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Distributed sampled-data control of Kuramoto–Sivashinsky equation☆

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ABSTRACT

The paper is devoted to distributed sampled-data control of nonlinear PDE system governed by 1-D Kuramoto–Sivashinsky equation. It is assumed that N sensors provide sampled in time spatially distributed (either point or averaged) measurements of the state over N sampling spatial intervals. Locally stabilizing sampled-data controllers are designed that are applied through distributed in space shape functions and zero-order hold devices. Given upper bounds on the sampling intervals in time and in space, sufficient conditions ensuring regional exponential stability of the closed-loop system are established in terms of Linear Matrix Inequalities (LMIs) by using the time-delay approach to sampled-data control and Lyapunov–Krasovskii method. As it happened in the case of diffusion equation, the descriptor method appeared to be an efficient tool for the stability analysis of the sampled-data Kuramoto–Sivashinsky equation. An estimate on the domain of attraction is also given. A numerical example demonstrates the efficiency of the results.

1. Introduction

Kuramoto–Sivashinsky equation (KSE) describes a variety of physical and chemical phenomena including magnetized plasmas, flame front propagation, viscous flow problems and chemical reaction–diffusion processes (see e.g. Kuramoto & Tsuzuki, 1975; Sivashinsky, 1977; Lunasin & Titi, 2017). Boundary control of 1-D KSE was studied in Coron and Lü (2015) and Liu and Krstic (2001). The local rapid stabilization problem for a controlled KSE on a bounded interval was considered in Coron and Lü (2015). In Liu and Krstic (2001), a Neumann feedback law was designed to guarantee L^2-global exponential stability and H^2-global asymptotic stability for small values of the anti-diffusion parameter.

Distributed control of KSE was studied in Armaou and Christofides (2000a, b), Christofides and Armaou (2000) and Lunasin and Titi (2017). In Armaou and Christofides (2000a, b), a finite-dimensional controller was designed on the basis of a finite-dimensional system that captures the dominant (slow) dynamics of the infinite-dimensional system. In Christofides and Armaou (2000), the problem of global exponential stabilization of the KSE subject to periodic boundary conditions was considered. In Lunasin and Titi (2017), a distributed finite-dimensional feedback controller based on either point or averaged measurements of the state was proposed.

For practical application of finite-dimensional controllers for partial differential equations (PDEs), their sampled-data implementation is important. Sampled-data control of PDEs is becoming a hot topic. Sampled-data control of KSE was studied in Ghantasala and El-Farra (2012), where model reduction approach was suggested, and the design was based on the finite-dimensional system that captures the dominant dynamics. The latter approach is a qualitative one without giving explicit bounds on the performance (e.g. decay rate) or on the domain of attraction of the closed-loop system.

Distributed sampled-data control of PDEs under the point or spatially averaged measurements was suggested in Bar Am and Fridman (2014), Fridman and Bar Am (2013) and Fridman and Blighovsky (2012), where LMI conditions for the exponential stability and L_2-gain analysis of the closed-loop systems were derived in the framework of time-delay approach to sampled-data control by employing appropriate Lyapunov functionals. However, the above results were *confined to diffusion equations and to globally Lipschitz nonlinearities*, where stabilization is global. Distributed sampled-data control of various classes of PDEs is of great interest.

In the present paper, we introduce distributed sampled-data control of 1-D nonlinear KSE with the Dirichlet or periodic boundary conditions. The sensors provide either point or averaged

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discrete-time measurements of the state. The controllers enter KSE through distributed in space shape functions and the control signals are generated by zero-order hold devices. As for the diffusion equation, we exploit the time-delay approach to sampled-data control and the descriptor method for Lyapunov–Krasovskii-based delay-dependent stability analysis (Fridman, 2001, 2014; Fridman & Orlov, 2009). In terms of LMIs, we give regional exponential stability conditions for the sampled-data closed-loop system and find a bound on the domain of attraction (i.e. on the set of initial conditions, starting from which the solutions are exponentially converging). Under the corresponding continuous-time controllers, we derive LMI conditions for the global exponential stability. Some preliminary results under point state measurements will be presented in Kang and Fridman (2018).

The paper is organized as follows. Problem formulation is given in Section 2. In Sections 3 and 4, continuous in time and sampled-data controllers under the point or averaged state measurements are constructed to stabilize the system. Section 5 contains a numerical example to illustrate the efficiency of the main results. Finally, some concluding remarks are presented in Section 6 and some proofs are given in the Appendix.

Notation. $L^2(0,L)$ stands for the Hilbert space of square integrable scalar functions $u(x)$ on $(0,L)$ with the corresponding norm $\|u\|_{L^2}=[\int_0^L u^2(x)dx]^{\frac{1}{2}}$. The Sobolev space $H^k(0,L)$ is defined as

$$H^k(0,L)=\{u: D^\alpha u\in L^2(0,L),\ \forall\, 0\le|\alpha|\le k\}$$

with norm $\|u\|_{H^k}=\left\{\sum_{0\le|\alpha|\le k}\|D^\alpha u\|_{L^2}^2\right\}^{\frac{1}{2}}$. Moreover,

$$H_0^k(0,L)=\{u\in H^k(0,L): u(0)=Du(0)=\cdots$$
$$=D^{k-1}u(0)=0,\ u(L)=Du(L)=\cdots=D^{k-1}u(L)=0\}.$$

2. Problem formulation and useful lemmas

We consider 1-D Kuramoto–Sivashinsky equation

$$u_t(x,t)+u_{xx}(x,t)+\nu u_{xxxx}(x,t)+u(x,t)u_x(x,t)$$
$$=\sum_{j=1}^N b_j(x)U_j(t),\ 0<x<L,\ t\ge 0, \tag{2.1}$$

subject to Dirichlet

$$u(0,t)=u(L,t)=0,\ u_x(0,t)=u_x(L,t)=0 \tag{2.2}$$

or to periodic

$$\frac{\partial^m u}{\partial x^m}(0,t)=\frac{\partial^m u}{\partial x^m}(L,t),\ m=0,1,2,3 \tag{2.3}$$

boundary conditions. Here ν is a positive constant, $u(x,t)$ is the state of KSE, and $U_j(t)\in\mathbb{R}, j=1,2,\ldots,N$ are the control inputs. Dirichlet boundary conditions were considered in Liu and Krstic (2001), whereas the periodic ones were studied in Armaou and Christofides (2000a, b) and Lunasin and Titi (2017). The open-loop system (2.1) (subject to $U_j(t)\equiv 0$) may become unstable if ν is small enough. Thus, for $L=2\pi$ if $\nu<1$ the open-loop system is unstable (see the example below).

As in Azouani and Titi (2014), Fridman and Bar Am (2013), Fridman and Blighovsky (2012) and Lunasin and Titi (2017), consider the points

$$0=x_0<x_1<\cdots<x_N=L$$

that divide $[0,L]$ into N sampling intervals $\Omega_j=[x_{j-1},x_j)$. Let

$$0=t_0<t_1<\cdots<t_k\cdots,\quad \lim_{k\to\infty}t_k=\infty$$

be sampling time instants. The sampling intervals in time and in space may be variable but bounded,

$$0\le t_{k+1}-t_k\le h,\ 0<x_j-x_{j-1}=\Delta_j\le\Delta,$$

where h and Δ are the corresponding upper bounds. The control inputs $U_j(t)$ enter (2.1) through the shape functions

$$\begin{cases} b_j(x)=1,\ x\in\Omega_j,\\ b_j(x)=0,\ x\notin\Omega_j,\end{cases} j=1,\ldots,N. \tag{2.4}$$

Sensors provide either point

$$y_{jk}=u(\bar{x}_j,t_k),\bar{x}_j=\frac{x_{j-1}+x_j}{2},\quad \begin{array}{l} j=1,\ldots,N,\\ k=0,1,2\ldots\end{array} \tag{2.5}$$

or averaged

$$y_{jk}=\frac{\int_{x_{j-1}}^{x_j}u(x,t_k)dx}{\Delta_j},\ j=1,\ldots,N,\ k=0,1,2\ldots \tag{2.6}$$

measurements of the state. Our main objective is to design for (2.1) an exponentially stabilizing sampled-data controller that can be implemented by zero-order hold devices:

$$U_j(t)=-\mu y_{jk},\ j=1,\ldots,N,\ t\in[t_k,t_{k+1}),\ k=0,1,\ldots, \tag{2.7}$$

where μ is a positive controller gain and y_{jk} is given by (2.5) or (2.6).

We formulate next some useful lemmas.[1]

Lemma 2.1 (*Poincaré Inequality Payne & Weinberger, 1960*). *Let $g\in H^1(0,L)$ be a scalar function with $\int_0^L g(x)dx=0$. Then*

$$\int_0^L g^2(x)dx\le\frac{L^2}{\pi^2}\int_0^L\left[\frac{dg}{dx}(x)\right]^2dx.$$

Lemma 2.2 (*Wirtinger Inequality and its Generalization Wang, 1994*). *Let $g\in H_0^1(0,L)$. Then the following inequality holds:*

$$\int_0^L g^2(x)dx\le\frac{L^2}{\pi^2}\int_0^L\left[\frac{dg}{dx}(x)\right]^2dx.$$

Moreover, if $g\in H_0^2(0,L)$, then

$$\int_0^L\left[\frac{dg}{dx}(x)\right]^2dx\le\frac{L^2}{\pi^2}\int_0^L\left[\frac{d^2g}{dx^2}(x)\right]^2dx.$$

Lemma 2.3 (*Halanay's Inequality Halanay, 1966 or p.138 of Fridman, 2014*). *Let $0<\delta_1<2\delta$ and let $V_1:[t_0-h,\infty)\to[0,\infty)$ be an absolutely continuous function that satisfies*

$$\dot{V}_1(t)\le-2\delta V_1(t)+\delta_1\sup_{-h\le\theta\le 0}V_1(t+\theta),\ t\ge t_0.$$

Then

$$V_1(t)\le e^{-2\alpha(t-t_0)}\sup_{-h\le\theta\le 0}V_1(t_0+\theta),\ t\ge t_0,$$

where α is a unique positive solution of

$$\alpha=\delta-\frac{\delta_1}{2}e^{2\alpha h}. \tag{2.8}$$

3. Continuous-time global stabilization

We will start with continuous in time results, where global stabilization can be achieved. Here the stability analysis is similar to Lunasin and Titi (2017), but differently from Lunasin and Titi (2017) we give a bound on the decay rate. Sampled-data controllers under the point/averaged measurements leading to regional stability will be presented in Section 4.

[1] It should be noted that the first Wirtinger's inequality in Lemma 2.2 is the one-dimensional Poincaré's inequality in Lemma 2.1 with optimal constant. This can be easily proved by the minimization principle of the nth eigenvalue.

In this section, we consider the stabilization of (2.1) with the Dirichlet or periodic boundary conditions under the continuous-time measurements

$$y_j(t) = u(\bar{x}_j, t),\ \bar{x}_j = \frac{x_{j-1}+x_j}{2},\ j = 1, \ldots, N, \tag{3.1}$$

or

$$y_j(t) = \frac{\int_{x_{j-1}}^{x_j} u(x,t)dx}{\Delta_j} \tag{3.2}$$

via a continuous-time control law

$$U_j(t) = -\mu y_j(t). \tag{3.3}$$

The control law (3.3) can be presented as

$$U_j(t) = -\mu[u(x,t) - f_j(x,t)],$$

where for (3.1)

$$f_j(x,t) = \int_{\bar{x}_j}^{x} u_\xi(\xi,t)d\xi, \tag{3.4}$$

and for (3.2)

$$f_j(x,t) = \frac{\int_{x_{j-1}}^{x_j}[u(x,t) - u(\zeta,t)]d\zeta}{\Delta_j}. \tag{3.5}$$

Then the closed-loop system (2.1), (3.3) has a form

$$u_t(x,t) + u_{xx}(x,t) + \nu u_{xxxx}(x,t) + u(x,t)u_x(x,t) = -\sum_{j=1}^{N} \mu b_j(x)[u(x,t) - f_j(x,t)], \quad t \geq 0. \tag{3.6}$$

We aim to find μ that enlarges Δ (leading to smaller number of sensors).

3.1. Well-posedness and stability of (3.6) subject to (2.2)

Let $H = L^2(0, L)$ be a Hilbert space with the inner product $\langle\cdot,\cdot\rangle$ and induced norm $\|\cdot\|_{L^2}$. We define an unbounded linear operator $A : D(A) \subset H \to H$ as follows:

$$\begin{cases} Af = -\nu f'''', \ \forall f \in D(A), \\ D(A) = H^4(0,L) \cap H_0^2(0,L). \end{cases} \tag{3.7}$$

It is well-known that A is a dissipative operator, and A generates an analytic semigroup $T(t)$.

The domain $H_1 = D(A) = A^{-1}H$ forms another Hilbert space with the graph inner product $\langle x, y\rangle_1 = \langle Ax, Ay\rangle$, $\forall x, y \in H_1$. The domain $D(A)$ is dense in H.

Operator $-A$ is positive implying that its square root $(-A)^{\frac{1}{2}}$ is also positive. We define the Hilbert space:

$$H_{\frac{1}{2}} = D((-A)^{\frac{1}{2}}) = H_0^2(0,L).$$

The norm of $H_{\frac{1}{2}}$ is endowed by the induced inner product:

$$\langle f, g\rangle_{\frac{1}{2}} = \langle(-A)^{\frac{1}{2}}f, (-A)^{\frac{1}{2}}g\rangle \quad \forall f, g \in H_{\frac{1}{2}},$$

$$\|f\|_{H_{\frac{1}{2}}} = \nu^{\frac{1}{2}}\left[\int_0^1 |f''(x)|^2 dx\right]^{\frac{1}{2}} \quad \forall f \in H_{\frac{1}{2}}. \tag{3.8}$$

Note that $H_{\frac{1}{2}}$ norm is equivalent to the inherent norm $\|\cdot\|_{H^2}$ of Sobolev space $H^2(0,L)$. We define the spaces $H_{-\frac{1}{2}}$ and H_{-1} as the dual spaces of $H_{\frac{1}{2}}$ and H_1 respectively, with respect to the pivot space H. Therefore, $H_{-\frac{1}{2}} = H^{-2}(0,L)$. Then we have $H_1 \subset H_{\frac{1}{2}} \subset H \subset H_{-\frac{1}{2}} \subset H_{-1}$, densely and with continuous embedding. All relevant material on fractional operator degrees can be found in Tucsnak and Weiss (2009) (see pp. 81–83).

The nonlinear term $F : H^2(0,L) \to L^2(0,L)$ is defined on functions $u(\cdot,t)$ according to

$$F(u(\cdot,t)) = -u(x,t)u_x(x,t) - u_{xx}(x,t) - \sum_{j=1}^{N} \mu b_j(x)[u(x,t) - f_j(x,t)], \ \ t \geq 0.$$

With the operator A at hand, the system (3.6) subject to (2.2) can be written as an evolution equation:

$$\begin{cases} \frac{d}{dt}u(\cdot,t) = Au(\cdot,t) + F(u(\cdot,t)), \\ u(\cdot,0) = u_0(\cdot). \end{cases} \tag{3.9}$$

Note that the nonlinear term F is locally Lipschitz continuous, that is, there exists a positive constant $l(K)$ such that the following inequality

$$\|F(u_1) - F(u_2)\|_{L^2} \leq l(K)\|(u_1 - u_2)\|_{H_0^2(0,L)}$$

holds for $u_1, u_2 \in H_0^2(0,L)$ with $\|u_1\|_{H_0^2(0,L)} \leq K$, $\|u_2\|_{H_0^2(0,L)} \leq K$. Thus, by Theorem 6.3.1 of Pazy (1983), we obtain that for any initial condition $u_0 \in H_0^2(0,L)$, there exists a unique local classical solution $u \in C([0,T), L^2(0,L)) \cap C^1((0,T), L^2(0,L))$ of (3.6) subject to (2.2), where $T = T(u_0) > 0$.

The following proposition provides conditions that guarantee the existence of solution of the closed-loop system (3.6) subject to (2.2) for all $t \geq 0$ as well as the global stability of this system:

Proposition 3.1. *Consider the closed-loop system (3.6) subject to (3.4) or (3.5) under the Dirichlet boundary conditions (2.2). Given a scalar $\alpha > 0$ and tuning parameter $\Delta > 0$, let there exist scalars $\mu > 0$, $\lambda \geq 0$ and $\lambda_1 > 0$ that satisfy the following LMI*

$$W = \begin{bmatrix} W_{11} & -1 - \frac{\lambda_1}{2} & \mu \\ * & -2\nu + \lambda & 0 \\ * & * & -\lambda_1\frac{\pi^2}{\Delta^2} \end{bmatrix} < 0, \tag{3.10}$$

where

$$W_{11} = -2\mu + 2\alpha - \lambda\frac{\pi^4}{L^4}. \tag{3.11}$$

Then a unique classical solution of the Dirichlet boundary value problem (3.6), (2.2) subject to (3.4) or (3.5) initialized by $u_0 \in H_0^2(0,L)$ exists for all $t \geq 0$ and the system is exponentially stable with a decay rate α, i.e. its classical solutions satisfy

$$\int_0^L u^2(x,t)dx \leq e^{-2\alpha t}\int_0^L u^2(x,0)dx, \ \ \forall t \geq 0. \tag{3.12}$$

If the LMI (3.10) is feasible with $\alpha = 0$, then the closed-loop system is exponentially stable with a small enough decay rate $\alpha_0 > 0$. Particularly, if $\frac{\Delta^2}{\pi^2} < \nu$ and $\mu = \frac{2\pi^2}{\Delta^2}$, then (3.10) is feasible with $\lambda = \alpha = 0$ and $\lambda_1 = 2$, meaning that the closed-loop system is exponentially stable.

Proof. See Appendix A. □

3.2. Extension to periodic boundary conditions

The well-posedness under the periodic boundary conditions can be established similar to the case of Dirichlet boundary conditions. Here instead of $H_0^2(0,L)$ we consider a Hilbert space $H_{per}^2 = \{f \in H^2(0,L) : f^{(m)}(0) = f^{(m)}(L),\ \ m = 0, 1\}$ with respect to the H^2 norm. By arguments of Proposition 3.1, where in (4.23) $\lambda = 0$, because Wirtinger's inequality is not applicable here, we arrive at the following result:

If the LMI condition of Proposition 3.1 holds with $\lambda = 0$, then there exists a unique solution of the periodic boundary value problem (3.6), (2.3) subject to (3.4) or (3.5) initialized by $u(\cdot, 0) \in H^2_{per}$ in the sense that $u \in C([0,\infty), L^2(0,L)) \cap C^1((0,\infty), L^2(0,L))$. Moreover, the solution satisfies (3.12) meaning that the closed-loop system (3.6), (2.3) subject to (3.4) or (3.5) is exponentially stable with a decay rate α.

Remark 3.1. In Lunasin and Titi (2017) stabilization of (2.1) was studied subject to the periodic boundary conditions (2.3) under the assumption $\int_0^L u_0(x)dx = 0$ (that implies also $\int_0^L u(x,t)dx = 0$). In this case Poincaré inequality allows to use (4.23) with $\lambda \geq 0$ as for the case of Dirichlet boundary conditions leading to LMI (3.10) with $\lambda \geq 0$. The stability conditions of (3.6) subject to (2.3) were given in Lunasin and Titi (2017) as $\mu > \frac{4}{\nu}$, $\nu > \mu\frac{\Delta^4}{\pi^4}$ (cf. (15) in Lunasin & Titi, 2017). The LMI condition (3.10) of Proposition 3.1 is less conservative (due to the use of $\lambda_1 > 0$ and $\lambda \geq 0$ instead of conservative use of Young inequality with a specific constant), and gives also a bound on the decay rate. The advantages of LMI condition (3.10) are illustrated in the example below (see Section 5).

4. Sampled-data regional stabilization

4.1. Sampled-data control under point measurements

By selecting the controller (2.7) subject to (2.5), we arrive at the closed-loop system

$$\begin{aligned} &u_t(x,t) + u_{xx}(x,t) + \nu u_{xxxx}(x,t) + u(x,t)u_x(x,t) \\ &= -\mu\sum_{j=1}^{N} b_j(x)u(\bar{x}_j, t_k),\ \ t\in[t_k,t_{k+1}),\ \ k=0,1,\ldots \end{aligned} \tag{4.1}$$

subject to (2.2) or (2.3).

4.1.1. Well-posedness and stability of (4.1) subject to (2.2)

We start with the well-posedness of the sampled-data closed-loop system (4.1) under the Dirichlet boundary conditions (2.2) initialized with $u_0(x) = u(x,0)$. We will use the step method for solution of time-delay systems (Bellman & Cooke, 1963; Fridman, 2014).

For $t \in [t_0, t_1]$, we consider the following equation:

$$\begin{cases} u_t(x,t) + u_{xx}(x,t) + \nu u_{xxxx}(x,t) + u(x,t)u_x(x,t) \\ = -\mu\sum_{j=1}^{N} b_j(x)u(\bar{x}_j, t_0), \\ u(0,t) = u(L,t) = u_x(0,t) = u_x(L,t) = 0. \end{cases} \tag{4.2}$$

We can present the system (4.2) as an evolution equation (3.9) with A defined by (3.7). Here the nonlinear term $F : H^2(0,L) \to L^2(0,L)$ is defined on functions $u(\cdot,t)$ according to

$$\begin{aligned} F(u(\cdot,t)) &= -u(x,t)u_x(x,t) - u_{xx}(x,t) \\ &\quad - \sum_{j=1}^{N}\mu b_j(x)u_0(\bar{x}_j),\ \ t\in[t_0,t_1). \end{aligned} \tag{4.3}$$

A function $u \in C([0,T]; H_0^2(0,L)) \cap L^2([0,T]; D(A))$ such that $\dot{u} \in L^2([0,T]; L^2(0,L))$ is called a strong solution of (3.9) with the nonlinearity F given by (4.3) if (3.9) holds almost everywhere on $[0,T]$.

From (4.3) it follows that the nonlinear term F is locally Lipschitz continuous. Thus, Theorem 3.3.3 of Henry (1981) is applicable to (3.9). Given any initial condition $u_0 \in H_0^2(0,L)$, there exists a unique local strong solution $u(\cdot,t) \in H_0^2(0,L)$ of (4.2) on some interval $[0,T] \subset [0,t_1]$, where $T = T(u_0) > 0$. From Theorem 6.23.5 of Krasnoselskii, Zabreiko, Pustylii, and Sobolevskii (1976), it follows that if this solution admits a priori estimate, then the solution exists on the entire interval $[0,t_1]$. The a priori estimate on the solutions starting from the domain of attraction will be guaranteed by the stability conditions that we will provide (see Theorem 4.1).

For the stability analysis, we present (4.1) as

$$\begin{aligned} &u_t(x,t) + u_{xx}(x,t) + \nu u_{xxxx}(x,t) + u(x,t)u_x(x,t) \\ &= -\mu u(x,t_k) + \mu\sum_{j=1}^{N} b_j(x)\int_{\bar{x}_j}^{x} u_\xi(\xi,t_k)d\xi, \\ &\qquad t\in[t_k,t_{k+1}),\ \ k=0,1,\ldots \end{aligned} \tag{4.4}$$

We will use an input delay approach to sampled-data control (Fridman, Seuret, & Richard, 2004; Mikheev, Sobolev, & Fridman, 1988), where the sampling time t_k is presented as delayed time $t - \tau(t)$ with $\tau(t) = t - t_k$ for $t \in [t_k, t_{k+1})$.

In order to derive the stability conditions for (4.4) we employ the following Lyapunov–Krasovskii functional

$$\begin{aligned} V_1(t) &= p_1\int_0^L u^2(x,t)dx + p_3\nu\int_0^L u_{xx}^2(x,t)dx \\ &\quad + r(t_{k+1}-t)\int_0^L\int_{t_k}^{t} e^{2\delta(s-t)}u_s^2(x,s)dsdx, \\ &\quad t\in[t_k,t_{k+1}),\ p_1>0,\ p_3>0,\ r>0. \end{aligned} \tag{4.5}$$

Here p_1 and p_3-terms are extensions of the corresponding terms of Fridman and Blighovsky (2012) to KSE, whereas r-term treats sampled-data control as introduced in Fridman (2010), and $\delta > 0$ stands for the decay rate. In the time-derivative of r-term we have a positive term $r(t_{k+1} - t)\int_0^L u_t^2(x,t)dx$ (see (4.13)). To compensate such a term in $\dot{V}_1$, we choose the p_3-term that guarantees convergence in H^2-norm (and not in H^1-norm like in Fridman and Blighovsky (2012) for the case of diffusion–reaction equation).

For convenience we define

$$\|u(\cdot,t)\|_V^2 = p_1\int_0^L u^2(x,t)dx + p_3\nu\int_0^L u_{xx}^2(x,t)dx, \tag{4.6}$$

where p_1 and p_3 are positive constants, and $u(\cdot,t) \in H_0^2(0,L)$. The choice of such norm is motivated by the Lyapunov–Krasovskii functional (4.5). By using Lyapunov–Krasovskii functional (4.5), in Theorem 4.1 we provide LMI conditions for regional exponential stability of (4.1) and for a bound on the domain of attraction.

Remark 4.1. To find a bound on the domain of attraction for system (4.4) subject to (2.2), we use positive invariance principle in Theorem 4.1: we derive stability conditions in terms of matrix inequalities that guarantee $V_1(t) \leq V_1(0)$ for all $t \geq 0$. These matrix inequalities ($\Theta_1 < 0$ and $\Theta_2 < 0$ with Θ_1 and Θ_2 defined by (4.10) and (4.11)) are affine in $u_x(x,t)$. Our objective is to guarantee that $\max_{x\in[0,L]}|u_x(x,t)|^2 < C^2$ for all $t \geq 0$. This allows to verify the matrix inequalities in the vertices $u_x = \pm C$ (see (4.8)). Therefore, if the initial condition satisfies $\|u_0\|_V < \sqrt{\frac{p_3\nu}{L}}C$, then from the Sobolev inequality we obtain the desired bound on u_x:

$$\begin{aligned} &\max_{x\in[0,L]}|u_x(x,t)|^2 \leq L\|u_{xx}(\cdot,t)\|_{L^2}^2 \leq \frac{L}{p_3\nu}V_1(t) \\ &\leq \frac{L}{p_3\nu}V_1(0) = \frac{L}{p_3\nu}\|u_0\|_V^2 < C^2. \end{aligned} \tag{4.7}$$

Now we are in a position to formulate our first main result:

Theorem 4.1. *Consider the closed-loop system (4.4) under the Dirichlet boundary conditions (2.2). Given positive scalars C, R, h, μ, Δ and $\delta_1 < 2\delta$, let there exist scalars $r > 0$, $\lambda \geq 0$, $p_i > 0$ (i=1,2,3) that satisfy the LMIs:*

$$\Theta_i|_{u_x=C} < 0,\ \ \Theta_i|_{u_x=-C} < 0,\ \ i=1,2, \tag{4.8}$$

and

$$\bar{\Theta} = \begin{bmatrix} -\delta_1 p_1 & \frac{\mu}{2}\frac{\Delta}{\pi}R^{-1}(p_2+p_3) \\ * & -\delta_1 p_3 \nu \end{bmatrix} < 0, \tag{4.9}$$

where

$$\Theta_1 = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ * & rh - 2p_3 + \frac{\Delta}{\pi}\mu R p_3 & \theta_{23} \\ * & * & \theta_{33} \end{bmatrix}, \tag{4.10}$$

$$\Theta_2 = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \mu p_2 h \\ * & -2p_3 + \frac{\Delta}{\pi}\mu R p_3 & \theta_{23} & \mu p_3 h \\ * & * & \theta_{33} & 0 \\ * & * & * & -re^{-2\delta h}h \end{bmatrix}, \tag{4.11}$$

$$\theta_{11} = 2\delta p_1 + \frac{\Delta}{\pi}\mu R p_2 - 2\mu p_2 - \lambda\frac{\pi^4}{L^4},$$
$$\theta_{12} = p_1 - p_2 - \mu p_3 - p_3 u_x,$$
$$\theta_{13} = -p_2,\ \theta_{23} = -p_3,\ \theta_{33} = 2\delta p_3 \nu - 2p_2\nu + \lambda.$$

Then for any initial function $u(\cdot,0) \in H_0^2(0,L)$ subject to $\|u(\cdot,0)\|_V < \sqrt{\frac{p_3\nu}{L}}C$, a unique strong solution of (4.4), (2.2) exists and satisfies

$$\begin{aligned} &p_1\int_0^L u^2(x,t)dx + p_3\nu\int_0^L u_{xx}^2(x,t)dx \\ &\le e^{-2\alpha t}\left[p_1\int_0^L u^2(x,0)dx + p_3\nu\int_0^L u_{xx}^2(x,0)dx\right] \end{aligned} \tag{4.12}$$

for all $t \ge 0$, where α is a unique positive solution of (2.8). Furthermore, if the strong inequalities (4.8) and (4.9) are feasible with $\delta = \frac{\delta_1}{2} > 0$, then the strong solutions of (4.4), (2.2) initialized with $u(\cdot,0) \in H_0^2(0,L)$ subject to $\|u(\cdot,0)\|_V < \sqrt{\frac{p_3\nu}{L}}C$ are exponentially converging with a small enough decay rate.

Proof. Step 1: It has been shown that a unique local strong solution of (4.2) exists on some interval $[0,T] \subset [0,t_1]$. From Theorem 6.23.5 of Krasnoselskii et al. (1976), it follows that if this solution admits a priori estimate, then the solution exists on the entire interval $[0,t_1]$. We will prove in Step 3 that if the LMIs (4.8), (4.9) are feasible, then the solution of (4.2) starting from $\|u(\cdot,0)\|_V < \sqrt{\frac{p_3\nu}{L}}C$ admits a priori bound. The latter guarantees the existence of the strong solution of (4.2) on the entire interval $[0,t_1]$. Then, by applying the same arguments step-by-step for $[t_k,t_{k+1}]$, $k = 1,2,\ldots$ we conclude that the strong solution exists for all $t \ge 0$.

Step 2. Assume formally that strong solutions of (4.4) subject to (2.2) starting from $\|u_0\|_V < \sqrt{\frac{p_3\nu}{L}}C$ exist for all $t \ge 0$. Differentiating V_1 along (4.4) subject to (2.2), we have

$$\begin{aligned} &\dot{V}_1(t) + 2\delta V_1(t) = 2p_1\int_0^L u(x,t)u_t(x,t)dx \\ &+ 2p_3\nu\int_0^L u_{xx}(x,t)u_{xxt}(x,t)dx + 2\delta p_1\int_0^L u^2(x,t)dx \\ &+ 2\delta p_3\nu\int_0^L u_{xx}^2(x,t)dx - r\int_0^L\int_{t_k}^t e^{2\delta(s-t)}u_s^2(x,s)dsdx \\ &+ r(t_{k+1}-t)\int_0^L u_t^2(x,t)dx, \end{aligned} \tag{4.13}$$

where $t \in [t_k, t_{k+1})$.

Denote

$$\rho(x,t) \triangleq \frac{1}{t-t_k}\int_{t_k}^t u_s(x,s)ds. \tag{4.14}$$

Here we understand $\lim_{t\to t_k^+}\rho(x,t) = u_t(x,t_k)$ and obtain

$$u(x,t) = u(x,t_k) + (t-t_k)\rho(x,t). \tag{4.15}$$

Jensen's inequality (Gu, Kharitonov, & Chen, 2003) yields

$$\begin{aligned} &-r\int_0^L\int_{t_k}^t e^{2\delta(s-t)}u_s^2(x,s)dsdx \\ &\le -re^{-2\delta h}\int_0^L \frac{1}{t-t_k}\left[\int_{t_k}^t u_s(x,s)ds\right]^2 dx \\ &= -re^{-2\delta h}(t-t_k)\int_0^L \rho^2(x,t)dx. \end{aligned} \tag{4.16}$$

We apply the descriptor method (Fridman, 2001, 2014; Fridman & Orlov, 2009) by adding to $\dot{V}_1 + 2\delta V_1$ the left-hand sides of the following equations:

$$\begin{aligned} &2\int_0^L [p_2 u(x,t) + p_3 u_t(x,t)][-u_t(x,t) - u_{xx}(x,t) \\ &- \nu u_{xxxx}(x,t) - u(x,t)u_x(x,t) - \mu u(x,t_k)]dx \\ &+ 2\mu\sum_{j=1}^N\int_{x_{j-1}}^{x_j}[p_2u(x,t) + p_3u_t(x,t)]\int_{\bar{x}_j}^x u_\xi(\xi,t_k)d\xi dx = 0, \end{aligned} \tag{4.17}$$

where $p_2 > 0$ is some scalar. This avoids substitution of u_t from (4.4) into the right-hand side of (4.13). Integration by parts and substitution of the Dirichlet boundary conditions (2.2) lead to

$$-2p_2\int_0^L u(x,t)[u(x,t)u_x(x,t)]dx = 0, \tag{4.18}$$

$$-2p_2\int_0^L u(x,t)[\nu u_{xxxx}(x,t)]dx = -2p_2\nu\int_0^L u_{xx}^2(x,t)dx, \tag{4.19}$$

and

$$\begin{aligned} &-2p_3\int_0^L u_t(x,t)[\nu u_{xxxx}(x,t)]dx \\ &= -2p_3\nu\int_0^L u_{xx}(x,t)u_{xxt}(x,t)dx. \end{aligned} \tag{4.20}$$

By adding to $\dot{V}_1 + 2\delta V_1$ the equality (4.17), and using (4.15), (4.16), (4.18), (4.19), (4.20), we obtain

$$\begin{aligned} &\dot{V}_1(t) + 2\delta V_1(t) \\ &= 2p_1\int_0^L u(x,t)u_t(x,t)dx - 2p_2\nu\int_0^L u_{xx}^2(x,t)dx \\ &+ 2\delta p_1\int_0^L u^2(x,t)dx + 2\delta p_3\nu\int_0^L u_{xx}^2(x,t)dx \\ &- re^{-2\delta h}(t-t_k)\int_0^L \rho^2(x,t)dx \\ &+ r(t_{k+1}-t)\int_0^L u_t^2(x,t)dx \\ &+ 2\int_0^L [p_2u(x,t) + p_3u_t(x,t)][-u_t(x,t) - u_{xx}(x,t) \\ &- \mu u(x,t) + \mu(t-t_k)\rho(x,t)]dx \\ &- 2p_3\int_0^L u_t(x,t)u(x,t)u_x(x,t)dx \\ &+ 2\mu\sum_{j=1}^N\int_{x_{j-1}}^{x_j}[p_2u(x,t) + p_3u_t(x,t)]\int_{\bar{x}_j}^x u_\xi(\xi,t_k)d\xi dx. \end{aligned} \tag{4.21}$$

From Young's and Wirtinger's inequalities, we have

$$\begin{aligned} &2\mu\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j}[p_2u(x,t)+p_3u_t(x,t)]\int_{\bar{x}_j}^{x}u_\xi(\xi,t_k)d\xi dx \\ &\le \mu\bar{R}\int_0^L[p_2u^2(x,t)+p_3u_t^2(x,t)]dx \\ &+\mu\bar{R}^{-1}(p_2+p_3)\frac{\Delta^2}{\pi^2}\int_0^L u_x^2(x,t_k)dx,\ \ \forall\bar{R}>0. \end{aligned} \tag{4.22}$$

From Lemma 2.2, the Wirtinger inequality implies

$$\lambda\int_0^L\left[u_{xx}^2(x,t)-(\frac{\pi^2}{L^2})^2u^2(x,t)\right]dx\ge 0, \tag{4.23}$$

where $\lambda\ge 0$.

Substituting (4.22) into the right-hand side of (4.21), adding (4.23) to $\dot{V}_1+2\delta V_1$, using Halanay's inequality and employing the inequality

$$\|u_x(\cdot,t_k)\|_{L^2}^2\le\|u(\cdot,t_k)\|_{L^2}\|u_{xx}(\cdot,t_k)\|_{L^2},$$

we obtain

$$\begin{aligned} &\dot{V}_1(t)+2\delta V_1(t)-\delta_1\sup_{\theta\in[-h,0]}V_1(t+\theta) \\ &\le\dot{V}_1(t)+2\delta V_1(t)-\delta_1V_1(t_k) \\ &\le(2p_1-2p_2-2\mu p_3)\int_0^L u(x,t)u_t(x,t)dx \\ &+(2\delta p_3\nu-2p_2\nu+\lambda)\int_0^L u_{xx}^2(x,t)dx \\ &-2\int_0^L[p_2u(x,t)+p_3u_t(x,t)]u_{xx}(x,t)dx \\ &+\left[2\delta p_1+\mu\bar{R}p_2-2\mu p_2-\lambda\frac{\pi^4}{L^4}\right]\int_0^L u^2(x,t)dx \\ &+\left[r(t_{k+1}-t)-2p_3+\mu\bar{R}p_3\right]\int_0^L u_t^2(x,t)dx \\ &-2p_3\int_0^L u_t(x,t)u(x,t)u_x(x,t)dx \\ &-re^{-2\delta h}(t-t_k)\int_0^L\rho^2(x,t)dx \\ &+\mu(p_2+p_3)\frac{\Delta^2}{\pi^2}\bar{R}^{-1}\|u(\cdot,t_k)\|_{L^2}\|u_{xx}(\cdot,t_k)\|_{L^2} \\ &+2\mu\int_0^L[p_2u(x,t)+p_3u_t(x,t)][(t-t_k)\rho(x,t)]dx \\ &-\delta_1p_1\int_0^L u^2(x,t_k)dx-\delta_1p_3\nu\int_0^L u_{xx}^2(x,t_k)dx. \end{aligned} \tag{4.24}$$

Set

$$\begin{aligned} &\eta=\mathrm{col}\{u(x,t),u_t(x,t),u_{xx}(x,t),\rho(x,t)\}, \\ &\eta_0=\mathrm{col}\{u(x,t),u_t(x,t),u_{xx}(x,t)\}, \\ &\bar{\eta}=\mathrm{col}\{\|u(\cdot,t_k)\|_{L^2},\|u_{xx}(\cdot,t_k)\|_{L^2}\}, \end{aligned} \tag{4.25}$$

and choose $R=\frac{\pi}{\Delta}\bar{R}$. Since $0\le t_{k+1}-t_k\le h$, from (4.24) it follows that

$$\begin{aligned} &\dot{V}_1(t)+2\delta V_1(t)-\delta_1\sup_{\theta\in[-h,0]}V_1(t+\theta) \\ &\le\int_0^L\frac{h-t+t_k}{h}\eta_0^T\Theta_1\eta_0+\frac{t-t_k}{h}\eta^T\Theta_2\eta+\bar{\eta}^T\bar{\Theta}\bar{\eta}dx, \\ &\qquad\qquad\forall t\in[t_k,t_{k+1}), \end{aligned} \tag{4.26}$$

where $\bar{\Theta}$, Θ_1 and Θ_2 are given by (4.9), (4.10), (4.11) respectively.

We first assume that

$$\max_{x\in[0,L]}|u_x(x,t)|<C,\ \ \forall t\ge 0. \tag{4.27}$$

Under the assumption (4.27), from (4.26) we obtain

$$\dot{V}_1(t)+2\delta V_1(t)-\delta_1\sup_{\theta\in[-h,0]}V_1(t+\theta)\le 0 \tag{4.28}$$

if $\Theta_1<0$, $\Theta_2<0$, $\bar{\Theta}<0$ hold for all $u_x\in(-C,C)$.

Matrices Θ_1 and Θ_2 given by (4.10), (4.11) are affine in u_x. Hence, $\Theta_1<0$ and $\Theta_2<0$ for all $u_x\in(-C,C)$ if these inequalities hold in the vertices $u_x=\pm C$ hold, i.e. if LMIs (4.8) are feasible.

We prove next that (4.27) holds. From (4.7) it follows that it is sufficient to show that $V_1(t)<\frac{p_3\nu}{L}C^2$. The initial condition $V_1(0)=\|u_0\|_V^2<\frac{p_3\nu}{L}C^2$ implies $\max_{x\in[0,L]}|u_x(x,0)|^2<C^2$. Let $t^*\in(0,\infty)$ be the smallest time instance such that $V_1(t^*)\ge\frac{p_3\nu}{L}C^2$. Since V_1 is continuous in time, we have $V_1(t^*)=\frac{p_3\nu}{L}C^2$ and $V_1(t)<\frac{p_3\nu}{L}C^2$ for $t\in[0,t^*)$. Together with (4.7) this implies $\max_{x\in[0,L]}|u_x(x,t)|^2<C^2$ for $t\in[0,t^*)$ and, therefore, the feasibility of (4.8) and (4.9) guarantees that (4.28) is true for $t\in[0,t^*)$. Hence, $V_1(t)\le e^{-2\alpha t}V_1(0)<\frac{p_3\nu}{L}C^2$ holds for $t\in[0,t^*]$, which contradicts to the definition of t^*. Thus, for all $t\ge 0$,

$$\|u_0\|_V<\sqrt{\tfrac{p_3\nu}{L}}C\Rightarrow(4.27)\Rightarrow(4.12).$$

Step 3: Now we continue to prove the well-posedness. When $k=0$, we obtain that if the LMIs conditions (4.8) and (4.9) are satisfied, then any strong solution of (4.2) initialized with $u_0\in H_0^2(0,L)$ subject to $\|u_0\|_V<\sqrt{\frac{p_3\nu}{L}}C$ admits a priori estimate

$$V_1(t)\le e^{-2\alpha t}V_1(0), \tag{4.29}$$

where α is the solution of (2.8). The latter bound guarantees the existence of these strong solutions for all $t\in[0,t_1]$. Then, by step method, the strong solution exists for all $t\ge 0$. Furthermore, Halanay's inequality implies (4.12) for all $t\ge 0$.

Note that the feasibility of strong inequalities (4.8) and (4.9) with $\delta=\frac{\delta_1}{2}>0$ implies their feasibility with a slightly larger $\bar{\delta}=\delta+\alpha_0>0$, where $\alpha_0>0$ is small. Therefore, if (4.8) and (4.9) hold with $\delta=\frac{\delta_1}{2}>0$, then the system (4.4), (2.2) is regionally exponentially stable with a small decay rate. □

4.2. Sampled-data control under the averaged measurements

Under the controller (2.7) subject to (2.6), the closed-loop system of (2.1) becomes:

$$\begin{aligned} &u_t(x,t)+u_{xx}(x,t)+\nu u_{xxxx}(x,t)+u(x,t)u_x(x,t) \\ &=-\mu\sum_{j=1}^{N}\left[b_j(x)\frac{\int_{x_{j-1}}^{x_j}u(x,t_k)dx}{\Delta_j}\right], \\ &\qquad t\in[t_k,t_{k+1}),\ k=0,1,\ldots \end{aligned} \tag{4.30}$$

subject to the Dirichlet or periodic boundary conditions.

For $j=1,\ldots,N$, $k=0,1,\ldots$ we consider the quantities

$$f_j(x,t)=u(x,t)-\frac{\int_{x_{j-1}}^{x_j}u(\zeta,t)d\zeta}{\Delta_j},$$

$$k_j(t)=\frac{1}{t-t_k}\frac{\int_{x_{j-1}}^{x_j}\int_{t_k}^{t}u_\xi(x,\xi)d\xi dx}{\Delta_j},$$

where by $k_j|_{t=t_k}$ we understand the following:

$$\lim_{t\to t_k^+}k_j(t)=\frac{\int_{x_{j-1}}^{x_j}u_t(x,t_k)dx}{\Delta_j}.$$

Similar to Bar Am and Fridman (2014), we use the following presentation of $U_j(t)$:

$$U_j(t) = -\mu[u(x,t) - f_j(x,t) - (t-t_k)k_j(t)]. \tag{4.31}$$

Theorem 4.2. *Consider the closed-loop system (4.30) under the Dirichlet boundary conditions (2.2). Given positive scalars C, h, μ, Δ and δ, let there exist scalars $r>0, \lambda\geq 0, \lambda_1>0, p_i>0\,(i=1,2,3)$ satisfy the linear matrix inequalities:*

$$\Phi_{i|_{u_x=C}} < 0,\ \Phi_{i|_{u_x=-C}} < 0,\ i=1,2 \tag{4.32}$$

where

$$\Phi_1 = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} \\ * & rh-2p_3 & \phi_{23} & \phi_{24} \\ * & * & \phi_{33} & \phi_{34} \\ * & * & * & \phi_{44} \end{bmatrix}, \tag{4.33}$$

$$\Phi_2 = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \mu p_2 h \\ * & -2p_3 & \phi_{23} & \phi_{24} & \mu p_3 h \\ * & * & \phi_{33} & \phi_{34} & 0 \\ * & * & * & \phi_{44} & 0 \\ * & * & * & * & -re^{-2\delta h}h \end{bmatrix}, \tag{4.34}$$

$$\begin{aligned} &\phi_{11} = -2p_2\mu + 2\delta p_1 - \lambda\frac{\pi^4}{L^4}, \\ &\phi_{12} = p_1 - p_2 - p_3\mu - p_3 u_x, \\ &\phi_{13} = -p_2 - \frac{\lambda_1}{2},\ \phi_{14} = \mu p_2,\ \phi_{23} = -p_3,\ \phi_{24} = \mu p_3, \\ &\phi_{33} = -2p_2\nu + 2\delta p_3\nu + \lambda,\ \phi_{34} = 0,\ \phi_{44} = -\frac{\lambda_1\pi^2}{\Delta^2}. \end{aligned}$$

Then for any initial function $u(\cdot,0)\in H_0^2(0,L)$ satisfying $\|u(\cdot,0)\|_V < \sqrt{\frac{p_3\nu}{L}}C$, a unique strong solution of (4.30), (2.2) exists and satisfies

$$\begin{aligned} &p_1\int_0^L u^2(x,t)dx + p_3\nu\int_0^L u_{xx}^2(x,t)dx \\ &\leq e^{-2\delta t}\left[p_1\int_0^L u^2(x,0)dx + p_3\nu\int_0^L u_{xx}^2(x,0)dx\right] \end{aligned} \tag{4.35}$$

for all $t\geq 0$. Furthermore, if the strong inequalities (4.32) are feasible with $\delta = 0$, then the strong solutions of (4.30), (2.2) initialized with $u(\cdot,0)\in H_0^2(0,L)$ subject to $\|u(\cdot,0)\|_V < \sqrt{\frac{p_3\nu}{L}}C$ are exponentially converging with a small enough decay rate.

Proof. See Appendix B. □

4.3. Extension to periodic boundary conditions

Finding a bound on the domain of attraction of the closed-loop system (4.1) or (4.30) under the periodic boundary conditions (2.3) will be based on the following useful Lemma:

Lemma 4.1. *Let $z(x)\in H^1(0,L)$, then*

$$\max_{x\in[0,L]} |z(x)|^2 \leq (1+\frac{1}{L})\|z(\cdot,t)\|_{L^2}^2 + \|z_x(\cdot,t)\|_{L^2}^2.$$

Proof. By Proposition 5.22 of Robinson (2001), $z(\cdot)\in H^1(0,L)$ implies $z(\cdot)\in C[0,L]$. Then by mean value theorem, there exists $c\in(0,L)$ such that

$$z(c) = \frac{1}{L}\int_0^L z(x)dx.$$

Then, by integration by parts and further application of Jensen's and Young's inequalities, for all $x_1\in[0,L]$ we have

$$\begin{aligned} z^2(x_1) &= z^2(c) + 2\int_c^{x_1} z(x)z_x(x)dx \\ &= \left[\frac{1}{L}\int_0^L z(x)dx\right]^2 + 2\int_c^{x_1} z(x)z_x(x)dx \\ &\leq \frac{1}{L}\int_0^L z^2(x)dx + \int_0^L z^2(x)dx + \int_0^L z_x^2(x)dx \\ &\leq (1+\frac{1}{L})\int_0^L z^2(x)dx + \int_0^L z_x^2(x)dx. \quad \square \end{aligned}$$

Denote

$$M = \max\{\frac{1}{2p_1}, \frac{3}{2p_3\nu}\}. \tag{4.36}$$

Similar to Section 3.2, Theorems 4.1 and 4.2, can be easily extended to periodic boundary conditions:

(i) If the conditions of Theorem 4.1 with $\lambda = 0$ hold, then for the initial function $u(\cdot,0)\in H_{per}^2$ satisfying $\|u(\cdot,0)\|_V < \sqrt{\frac{L}{(L+1)M}}C$, a unique strong solution of (4.1) under the periodic boundary conditions (2.3) exists and satisfies (4.12).

(ii) If the conditions of Theorem 4.2 with $\lambda = 0$ hold, then for the initial function $u(\cdot,0)\in H_{per}^2$ satisfying $\|u(\cdot,0)\|_V < \sqrt{\frac{L}{(L+1)M}}C$, a unique strong solution of (4.30) under the periodic boundary conditions (2.3) exists and satisfies (4.35).

Proof of (i) The proof is similar to the case of Dirichlet boundary conditions. Due to the periodic boundary conditions, the Wirtinger inequality is not applicable (cf. (4.23)). Therefore, by arguments of Theorem 4.1, we obtain that if the conditions of Theorem 4.1 with $\lambda = 0$ hold, then (4.28), (4.29) are satisfied (and, thus, (4.12) holds) provided that (4.27) holds for all $t\geq 0$.

By Lemma 4.1 and Young's inequality, using the inequality $\|u_x(\cdot,t)\|_{L^2}^2 \leq \|u(\cdot,t)\|_{L^2}\|u_{xx}(\cdot,t)\|_{L^2}$ we have

$$\begin{aligned} &\max_{x\in[0,L]}|u_x(x,t)|^2 \leq (1+\frac{1}{L})\|u_x(\cdot,t)\|_{L^2}^2 + \|u_{xx}(\cdot,t)\|_{L^2}^2 \\ &\leq (1+\frac{1}{L})\|u(\cdot,t)\|_{L^2}\|u_{xx}(\cdot,t)\|_{L^2} + \|u_{xx}(\cdot,t)\|_{L^2}^2 \\ &\leq (1+\frac{1}{L})\left[\frac{1}{2}\|u(\cdot,t)\|_{L^2}^2 + (\frac{1}{2}+1)\|u_{xx}(\cdot,t)\|_{L^2}^2\right] \\ &\leq (1+\frac{1}{L})M\left[p_1\|u(\cdot,t)\|_{L^2}^2 + p_3\nu\|u_{xx}(\cdot,t)\|_{L^2}^2\right] \\ &= (1+\frac{1}{L})M\|u(\cdot,t)\|_V^2 \leq (1+\frac{1}{L})MV_1(t), \end{aligned} \tag{4.37}$$

where M is given by (4.36). Therefore, if the initial function $u(\cdot,0)\in H_{per}^2$ satisfies $\|u(\cdot,0)\|_V < \sqrt{\frac{L}{(L+1)M}}C$, then

$$\max_{x\in[0,L]}|u_x(x,0)|^2 \leq (1+\frac{1}{L})M\|u(\cdot,0)\|_V^2 < C^2.$$

Due to (4.37), the LMI conditions of Theorem 4.1 with $\lambda = 0$ guarantee that

$$\begin{aligned} &\max_{x\in[0,L]}|u_x(x,t)|^2 \leq (1+\frac{1}{L})MV_1(t) \\ &\leq (1+\frac{1}{L})MV_1(0) = (1+\frac{1}{L})M\|u(\cdot,0)\|_V^2 < C^2 \end{aligned}$$

meaning that (4.27) holds.

Proof of (ii) follows arguments of (i).

Remark 4.2. The presented results can be extended to the case of time-varying input delay. Similar to Bar Am and Fridman (2014)

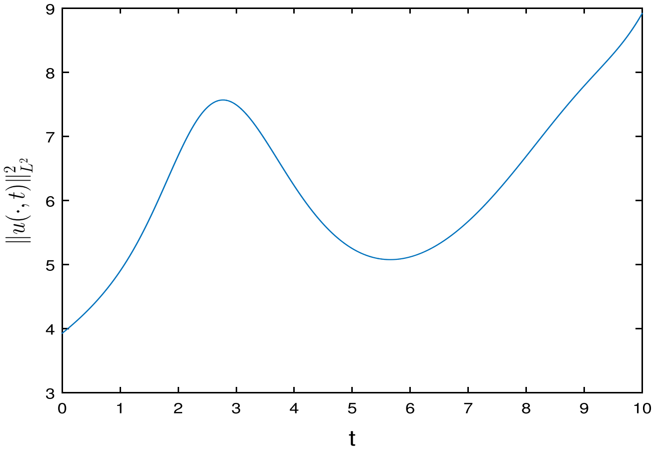

Fig. 1. Open-loop system (without control input).

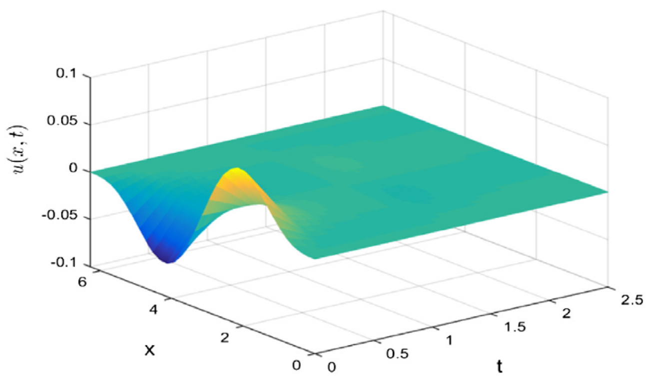

Fig. 2. Closed-loop system with $x_j - x_{j-1} = \frac{\pi}{8}$, $t_{k+1} - t_k = 0.28$ under the averaged measurements.

and Fridman and Blighovsky (2012), LMI stability conditions for KSE with time-varying delay can be derived. However, a special attention should be paid to the bound on the domain of attraction. This is because of the initial time interval, where the system is in the open-loop (see Liu & Fridman, 2014).

5. Example

Consider KSE (2.1) under the Dirichlet boundary conditions (2.2) with $L = 2\pi$ and $\nu = 0.5 < 1$:

$$u_t(x,t) + u_{xx}(x,t) + 0.5u_{xxxx}(x,t) + u(x,t)u_x(x,t)$$
$$= \sum_{j=1}^{N} b_j(x)U_j(t), \; 0 < x < 2\pi, \; t \geq 0.$$

Fig. 1 demonstrates the time evolution of the L^2-norm $\|u(\cdot,t)\|_{L^2}^2$ for the open-loop system initialized by $u(x,0) = (1-\cos x)\sin x$ $(0 \leq x \leq 2\pi)$. It is seen that the open-loop system is unstable.

(i) We start with the continuous in time controller under the point and averaged measurements. Verifying the LMI condition of Proposition 3.1 with $\alpha = 0$ by Yalmip Toolbox of Matlab, we find that the closed-loop system preserves the exponential stability till $\Delta \leq 2.2$ (this corresponds to $\mu = 3.0853$, $\lambda_1 = 1.5131$ and $\lambda = 2.2905 * 10^{-7}$). Note that for $\Delta \leq 2.2$, the condition $\frac{\Delta^2}{\pi^2} \leq 0.49 < \nu$ of particular case of Proposition 3.1 is satisfied, and the corresponding bound for $\mu = \frac{2\pi^2}{\Delta^2}$ is essentially larger: $\mu = 4.07 > 3.0853$. For both values of μ, the condition of Lunasin and Titi (2017) $\mu > 8$ is not satisfied.

Using Proposition 3.1 with $\alpha = 0$, $\mu > 4/\nu = 8$ that satisfies the condition of Lunasin and Titi (2017), we find that the closed-loop system is exponentially stable till $\Delta \leq 2.1$ (allowing 3 sensors under the point measurements). This corresponds to $\mu = 8.0004$, $\lambda_1 = 2.7795$ and $\lambda = 3.3738 * 10^{-5}$. Here we added the restriction $\mu > 4/\nu$ (that is used to compare with Lunasin & Titi, 2017). The condition $\mu\frac{\Delta^4}{\pi^4} < \nu$ of Lunasin & Titi (2017) with the same $\mu = 8.0004$ leads to essentially smaller $\Delta < 1.56$ (that requires 5 point sensors) meaning that the LMIs conditions of Proposition 3.1 are less restrictive.

(ii) Consider next the closed-loop system (4.30) subject to (2.2) under the sampled-data control law (2.7) with the averaged measurements. To enlarge the sampling time interval, we choose $\mu = 3.0853$ (the smallest μ found from the LMI of Proposition 3.1). We verify LMI conditions of Theorem 4.2 with $\delta = 0.0002$ and $C = h = 0$ and find that the LMIs are feasible till $\Delta \leq 2.2$ (as in the continuous-time case). For $\delta = 0.2, C = 1$ and $\Delta = \frac{\pi}{8}$, we find that the closed-loop system preserves the exponential stability within a given domain of initial conditions $\|u(\cdot,0)\|_V < \sqrt{\frac{p_3\nu}{L}}C = 0.58$ for $t_{k+1} - t_k \leq h = 0.28$.

Next, a finite difference method is applied to compute the displacement of the closed-loop system (4.30) under the Dirichlet boundary conditions (2.2). We choose initial condition $u(x,0) = 0.05(1-\cos x)\sin x$, $0 \leq x \leq 2\pi$ satisfying $\|u(\cdot,0)\|_V < 0.58$. The steps of space and time are taken as $\frac{\pi}{16}$ and 0.0001, respectively. Simulations of solutions under the sampled-data in time and in space controller $U_j(t) = -3.0853\frac{\int_{x_{j-1}}^{x_j} u(x,t_k)dx}{\Delta_j}$ with $x_j - x_{j-1} = \Delta_j = \frac{\pi}{8}, j = 1, \ldots, 16, t_{k+1} - t_k = 0.28$, where the spatial domain is divided into sixteen sub-domains, show that the closed-loop system is exponentially stable (see Fig. 2). By enlarging the sampling period, we find that the system becomes unstable for $t_{k+1} - t_k \geq 1$ (see Fig. 3).

(iii) Consider now the sampled-data controller under the point measurements. Here we choose the same value of μ as in (ii). For sampled-data control law (2.7) with the point measurements, we verify LMI conditions of Theorem 4.1 with $\delta_1 = 0.5$, $\delta = 0.26$, $R = 1, C = 1$ and $\Delta = \pi/8$. We find that the closed-loop system is exponentially stable for $t_{k+1} - t_k \leq h = 0.23$ for any initial values satisfying $\|u(\cdot,0)\|_V < \sqrt{\frac{p_3\nu}{L}}C = 0.5$. For $h = 0.23$, the above controller locally exponentially stabilizes the closed-loop system with a decay rate $\alpha = 0.0089$. Table 1 shows the maximum values of h that preserve the exponential stability. The corresponding value of δ_1 is given by $\delta_1 = 0.5$, whereas the values of $\delta > \frac{\delta_1}{2}$ are chosen to be close to $\frac{\delta_1}{2}$. The latter leads to a small decay rate α but enlarges the sampling intervals.

Table 1
Max. values of h preserving the stability of the closed-loop system in Example 1.

δ	0.4	0.3	0.26
h	0.2	0.22	0.23

We proceed further with the numerical simulations of the solutions of the closed-loop system (4.1) subject to (2.2) under the sampled in time and in space controller $U_j(t) = -3.0853u(\bar{x}_j, t_k)$ with $x_j - x_{j-1} = \frac{\pi}{8}, j = 1, \ldots, 16, t_{k+1} - t_k = 0.23$. Here we choose the same initial conditions and the same value of μ as in (ii). The simulations show that the state of KSE converges to zero (see Fig. 4). Simulations of the solutions confirm the theoretical results that follow from LMIs.

As expected (because averaged measurements use more information on the state), the averaged measurements allow larger sampling intervals than point measurements.

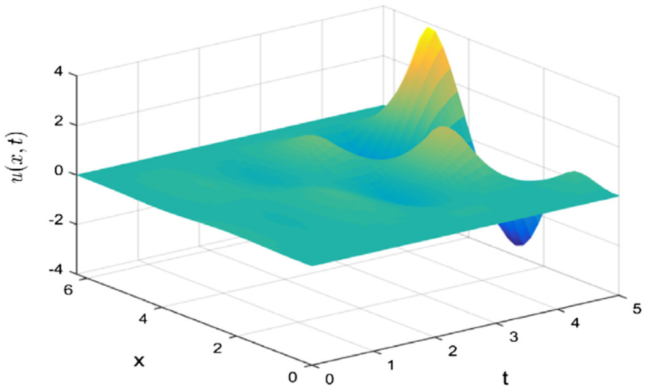

Fig. 3. Closed-loop system with $x_j - x_{j-1} = \frac{\pi}{8}$, $t_{k+1} - t_k = 1$ under the averaged measurements.

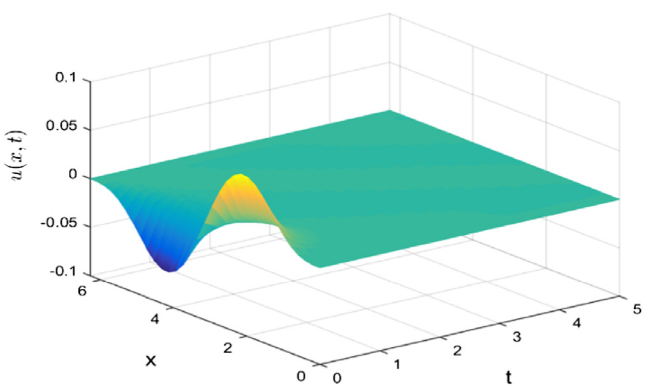

Fig. 4. Closed-loop system with $x_j - x_{j-1} = \frac{\pi}{8}$, $t_{k+1} - t_k = 0.23$ under the point measurements.

6. Conclusion

Distributed control of KSE under the point or averaged state measurements was initiated in Lunasin and Titi (2017). However, for practical use of such controllers, their sampled-data implementation was missing. This paper provided distributed sampled-data control of KSE under the point or averaged discrete-time state measurements and either Dirichlet or periodic boundary conditions. By using the time-delay approach to sampled-data control and constructing an appropriate Lyapunov–Krasovskii functional, sufficient conditions for the regional exponential stabilization were derived in terms of LMIs.

The presented method gives efficient tools for sampled-data observer design. Another interesting, yet technically challenging, open question is design of observer-based sampled-data controllers for distributed parameter systems, which may be a topic for future research.

Appendix A. Proof of Proposition 3.1

The proof of this proposition consists of several steps.

Step 1: It has been shown that the local classical solution of (3.6) under the Dirichlet boundary conditions (2.2) exists in the sense that $u \in C([0,T), L^2(0,L)) \cap C^1((0,T), L^2(0,L))$, where $T = T(u_0) > 0$. From Theorem 6.23.5 of Krasnoselskii et al. (1976), it follows that if this solution admits a priori estimate, then the solution exists for any $T > 0$. We will prove in Step 3 that if the LMI (3.10) is feasible, then the solution of (3.6) subject to (2.2) admits a priori bound, which guarantees the existence of the solution of (3.6) subject to (2.2) for all $t \geq 0$.

Step 2: Assume formally that solution of (3.6) subject to (2.2) exists for all $t \geq 0$ and choose the Lyapunov function of the form:

$$V(t) = \int_0^L u^2(x,t)dx. \tag{A.1}$$

Differentiating V along (3.6) subject to (2.2) and integrating by parts, we find

$$\begin{aligned}\dot{V}(t) &= 2\int_0^L u(x,t)u_t(x,t)dx \\ &= -2\int_0^L u(x,t)u_{xx}(x,t)dx - 2\nu\int_0^L u_{xx}^2(x,t)dx \\ &+ 2\mu\sum_{j=1}^N \int_{x_{j-1}}^{x_j} u(x,t)f_j(x,t)dx \\ &- 2\mu\int_0^L u^2(x,t)dx.\end{aligned} \tag{A.2}$$

(a) For the case of the distributed control under the point measurements, $f_j(x,t)$ is given by (3.4).

Applying Wirtinger's inequality, we obtain

$$\begin{aligned}\int_{x_{j-1}}^{x_j} f_j^2(x,t)dx &= \int_{x_{j-1}}^{x_j}\left[\int_{\bar{x}_j}^{x} u_\xi(\xi,t)d\xi\right]^2 dx \\ &\leq \frac{\Delta^2}{\pi^2}\int_{x_{j-1}}^{x_j} u_x^2(x,t)dx.\end{aligned} \tag{A.3}$$

(b) For the case of the distributed control under the averaged measurements, $f_j(x,t)$ is given by (3.5).

Since

$$\int_{x_{j-1}}^{x_j}\left[u(x,t) - \frac{\int_{x_{j-1}}^{x_j} u(\zeta,t)d\zeta}{\Delta_j}\right]dx = 0,$$

the Poincaré inequality and Jensen inequality allow to obtain

$$\begin{aligned}\int_{x_{j-1}}^{x_j} f_j^2(x,t)dx &= \int_{x_{j-1}}^{x_j}\left[u(x,t) - \frac{\int_{x_{j-1}}^{x_j} u(\zeta,t)d\zeta}{\Delta_j}\right]^2 dx \\ &\leq \frac{\Delta^2}{\pi^2}\int_{x_{j-1}}^{x_j} u_x^2(x,t)dx.\end{aligned} \tag{A.4}$$

Hence, for both cases (a) and (b), from (A.3) and (A.4) the following inequality

$$\int_{x_{j-1}}^{x_j} f_j^2(x,t)dx \leq \frac{\Delta^2}{\pi^2}\int_{x_{j-1}}^{x_j} u_x^2(x,t)dx \tag{A.5}$$

holds.

Note that integration by parts of $-uu_{xx}$ yields

$$-\int_0^L u(x,t)u_{xx}(x,t)dx = \int_0^L u_x^2(x,t)dx. \tag{A.6}$$

Multiplying the inequality (A.5) by some constant $\lambda_1 > 0$, summing and using (A.6) we obtain

$$\begin{aligned}&\sum_{j=1}^N \lambda_1\left[\int_{x_{j-1}}^{x_j} u_x^2(x,t)dx - \frac{\pi^2}{\Delta^2}\int_{x_{j-1}}^{x_j} f_j^2(x,t)dx\right] \\ &= -\lambda_1\int_0^L u(x,t)u_{xx}(x,t)dx \\ &\quad - \sum_{j=1}^N \frac{\lambda_1\pi^2}{\Delta^2}\int_{x_{j-1}}^{x_j} f_j^2(x,t)dx \geq 0.\end{aligned} \tag{A.7}$$

Applying S-procedure, we add to $\dot{V}(t)+2\alpha V(t)$ the left-hand sides of (4.23) and (A.7). This leads to

$$\dot{V}(t)+2\alpha V(t)\le\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j}\varphi^T W\varphi dx<0 \tag{A.8}$$

if $W<0$ holds, where

$\varphi=\mathrm{col}\{u(x,t),u_{xx}(x,t),f_j(x,t)\}$

and W is given by (3.10).

Note that the feasibility of the LMI (3.10) with $\alpha=0$ implies its feasibility with a small enough $\alpha_0>0$. Therefore, if the LMI (3.10) holds for $\alpha=0$, then the system (3.6), (2.2) subject to (3.4) or (3.5) is exponentially stable with a small decay rate $\alpha_0>0$.

For $\lambda=\alpha=0$, by application of Schur complement theorem, the strict LMI (3.10) is feasible iff the following inequalities hold:

$$\mu^2(\lambda_1\frac{\pi^2}{\Delta^2})^{-1}-2\mu+(1+\frac{\lambda_1}{2})^2(2\nu)^{-1}<0. \tag{A.9}$$

Minimizing the left-hand side of the latter inequality in μ, we find that for $\mu=\lambda_1\frac{\pi^2}{\Delta^2}$ this inequality is reduced to $\frac{\Delta^2}{\pi^2}<\frac{2\lambda_1}{(1+\frac{\lambda_1}{2})^2}\nu$. Note that $\max_{\lambda_1}\frac{2\lambda_1}{(1+\frac{\lambda_1}{2})^2}=1$ (this corresponds to $\lambda_1=2$). Therefore, the system (3.6), (2.2) subject to (3.4) or (3.5) is exponentially stable if $\frac{\Delta^2}{\pi^2}<\nu$ and $\mu=\frac{2\pi^2}{\Delta^2}$.

Step 3: We obtain that if the LMI condition (3.10) is satisfied, then the solution (3.6), (2.2) subject to (3.4) or (3.5) initialized with $u_0\in H_0^2(0,L)$ on $[0,T)$ admits a priori estimate

$V(t)\le e^{-2\alpha t}V(0).$

Thus, continuation of this solution of (3.6), (2.2) subject to (3.4) or (3.5) under a priori bound to entire interval $[0,\infty)$ follows from Theorem 6.23.5 of Krasnoselskii et al. (1976). Furthermore, the inequality (A.8) implies (3.12) for all $t\ge 0$, which completes the proof.

Appendix B. Proof of Theorem 4.2

For the case of sampled-data controller under the averaged measurements, by arguments of Theorem 4.1, the well-posedness of (4.30) subject to (2.2) can be established via the step method.

Step 1: For $[t_0,t_1]$, we consider the following equation:

$$\begin{cases} u_t(x,t)+u_{xx}(x,t)+\nu u_{xxxx}(x,t)+u(x,t)u_x(x,t)\\ =-\mu\sum_{j=1}^{N}\left[b_j(x)\frac{\int_{x_{j-1}}^{x_j}u(\zeta,t_0)d\zeta}{\Delta_j}\right],\\ u(0,t)=u(L,t)=u_x(0,t)=u_x(L,t)=0.\end{cases} \tag{B.1}$$

We define the nonlinear term $F:H^2(0,L)\to L^2(0,L)$ according to

$$\begin{aligned}F(u(\cdot,t))&=-u(x,t)u_x(x,t)-u_{xx}(x,t)\\ &\quad-\mu\sum_{j=1}^{N}\left[b_j(x)\frac{\int_{x_{j-1}}^{x_j}u_0(\zeta)d\zeta}{\Delta_j}\right],\ t\in[t_0,t_1].\end{aligned}$$

The nonlinear term F is locally Lipschitz continuous. Thus, there exists a unique local strong solution $u(\cdot,t)\in H_0^2(0,L)$ of (4.30) subject to (2.2) on some interval $[0,T]\subset[0,t_1]$, where $T=T(u_0)>0$. By Theorem 6.23.5 of Krasnoselskii et al. (1976), we only need to prove that if the LMIs (4.32) are feasible, the solution of (4.30) subject to (2.2) starting from $\|u(\cdot,0)\|_V<\sqrt{p_3\nu}C$ admits a priori bound, which guarantees the existence of a strong solution of (4.30) subject to (2.2) on the entire interval $[0,t_1]$. Then, by applying the same arguments step-by-step for $[t_k,t_{k+1}]$, $k=1,2,\ldots$ we conclude that the strong solution exists for all $t\ge 0$.

Step 2: Assume formally that strong solution of (4.30) subject to (2.2) starting from $\|u_0\|_V<\sqrt{\frac{p_3\nu}{L}}C$ exists for all $t\ge 0$ and choose the same Lyapunov function V_1 as in (4.5). Differentiating V_1 along (4.30) subject to (2.2), we obtain the inequalities (4.13) and (4.16). Taking into account (4.31), we further apply the descriptor method to (4.30), where we add to $\dot{V}_1+2\delta V_1$ the left-hand side of the following equation

$$\begin{aligned}&2\int_0^L[p_2u(x,t)+p_3u_t(x,t)][-u_t(x,t)-u_{xx}(x,t)\\ &-\nu u_{xxxx}(x,t)-u(x,t)u_x(x,t)-\mu u(x,t)]dx\\ &+2\mu\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j}[p_2u(x,t)+p_3u_t(x,t)][f_j(x,t)\\ &+(t-t_k)k_j(t)]dx=0.\end{aligned} \tag{B.2}$$

Integration by parts and substitution of the Dirichlet boundary conditions (2.2) lead to (4.18), (4.19) and (4.20). Adding the left-hand side of the expression (B.2) into the right-hand side of (4.13), we obtain

$$\begin{aligned}&\dot{V}_1(t)+2\delta V_1(t)\\ &=2p_1\int_0^L u(x,t)u_t(x,t)dx-2p_2\nu\int_0^L u_{xx}^2(x,t)dx\\ &-re^{-2\delta h}\int_0^L\frac{1}{t-t_k}\left[\int_{t_k}^t u_s(x,s)ds\right]^2dx\\ &+r(t_{k+1}-t)\int_0^L u_t^2(x,t)dx+2\delta p_1\int_0^L u^2(x,t)dx\\ &+2\delta p_3\nu\int_0^L u_{xx}^2(x,t)dx\\ &+2\int_0^L[p_2u(x,t)+p_3u_t(x,t)][-u_t(x,t)-u_{xx}(x,t)\\ &-\mu u(x,t)]dx-2p_3\int_0^L u_t(x,t)u(x,t)u_x(x,t)dx\\ &+2\mu\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j}[p_2u(x,t)+p_3u_t(x,t)][f_j(x,t)\\ &+(t-t_k)k_j(t)]dx.\end{aligned} \tag{B.3}$$

From Jensen's inequality, we have

$$\begin{aligned}&-re^{-2\delta h}\int_0^L\frac{1}{t-t_k}\left[\int_{t_k}^t u_s(x,s)ds\right]^2dx\\ &=-\frac{r}{t-t_k}e^{-2\delta h}\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j}\left[\int_{t_k}^t u_s(x,s)ds\right]^2dx\\ &\le -r(t-t_k)e^{-2\delta h}\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j}k_j^2dx.\end{aligned} \tag{B.4}$$

To bound the term "$\int_0^L u_t(x,t)u(x,t)u_x(x,t)dx$" in (B.3), we need to bound $u_x(x,t)$ by some constant $C>0$ for all $t\ge 0$ and $x\in[0,L]$. Similar to Selivanov & Fridman (2016), we first assume that (4.27) is satisfied.

Set

$$\begin{aligned}&\eta_1=\mathrm{col}\{u(x,t),u_t(x,t),u_{xx}(x,t),f_j(x,t)\},\\ &\eta_2=\mathrm{col}\{u(x,t),u_t(x,t),u_{xx}(x,t),f_j(x,t),k_j\}.\end{aligned} \tag{B.5}$$

Applying S-procedure, we add the left-hand sides of (4.23) and (A.7) to $\dot{V}_1(t) + 2\delta V_1(t)$. Substituting (B.4) into (B.3), we obtain

$$\dot{V}_1(t) + 2\delta V_1(t) \leq \sum_{j=1}^{N} \int_{x_{j-1}}^{x_j} \frac{h - t + t_k}{h} \eta_1^T \Phi_1 \eta_1 + \frac{t - t_k}{h} \eta_2^T \Phi_2 \eta_2 dx < 0$$

if $\Phi_1 < 0$ and $\Phi_2 < 0$ hold for all $u_x \in (-C, C)$, where Φ_1 and Φ_2 are given by (4.33), (4.34) respectively.

Similarly to Theorem 4.1, LMIs (4.32) yield $\Phi_1 < 0$ and $\Phi_2 < 0$ for all $u_x \in (-C, C)$, and

$$V_1(t) \leq e^{-2\delta t} V_1(0) \quad \forall t \geq 0$$

for initial conditions $u(\cdot, 0) \in H_0^2(0, L)$ satisfying $\|u(\cdot, 0)\|_V < \sqrt{\frac{p_3 \nu}{L}} C$. Moreover, (4.27) is satisfied.

Step 3: By the arguments of the proof of Step 3 in Theorem 4.1, if the LMIs (4.32) are satisfied, then there exists a strong solution for $t \in [0, t_1]$ initialized with $\|u(\cdot, 0)\|_V < \sqrt{\frac{p_3 \nu}{L}} C$. Then, by applying the same arguments step-by-step for $[t_k, t_{k+1}]$, $k = 1, 2, \ldots$ we conclude that the strong solution exists for all $t \geq 0$.

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