsilon \in R$, where ε is a tuning scalar parameter. Note that P_2 is non-singular due to the fact that the only matrix which can be negative definite in the second block on the diagonal of (4.1) is $-\varepsilon(P_2 + P_2^{\mathrm{T}})$. Defining

$$
\bar{P} = P_2^{-1}, \left[\bar{P}_1 \quad \bar{Y}_{ij} \quad \bar{Y}_{2ja} \quad \bar{S}_i \quad \bar{U} \quad \bar{R}_i \quad \bar{R}_{ia} \quad \bar{Z}_{ik} \quad \bar{Z}_{2ka}\right]
$$

$$
= \bar{P}^{\mathrm{T}}\left[P_1\bar{P} \quad Y_{ij}\bar{P} \quad Y_{2ja}\bar{P} \quad S_i\bar{P} \quad U\bar{P} \quad R_i\bar{P} \quad R_{ia}\bar{P} \quad Z_{ik}\bar{P} \quad Z_{2ka}\bar{P}\right],
$$

$$
i = 1, 2, \ j = 1, 2, \ k = 1, 2, 3,
$$

and $W = K\bar{P}$, multiplying (4.1) by $\mathrm{diag}\{\bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, \bar{P}, I_q, I_l\}$ and its transpose, from the right and the left, respectively, and multiplying (3.7b), (3.13b) by $\mathrm{diag}\{\bar{P}, \bar{P}, \bar{P}\}$ and its transpose, from the right and the left, we obtain:

THEOREM 2 For a prescribed $\gamma > 0$, consider (2.1a) with delays given by (2.2), where $\eta_1(t)$ is a piecewise-continuous function satisfying $|\eta_1(t)| \leqslant \mu_1 \leqslant h_1$ and $\eta_2(t)$ is a differentiable function satisfying (2.4), where $i = 2$. Under the state-feedback law $u = Kx(t)$, the system (2.1a) is asymptotically stable and for a prescribed scalar γ, $J < 0, \forall 0 \neq w(t) \in \mathcal{L}_2, \phi = 0$, if for some tuning scalar parameter ε there exist $n \times n$ matrices $0 < \bar{P}_1, \bar{P}, \bar{S}_i, \bar{U}, \bar{S}_{2a}, \bar{Y}_{ij}, \bar{Y}_{2ja}, \bar{Z}_{2ka}, \bar{Z}_{ik}, \bar{R}_i, \bar{U}, \bar{R}_{ia} > 0, i = 1, 2, j = 1, 2, k = 1, 2, 3, W \in R^{m \times n}$ that satisfy (3.7b), (3.13b) with $i = 2$ and the

following LMIs:

$$
\begin{bmatrix}
\begin{bmatrix} \Phi_1 & \Phi_2 \\ \Phi_2^{\mathrm{T}} & \Phi_3 \end{bmatrix} & \begin{bmatrix} A_1\bar{P} - \bar{Y}_{11}^{\mathrm{T}} \\ \varepsilon A_1\bar{P} - \bar{Y}_{12}^{\mathrm{T}} \end{bmatrix} & \begin{bmatrix} A_2\bar{P} - \bar{Y}_{21a}^{\mathrm{T}} \\ \varepsilon A_2\bar{P} - \bar{Y}_{22a}^{\mathrm{T}} \end{bmatrix} & \begin{bmatrix} F\bar{P} \\ \varepsilon F\bar{P} \end{bmatrix} & \mu_1 \begin{bmatrix} A_1\bar{P} \\ \varepsilon A_1\bar{P} \end{bmatrix} & \begin{bmatrix} \bar{Y}_{21a}^{\mathrm{T}} - \bar{Y}_{21}^{\mathrm{T}} \\ \bar{Y}_{22a}^{\mathrm{T}} - \bar{Y}_{22}^{\mathrm{T}} \end{bmatrix} \\
* & -\bar{S}_1 & 0 & 0 & 0 & 0 \\
* & * & -(1-d_2)\bar{S}_{2a} & 0 & 0 & 0 \\
* & * & * & -(1-d_0)\bar{U} & 0 & 0 \\
* & * & * & * & -\mu_1\bar{R}_{1a} & 0 \\
* & * & * & * & * & -\bar{S}_2 \\
* & * & * & * & * & * \\
* & * & * & * & * & *
\end{bmatrix}
$$

$$
\begin{bmatrix}
\begin{bmatrix} B_1 \\ \varepsilon B_1 \end{bmatrix} & \begin{bmatrix} \bar{P}^{\mathrm{T}}C^{\mathrm{T}} + W^{\mathrm{T}}D^{\mathrm{T}} \\ 0 \end{bmatrix} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
-\gamma^2 I_q & 0 \\
* & -I_l
\end{bmatrix} < 0,
\tag{4.4}
$$

$$
\begin{bmatrix} \bar{R}_i & \bar{Y}_{i1} & \bar{Y}_{i2} \\ * & \bar{Z}_{i1} & \bar{Z}_{i2} \\ * & * & \bar{Z}_{i3} \end{bmatrix} > 0, \quad i = 1, 2, \qquad
\begin{bmatrix} \bar{R}_{2a} & \bar{Y}_{21a} & \bar{Y}_{22a} \\ * & \bar{Z}_{21a} & \bar{Z}_{22a} \\ * & * & \bar{Z}_{23a} \end{bmatrix} > 0,
$$

where

$$
\Phi_1 = \bar{P}^{\mathrm{T}}A_0^{\mathrm{T}} + A_0\bar{P} + B_2 W + W^{\mathrm{T}}B_2^{\mathrm{T}} + \sum_{i=1}^{2}[\bar{Y}_{i1} + \bar{Y}_{i1}^{\mathrm{T}} + \bar{S}_i + h_i\bar{Z}_{i1}] + \bar{S}_{2a} + \mu_2\bar{Z}_{21a},
$$

$$
\Phi_2 = \bar{P}_1 - \bar{P} + \varepsilon(\bar{P}^{\mathrm{T}}A_0^{\mathrm{T}} + W^{\mathrm{T}}B_2^{\mathrm{T}}) + \sum_{i=1}^{2}[\bar{Y}_{i2} + h_i\bar{Z}_{i2}] + \mu_2\bar{Z}_{22a},
$$

$$
\Phi_3 = -\varepsilon(\bar{P}^{\mathrm{T}} + \bar{P}) + \bar{U} + \sum_{i=1}^{2}h_i(\bar{R}_i + \bar{Z}_{i3}) + \mu_2\bar{Z}_{23a} + 2\mu_1\bar{R}_{1a} + \mu_2\bar{R}_{2a}.
$$

The state-feedback gain is then given by

$$
K = W\bar{P}^{-1}.
\tag{4.4c}
$$

EXAMPLE 3 We address the problem of finding a H_∞ state-feedback controller for (2.1a) with one delay, where

$$A_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = [0, 1], \quad D = 0.1.$$

By descriptor method, the system is stabilizable for all fast-varying small delays $\tau(t) \in [0, 1)$ (Fridman & Shaked, 2003). Hence, the existing method for fast-varying delays $\tau(t) \geqslant 1$ is not applicable. For constant delays by descriptor method with iterative search of gain (Gao & Wang, 2003) the system is stabilizable for $\tau \in [0, 3.2]$.

Choosing e.g. $h = 2, |\eta(t)| \leqslant 0.2, \varepsilon = 1$ and applying Theorem 2, we find that the gain $K = -[74.8 \ 105.5]$ stabilizes the system for all time-varying delays $\tau(t) \in [1.8, 2.2]$ and leads to $\gamma = 6$.

5. Conclusions

A new Lyapunov–Krasovskii technique is introduced for stability and control of linear systems with uncertain time-varying delay in the case when the nominal value of the delay is constant and non-zero. The following construction of LKF is suggested: to a nominal LKF, which is appropriate to the nominal system (with nominal delays), terms are added that correspond to the perturbed system and that vanish when the delay perturbations approach 0. In the present paper the nominal descriptor-type LKF is considered. The method is applied to the stability and state-feedback H_∞ control problem. Sufficient LMI conditions are derived which are affine in the system matrices and, thus, the results for the case of the systems with polytopic-type uncertainties are straightforward. As a by-product, new criteria are also derived for the case of *small* fast-varying delays from $[0, \mu]$. Illustrative examples show the efficiency of the method.

The obtained LMIs may be feasible if the nominal LMIs (for the system with nominal values of the delay) based on descriptor method are feasible. If the latter assumption does not hold, the other nominal LKF (e.g. the *complete* LKF) should be applied. This case is currently under study. The new Lyapunov technique may be applied to the discrete-time delay systems.

Acknowledgements

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