

A Switched System Approach to Exponential Stabilization Through Communication Network

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Abstract—This paper considers a networked control loop, where the plant is a “slave” part, and the remote controller and observer constitute the “master”. Since the performance of networked control systems (NCS) depends on the quality of service (QoS) available from the network, it is worth to design a controller that takes into account qualitative information on the QoS in real-time. The goal of the design is to provide a controller that guarantees the following two things: 1) high performances (here expressed by exponential decay rates) when the QoS remains globally the same and 2) global stability when the QoS changes. In order to guarantee the global stability, the controller will switch by respecting a dwell time constraint. The dwell time parameters are obtained by using the switched system theories and the obtained conditions are linear matrix inequalities. An experiment illustrates how the controller can be implemented for a control over Internet application (remote control of a small robot).

Index Terms—Gain scheduling, linear matrix inequality (LMI), Lyapunov-Krasovskii method, networked control systems, time delay approach.

I. INTRODUCTION

WITH the development of computer networks and of communication technologies, real-time control over networks became possible and attracted a lot of attention [see [7], [12], [33], and [38] for a general overview on control trends and approaches for networked control systems (NCS)]. At the same time, in addition to the resulting gain of flexibility, the other expected performances (speed, robustness) also keep growing, and such a demand has to cope with the perturbations the networks induce. Data-packet loss and disorder, time lags depending on the traffic load, asynchronism, bandwidth limitation, belong to such classical drawback of communication networks. This is why real-time control applications classically prefer token ring local area networks, whereas cheaper

technologies such as Internet and Ethernet are still limited to less demanding applications such as remote monitoring. The present work aims at both guaranteeing and improving the real-time control performances achievable with classical networks that allow for sending time-stamped packets, including Internet/Ethernet, wireless LAN, Bluetooth, Zigbee, etc.

Many authors already classified the perturbations induced by communication networks and most of them can be regrouped into time-lag effects (see [7], [12], [19], [33], and [38]). Such network-induced delays vary depending on the network hardware, the different protocols, data-packet losses and disorder. This can cause poor performance, instability or danger (see for instance of [8, ch. 1], [27], and the references herein). A variety of stability and control techniques have been developed for general time-delay systems [2], [9], [19], [26]. Many of these techniques consider constant delays. Their adaptation to NCS is either based on simplifying assumptions (considering the time delay as constant [4], [14], [20], [39] is unrealistic in our case, due to the dynamic character of the network), or lead to technical solutions that make the delay become constant: A delay maximizing strategy [3], [16], [23] (“virtual delay”, “buffer”, or “waiting” strategy) can be carried out so to make the delay become constant and known. However, it is obvious that maximizing the delay up to its largest value decreases the speed performance of the remote system. Concerning time-varying delays, several other results were developed. Among them, [1] and [22] considered a communication delay which value is less than the sensor and controller sampling periods. In the Internet case, this constraint leads to increase the sampling periods up to the maximal network delay, which may be constraining for applications with fast dynamics. Predictor-based techniques were also generalized to variable delays [34] but, in the Internet case, the network delays cannot be modelled nor predicted and this lack of knowledge does not allow for concluding. In [29] and [37], LMIs allow for guaranteeing the stability of the closed-loop systems despite any variation of the communication delays, provided they stay within some interval, e.g., $[h_{\min}, h_{\max}]$. In [37], the results are based on Lyapunov-Razumikhin functions for the continuous-time case, which leads to stability without additional performance evaluation. In [29] Lyapunov-Krasovskii functionals (LKF) are applied and exponential stabilization is considered.

The structure of NCS that we consider is depicted on Fig. 1. It is a single feedback loop which corresponds to the classical simplification of more general NCS, while still capturing many important characteristics of NCS (see [12]). There, a plant (namely, the Slave) is controlled by a remote controller and observer (the

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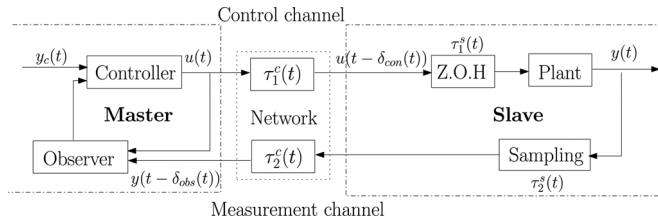


Fig. 1. Feedback NCS with observer-based controller.

Master) via a network. The two communication delays are denoted $\tau_1^c(t)$ for the control channel (from Master to Slave) and $\tau_2^c(t)$ for the measurement channel (Slave to Master). Inspired by [29], we limit the waiting delay strategy (aiming at obtaining a constant, virtual delay as recalled above [3], [16], [23]) to the Slave side, while on the side of Master there is no such additional delay. In this way, the measurement channel data can be used directly by the Master as soon as received from the Slave. It will be shown that this structure allows the Master to reconstruct the present Slave's state despite the delays. Note that it could be possible to get rid of the waiting strategy in both the control and measurement channel, and thus to allow a more reactive communication. Such a solution was proposed in [30] at the price of an additional complexity of the LMI involved in the observer design. In the present paper, we prefer to avoid this additional complexity and to focus on the switched gain control, which effects on the performance more essentially.

A main novelty of the present paper lies in the introduction of a *QoS-dependent* controller structure. In the existing works [7], the stabilization is obtained on the basis of the two bounds h_{\min} and h_{\max} , disregarding the way the delays vary between these extrema. If the actual delays stay a long time near to h_{\min} , the robust control design keeps taking into account the worst case h_{\max} (highest delay, lowest QoS). Such a cautious solution generally yields poor speed performance and we prefer to design a switching controller that adapts to different intervals of delay variation.

In this case, the usual techniques for time-delay systems cannot be directly applied and one has to go to switched systems techniques. Switched systems are dynamical hybrid systems consisting of a family of continuous-time subsystems and a logical rule that orchestrates the switching between them [13], [17]. It is well established that the stability of the subsystems themselves is not sufficient for the stability of the overall system and Lyapunov techniques constitute the basis of the global analysis. In what concerns switched time-delay systems, single [15], [31] and multiple [21] Lyapunov-Krasovskii Functionals (LKFs) methods are the most frequently used for the stabilization of the whole system. A common LKF can ensure the stability with arbitrary switching, but it does not necessarily exist. As an alternative, dwell-time-based switching is considered in [11], [31], [35], and [36]. These results on switched time-delay systems consider switched linear systems with the same delays **in all the modes**.

In this paper, we consider a two-modes switched system with a time-varying delay. We assume that the control input delay δ may belong to two intervals: it is either *small* (belonging to $[h_1, h_2)$) or *big* (belonging to $[h_2, h_3)$). Similarly, we assume

that the measurement time-varying delay is either small or big. We derive novel conditions for the exponential stability of switched systems with switched delays and coefficients. Next, we solve the output-feedback stabilization problem by designing switched gains for the controller and for the observer. These gains are switched correspondingly to delay switches (such delays are known from the round-trip-time (RTT) measurement and a time-stamp strategy). Of course, more numerous modes could be considered with an increased, offline computation effort, but this is not to be developed in the present paper.

Finally, differently from the existing works on NCS, we illustrate the efficiency of our theoretical results by experimental ones, where a light-inertia robot is controlled via Internet.

This paper is organized as follows. The features of NCS as well as the problem statement are described in Section II. Section III addresses the exponential stability analysis and the design of switched time-delay systems. Exponential stability conditions under arbitrary switching is deduced. In Section IV, multiple LKFs are adapted in the exponential stabilization theory to get a better performance and the global stability is guaranteed by adding the minimum dwell-time into the switched system. The experimental example is given in Section V and confirms the theoretical results.

II. PROBLEM STATEMENT

A. Description of the Considered System

In the NCS of Fig. 1, the process (“Slave”) is a low energy consumption system, the embedded computer of which has a limited computation power. Thus, the control and observation complexity is concentrated on the remote Master computer (the “Master”). This structure makes our theory and application adaptable to sensor and actuator NCSs.

The transmission protocol UDP [24] is applied to communicate the data packets between Master and Slave. It is known that, compared to TCP/IP which retransmits the dropped out data packets, UDP suffers from a lack of reliability. However, in our situation, transmitting fresh data (control data and sampled measurement data) is preferable to retransmitting old ones. Finally, UDP is consuming much less time and is preferred to TCP.

B. Three Delay Sources

In Fig. 1, the control channel total delay (Master-to-Slave) $\delta_{con}(t)$ results from the addition of the following three (variable) delay sources: 1) the communication through the Internet; 2) the data-sampling (see [6]); and 3) the possible packet losses (which can be seen as longer sampling in the case of UDP, see [12]). The same phenomenon stands for the measuring channel delay (Slave-to-Master) which total value is denoted $\delta_{obs}(t)$.

1) *Communication*: The time-delays of communication $\tau_i^c(t)$ are variable and not bounded, it is reasonable to assume an upper-bound value since if the time-delay surpasses this value, the packet can be treated as lost.

2) *Sampling and Packet Dropout*: The real remote system, including Master, Slave, and Network, must involve some data sampling. However, following [6] and [28], this phenomenon is

equivalent to a time-varying, discontinuous delay. If the sampling period is not negligible, it constitutes a disturbance that should be considered in the stabilization design [37]. If some packet p_{t_k} containing the Slave's output (or Master's control data) sampled at t_k is lost, or arrives later than the packet $p_{t_{k+1}}$, then the other part only considers the most recent data (i.e., those from $p_{t_{k+1}}$). The phenomenon acts as a longer sampling period. N is the maximum number of consecutive loss of packet. The packet loss sampling and the sampling induced delay is $\tau_i^s(t) \leq (N+1)T$, $i = 1, 2$.

In conclusion, the total delays are treated as variable and bounded

$$\begin{aligned} \delta_{\text{con}}(t) &= \tau_1^c(t) + \tau_1^s(t) \\ \delta_{\text{obs}}(t) &= \tau_2^c(t) + \tau_2^s(t). \end{aligned} \quad (1)$$

C. Output Stabilization

1) *Stabilization for Closed-Loop System*: The master-slave exchanges are network packets composed of data information and a time-stamp. From the master to the slave, the time stamps represent the instant when the master would like to apply the control input. In the other way, the time stamps represent the time t_o that the measurement has been done. The clocks of the master and the slave are synchronized before powering up the system. The network time protocol (NTP) [18] is used for this purpose.

The Slave system, is constituted by the plant and a communication system. The communication system is in charge of re-ordering the packets and applying the control value in the right moment t_c given by the master. It is also in charge of acquiring the measurement and send them periodically to the master. The measurement time is noted as t_o by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t_c) \\ y(t) = Cx(t) \end{cases} \quad (2)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ are, respectively, the state, the input and the output vector. t_c is the time of the last update of the control value.

The master part is an observer-based state-feedback controller with a communication structure. This latter compares the newest measurements with its respective estimation done at the time t_o , i.e., when the data was acquired, and sends it to the observer structure. The same method is used to send the input value to the observer, ensuring the synchronization of the observer and the plant. This is possible thanks to the slave buffer, which allows the master to choose the instant t_c that the plant will apply the control value. The delay induced by this buffer strategy must be larger than the maximum delay induced by the communication delay $\tau_1^c(t)$, i.e., $t_c = t + \text{buffer} \geq t + \max(\tau_1^c(t))$. The last task is to send the control values and the corresponding appliance time to the slave.

The output controller (Master) is based on the following observer:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t_c) + L(y(t_o) - \hat{y}(t_o)) \\ \hat{y}(t) = C\hat{x}(t) \\ u(t) = K\hat{x}(t) + ky_c(t) \end{cases} \quad (3)$$

where $y_c(t)$ is the desired setpoint.

The estimation error $e(t) = x(t) - \hat{x}(t)$ is given by

$$\dot{e}(t) = Ae(t) - LCe(t_o). \quad (4)$$

Note that the error system does not depend on u nor x . This ensures that the separation principle is applicable and allows for reducing the study of the closed-loop dynamic to two smaller stabilization problems. These two problems can be formulated as delay-system stabilization problems [29]

$$1) \quad \dot{x}(t) = Ax(t) + BKx(t - \delta_{\text{con}}(t)) \quad (5)$$

$$2) \quad \dot{e}(t) = Ae(t) - LCe(t - \delta_{\text{obs}}(t)) \quad (6)$$

where $\delta_{\text{con}}(t) = t - t_c$, $\delta_{\text{con}}(t) \in [h_{\min}, h_{\max}]$, and $\delta_{\text{obs}}(t) = t - t_o$, $\delta_{\text{obs}}(t) \in [h_{\min}, h_{\max}]$ being the resulting delays described by the following.

2) *QoS Adaptation With Switched Controller*: In order to enhance the performance of the system and take into account the network QoS into the control, the controller switches to the mode i according to the value of the delay. The controller has the information to which predefined subset S_i of $[h_{\min}, h_{\max}]$ belongs the delays. In the following, it is assumed that the two delays $\{\delta_{\text{con}}(t), \delta_{\text{obs}}(t)\}$ are always in the same subset S_i , i.e., $\delta_{\text{con}}(t) \in S_i \Leftrightarrow \delta_{\text{obs}}(t) \in S_i$. This property can be fulfilled by adjusting the waiting time on the control channel with the measurement of the delays on each channels (i.e., $\max(\delta_{\text{con}}(t), \delta_{\text{obs}}(t))$ is used to decide of the size of the control channel buffer). The number of modes considered is two, i.e., $i \in \{1, 2\}$. The two subsets are defined as: $S_1 = [h_1, h_2)$ and $S_2 = [h_2, h_3]$, where $\delta(t) \in S_1$ represents a small delay, and $\delta(t) \in S_2$ represents a big delay. To cover all possible delays, the bounds of the sets are chosen as $h_1 = h_{\min}$ and $h_3 = h_{\max}$ then $S_1 \cup S_2 = [h_{\min}, h_{\max}]$.

Remark 1: Only two modes are considered and the same delay subsets are chosen for the controller and the observer for sake of clarity. It is obvious that the results presented here can be extended to more complex cases where the subsets are different for the two delays and where more modes are considered. This is not detailed in this paper due to the heavy burden it may cause to the comprehension.

The resulting controller differs slightly from the non switching one. In the i th mode, the time t_c of appliance of the control value is based on the maximum of the set S_i , h_{i+1} . It is given by $t_c = t + h_{i+1} - T$. The delay is detected by analyzing the time stamps in the packets, or by measuring the RTT. This introduce a delay in the delay measurement. According to this measure, the switching strategy have the following properties.

- 1) As soon as a bigger delay is detected the controller must switch to the corresponding mode to minimize disturbances due to a bad size of buffer (buffer \downarrow delay).
- 2) The controller has to follow the dwell time requirements when the detected delay becomes smaller, i.e., in a mode i smaller than the current mode. During the dwell time, the controller artificially enlarge the delay to meet the properties of the current mode.

In its i th mode, the controller is given by

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t - \hat{\delta}_{\text{con}}^i(t)) \\ \quad + L_i(y(t - \delta_{\text{obs}}^i(t)) - \hat{y}(t - \delta_{\text{obs}}^i(t))) \\ \hat{y}(t) = C\hat{x}(t) \\ u(t) = K_i\hat{x}(t) + ky_c(t) \end{cases} \quad (7)$$

where $\hat{\delta}_{\text{con}}^i(t)$ is predicted control delay implied by the buffer. $\hat{\delta}_{\text{con}}^i(t) = \delta_{\text{con}}^i(t)$ when the communication delay is less or equal to the buffer, otherwise $\hat{\delta}_{\text{con}}^i(t) \neq \delta_{\text{con}}^i(t)$ and is not in the same set than $\delta_{\text{con}}^i(t)$. This latter case occurs when the delay moves from the set S_1 to S_2 , making the buffer smaller than the delay, disturbing the observer (10).

Remark 2: An important feature of this controller is that it has the ability to maintain artificially the delay at an higher value simply by increasing the buffer size. So even if the communication delay is small, we can force δ_{con} to be big, i.e., it belongs to S_2 . The global stability will be ensured by a correct use of this ability.

Since a delay is either in S_1 or in S_2 , one naturally introduces $\chi : \mathbb{R} \rightarrow \{0, 1\}$, the characteristic function of S_1 , defined by

$$\chi_{S_1}(s) = \begin{cases} 1, & \text{if } s \in S_1 \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

The characteristic function of S_2 is $1 - \chi$. Since it is assumed that the two delays are always in the same subsets we have $\chi(\delta_{\text{con}}(t)) = \chi(\delta_{\text{obs}}(t))$.

Now, considering the control problem (2) together with this 2-modes gain switching strategy, the stabilization problems of observer and controller can be rewritten as follows:

$$1) \quad \dot{x}(t) = Ax(t) + \chi_{S_1}(\delta_{\text{con}}(t))BK_1\hat{x}(t - \delta_{\text{con}}(t)) \\ + (1 - \chi_{S_1}(\delta_{\text{con}}(t)))BK_2\hat{x}(t - \delta_{\text{con}}(t)) \quad (9)$$

$$2) \quad \dot{e}(t) = Ae(t) - \chi_{S_1}(\delta_{\text{obs}}(t))L_1Ce(t - \delta_{\text{obs}}(t)) \\ - (1 - \chi_{S_1}(\delta_{\text{obs}}(t)))L_2Ce(t - \delta_{\text{obs}}(t)) \\ - Bu(t - \delta_{\text{con}}^i(t)) + Bu(t - \hat{\delta}_{\text{con}}^i(t)). \quad (10)$$

It must be notified that the separation principle does not hold in this case because of the term $-Bu(t - \delta_{\text{con}}^i(t)) + Bu(t - \hat{\delta}_{\text{con}}^i(t))$. This term equals zero except when a switch from mode 1 to mode 2 occurs. This means that the separation principle holds if there are no switches.

III. EXPONENTIAL OUTPUT-FEEDBACK STABILIZATION OF SWITCHED TIME-DELAY SYSTEM

This part provides two results in the general framework of time delay system with switches. The first one gives sufficient conditions for guaranteed performance analysis of the modes of a switched system with delays. The second subsection provides a controller design procedure guaranteeing exponential stability of each mode when no switch occurs. These two results are important to design a switching strategy guaranteeing stability and performances.

A. Exponential Stability of Switched Time-Delay Systems

Consider the switched system

$$\dot{x}(t) = Ax(t) + \chi_{S_1}(\tau)A_1x(t - \tau(t)) \\ + (1 - \chi_{S_1}(\tau))A_2x(t - \tau(t)) \quad (11)$$

where the delays $\tau \in S_1 \cup S_2$ are assumed to be fast-varying (no restrictions on the delay-derivative) within these bounds.

The easy way to guarantee the stability of a switched system with arbitrary switches is to find a common LKF for each mode. This has a negative effect on the decay rate α which will be the same in all the modes (the worse one) and it is not the goal of QoS-based controller. Another way to deal with switch system is to consider a LKF for each mode and compute some minimum dwell time requirement for stability and performances. This allows performances adaptation according to the current QoS and has a reduced conservatism (in term of finding gains and LKF for some α).

For the present problem, the stability analysis is performed by using a pair of LKFs for each mode. In order to compute a dwell time condition, they must be easy to compare so they have the same structure

$$V_j(t, x_t, \dot{x}_t) \\ = x^T(t)P_jx(t) + \sum_{i=0}^2 \int_{t-h_{i+1}}^{t-h_i} e^{2\alpha_j(s-t)} x^T(s)S_{ij}x(s)ds \\ + \sum_{i=0}^2 (h_{i+1} - h_i) \int_{-h_{i+1}}^{-h_i} \int_{t+\theta}^t e^{2\alpha_j(s-t)} \dot{x}^T(s)R_{ij}\dot{x}(s)dsd\theta \quad (12)$$

where j represents the mode and $h_0 = 0$.

Remark 3: These functionals are taking into account the two possible subsets for the delay. It has no repercussions on the conservatism even in the case where the delay is considered to be in a given subset (for example the delay is considered to belong to S_1). The proof is based on setting the appropriate matrices close to zero.

The following condition along the trajectories of (11)

$$\dot{V}_j(t, x_t, \dot{x}_t) + 2\alpha_j V_j(t, x_t, \dot{x}_t) \leq 0 \quad (13)$$

implies some decay rate on the LKF

$$V_j(t, x_t, \dot{x}_t) \leq e^{-2\alpha_j(t-t_0)} V_j(t, x_{t_0}, \dot{x}_{t_0}) \quad \forall t_0 \in \mathbb{R}. \quad (14)$$

The latter implies exponential stability of (11) since

$$x^T(t)P_jx(t) \leq V_j(t, x_t, \dot{x}_t) \leq e^{-2\alpha_j(t-t_0)} V_j(t, x_{t_0}, \dot{x}_{t_0}) \\ \leq e^{-2\alpha_j(t-t_0)} V_j(t, x_{t_0}, \dot{x}_{t_0})|_{\alpha_j=0}.$$

The following theorem gives conditions guaranteeing exponential stability with prescribed decay rate in each mode separately. It DOES NOT ensure stability of the system (11) for arbitrary switches.

Theorem 1: Given $\alpha_j > 0$, $j = 1, 2$, if there exist $n \times n$ -matrices $P_j > 0$, $R_{ij} > 0$, $S_{ij} > 0$, $i = 0, 1, 2$, P_{2j} , P_{3j} , Y_{1j} , and Y_{2j} such that the LMIs (15), (16) with (17) are feasible, then each mode $j = 1, 2$ of switched delay system (11) is exponentially stable with the rate α_j for all fast-varying delays $\tau \in [h_j, h_{j+1}]$. See (15) and (16) at the bottom of the next page, holds, where

$$\tilde{R}_{ij} = e^{-2\alpha_j(h_{i+1})} R_{ij} \\ \tilde{S}_{ij} = e^{-2\alpha_j h_{i+1}} S_{ij}$$

$$\begin{aligned}
 \hat{S}_{ij} &= e^{-2\alpha_j h_i} S_{ij} \\
 \Phi_{11j} &= A^T P_{2j} + P_{2j}^T A + S_{0j} - e^{-2\alpha_j h_1} R_{0j} + 2\alpha_j P_j \\
 \Phi_{12j} &= P_j - P_{2j}^T + A^T P_{3j} \\
 \Phi_{22j} &= -P_{3j} - P_{3j}^T + \sum_{i=0}^2 (h_{i+1} - h_i)^2 R_{ij}. \quad (17)
 \end{aligned}$$

Proof: Computing (13) with the LKFs (12) gives

$$\begin{aligned}
 &\dot{V}_j(t, x_t, \dot{x}_t) + 2\alpha_j V_j(t, x_t, \dot{x}_t) \\
 &\leq 2x^T(t)P_j\dot{x}(t) + 2\alpha_j x^T(t)P_j x(t) \\
 &\quad + \dot{x}^T(t) \left[\sum_{i=0}^2 (h_{i+1} - h_i)^2 R_{ij} \right] \dot{x}(t) \\
 &\quad - \sum_{i=0}^2 (h_{i+1} - h_i) e^{-2\alpha_j (h_{i+1})} \int_{t-h_{i+1}}^{t-h_i} \dot{x}^T(s) R_{ij} \dot{x}(s) ds \\
 &\quad + \sum_{i=0}^2 x(t-h_i)^T e^{-2\alpha_j h_i} S_{ij} x(t-h_i) \\
 &\quad - \sum_{i=0}^2 e^{-2\alpha_j h_{i+1}} x^T(t-h_{i+1}) S_{ij} x(t-h_{i+1}). \quad (18)
 \end{aligned}$$

1) We start with the case of $\chi = 1$, i.e., $\tau \in [h_1, h_2)$, $i = 0, 2$, $j = 1$. Note that

$$\begin{aligned}
 \int_{t-h_2}^{t-h_1} \dot{x}^T(s) R_{11} \dot{x}(s) ds &= \int_{t-h_2}^{t-\tau(t)} \dot{x}^T(s) R_{11} \dot{x}(s) ds \\
 &\quad + \int_{t-\tau(t)}^{t-h_1} \dot{x}^T(s) R_{11} \dot{x}(s) ds.
 \end{aligned}$$

Applying the Jensen's inequality [9] for $i = 0, 2$ gives

$$\begin{aligned}
 &\int_{t-h_2}^{t-\tau(t)} \dot{x}^T(s) R_{11} \dot{x}(s) ds \\
 &\geq \frac{1}{h_2 - h_1} \int_{t-h_2}^{t-\tau(t)} \dot{x}^T(s) ds R_{11} \int_{t-h_2}^{t-\tau(t)} \dot{x}(s) ds
 \end{aligned}$$

$$\begin{aligned}
 &\int_{t-\tau(t)}^{t-h_1} \dot{x}^T(s) R_{11} \dot{x}(s) ds \\
 &\geq \frac{1}{h_2 - h_1} \int_{t-\tau(t)}^{t-h_1} \dot{x}^T(s) ds R_{11} \int_{t-\tau(t)}^{t-h_1} \dot{x}(s) ds \\
 &\int_{t-h_{i+1}}^{t-h_i} \dot{x}^T(s) R_{i1} \dot{x}(s) ds \\
 &\geq \frac{1}{h_{i+1} - h_i} \int_{t-h_{i+1}}^{t-h_i} \dot{x}^T(s) ds R_{i1} \int_{t-h_{i+1}}^{t-h_i} \dot{x}(s) ds. \quad (19)
 \end{aligned}$$

Then, denoting $v_{11} = \int_{t-\tau(t)}^{t-h_1} \dot{x}(s) ds$, $v_{12} = \int_{t-h_2}^{t-\tau(t)} \dot{x}(s) ds$, we obtain

$$\begin{aligned}
 &\dot{V}_1(t, x_t, \dot{x}_t) + 2\alpha_1 V_1(t, x_t, \dot{x}_t) \\
 &\leq 2x^T(t)P_1\dot{x}(t) + 2\alpha_1 x^T(t)P_1 x(t) \\
 &\quad + \dot{x}^T(t) \sum_{i=0}^2 (h_{i+1} - h_i)^2 R_{i1} \dot{x}(t) \\
 &\quad + \sum_{i=0}^2 e^{-2\alpha_1 h_i} x^T(t-h_i) S_{i1} x(t-h_i) \\
 &\quad - \sum_{i=0}^2 e^{-2\alpha_1 h_{i+1}} x^T(t-h_{i+1}) S_{i1} x(t-h_{i+1}) \\
 &\quad - [x(t) - x(t-h_1)]^T e^{-2\alpha_1 h_1} R_{01} [x(t) - x(t-h_1)] \\
 &\quad - [x(t-h_2) - x(t-h_3)]^T e^{-2\alpha_1 h_3} R_{21} [x(t-h_2) - x(t-h_3)] \\
 &\quad - v_{11}^T e^{-2\alpha_1 h_2} R_{11} v_{11} - v_{12}^T e^{-2\alpha_1 h_2} R_{11} v_{12}. \quad (20)
 \end{aligned}$$

We use further the descriptor method [5]. The following quantity:

$$\begin{aligned}
 0 &= 2[x^T(t)P_{21}^T + \dot{x}^T(t)P_{31}^T] \\
 &\quad \times [Ax(t) + A_1 x(t-h_1) - A_1 v_{11} - \dot{x}(t)]
 \end{aligned}$$

with some matrices P_{21} , P_{31} of the appropriate size, is added into the right-hand side of (20). Following [10], we also add free weighting matrices

$$\Phi|_{\chi=1} = \begin{bmatrix} \Phi_{111} & \Phi_{121} & \tilde{R}_{01} + P_{21}^T A_1 - Y_{11}^T & Y_{11}^T & 0 & [Y_{11}^T - P_{21}^T A_1] & Y_{11}^T \\ * & \Phi_{221} & P_{31}^T A_1 - Y_{21}^T & Y_{21}^T & 0 & [Y_{21}^T - P_{31}^T A_1] & Y_{21}^T \\ * & * & \hat{S}_{11} - (\tilde{S}_{01} + \tilde{R}_{01}) & 0 & 0 & 0 & 0 \\ * & * & * & \hat{S}_{21} - (\tilde{S}_{11} + \tilde{R}_{21}) & \tilde{R}_{21} & 0 & 0 \\ * & * & * & * & -(\tilde{S}_{21} + \tilde{R}_{21}) & 0 & 0 \\ * & * & * & * & * & -\tilde{R}_{11} & 0 \\ * & * & * & * & * & * & -\tilde{R}_{11} \end{bmatrix} < 0 \quad (15)$$

$$\Phi|_{\chi=0} = \begin{bmatrix} \Phi_{112} & \Phi_{122} & \tilde{R}_{02} & P_{22}^T A_2 - Y_{12}^T & Y_{12}^T & [Y_{12}^T - P_{22}^T A_2] & Y_{12}^T \\ * & \Phi_{222} & 0 & P_{32}^T A_2 - Y_{22}^T & Y_{22}^T & [Y_{22}^T - P_{32}^T A_2] & Y_{22}^T \\ * & * & \hat{S}_{12} - (\tilde{S}_{02} + \tilde{R}_{02} + \tilde{R}_{12}) & \tilde{R}_{12} & 0 & 0 & 0 \\ * & * & * & \hat{S}_{22} - (\tilde{S}_{12} + \tilde{R}_{12}) & 0 & 0 & 0 \\ * & * & * & * & -\tilde{S}_{22} & 0 & 0 \\ * & * & * & * & * & -\tilde{R}_{22} & 0 \\ * & * & * & * & * & * & -\tilde{R}_{22} \end{bmatrix} < 0 \quad (16)$$

$$0 = 2[x^T(t)Y_{11}^T + \dot{x}^T(t)Y_{21}^T] \\ \times [-x(t-h_1) + x(t-h_2) + v_{11} + v_{12}].$$

Setting $\eta_j(t) = \text{col}\{x(t), \dot{x}(t), x(t-h_1), x(t-h_2), x(t-h_3), v_{11}, v_{12}\}$, the inequality

$$\dot{V}_1(t, x_t, \dot{x}_t) + 2\alpha_1 V_1(t, x_t, \dot{x}_t) \leq \eta_1^T(t) \Phi \eta_1(t) < 0 \quad (21)$$

is satisfied if the LMI (15) holds.

2) *Continuing with the case* $\chi = 0$, i.e., $\tau \in [h_2, h_3]$, by applying the same method with $j = 2$ and $i = 0, 1$, we obtain:

$$\begin{aligned} & \dot{V}_2(t, x_t, \dot{x}_t) + 2\alpha_2 V_2(t, x_t, \dot{x}_t) \\ & \leq 2x^T(t)P_2\dot{x}(t) + 2\alpha_2 x^T(t)P_2x(t) \\ & \quad + \dot{x}^T(t) \sum_{i=0}^2 (h_{i+1} - h_i)^2 R_{i2} \dot{x}(t) \\ & \quad + \sum_{i=0}^2 x^T(t-h_i) \hat{S}_{i2} x(t-h_i) \\ & \quad - \sum_{i=0}^2 x^T(t-h_{i+1}) \tilde{S}_{i2} x(t-h_{i+1}) \\ & \quad - [x(t) - x(t-h_1)]^T \tilde{R}_{02} [x(t) - x(t-h_1)] \\ & \quad - [x(t-h_2) - x(t-h_1)]^T \tilde{R}_{21} [x(t-h_2) - x(t-h_1)] \\ & \quad - v_{j1}^T \tilde{R}_{22} v_{j1} - v_{j2}^T \tilde{R}_{22} v_{j2}. \end{aligned} \quad (22)$$

The (21) with $j = 2$ is then satisfied if the LMI (16) holds.

This concludes the proof of Theorem 1. \blacksquare

Remark 4: Note that the Theorem 1 can also be easily applied to arbitrary switches by using the same LKF for both the switching modes, i.e., fixing $\alpha_1 = \alpha_2 = \alpha$, $P_1 = P_2$, $S_{i1} = S_{i2}$, $R_{i1} = R_{i2}$, $P_{21} = P_{22}$, $P_{31} = P_{32}$, ($i = 0, 1, 2$).

Form this remark, the following result is obtained.

Corollary 1: Given $\alpha > 0$, if there exist $n \times n$ -matrices $P_1 = P_2 > 0$, $R_{i1} = R_{i2} > 0$, $S_{i1} = S_{i2} > 0$, $i = 0, 1, 2$,

$P_{21} = P_{22}$, $P_{31} = P_{32}$, Y_{j1} , and Y_{j2} , $j = 1, 2$ such that the LMIs (15), (16) with notations given in (17) are feasible, then the arbitrary switching delay system (11) is exponentially stable with the rate α for all fast-varying delays $\tau \in [h_j, h_{j+1}]$, $j = 1, 2$.

B. Strategy for the Output Stabilization

The results presented here are direct consequence of theorem 1. They provide a design procedure to guarantee prescribed performances in each mode. As it was the same case as in Theorem 1, it DOES NOT ensure the stability when arbitrary switches occur. That is why it is assumed that in (9), $\hat{x}(t) = x(t)$, and in (10), $\hat{\delta}_{\text{con}}^i(t) = \delta_{\text{con}}^i(t)$ (separation principle holds). The first lemma is used for the state feedback controller design procedure and the second for the observer design procedure.

Lemma 1: Given some scalars $\alpha_j > 0$ and $\epsilon_j > 0$, $j = 1, 2$ if there exist some matrices $P_{1j} > 0$, $R_{ij} > 0$, $S_{ij} > 0$, P_{2j} , P_{3j} , Y_{ij} , and M_j such that the LMIs (23) and (24) hold, then each mode j of the closed loop (9) with $\hat{x}(t) = x(t)$ is exponentially stabilized with the decay rate α_j . The control gains are given by $K_j = M_j P_{2j}^{-1}$. See (23) and (24) at the bottom of the page, with

$$\begin{aligned} \tilde{R}_{ij} &= e^{-2\alpha_j h_{i+1}} R_{ij} \\ \tilde{S}_{ij} &= e^{-2\alpha_j h_{i+1}} S_{ij} \\ \hat{S}_{ij} &= e^{-2\alpha_j h_i} S_{ij} \\ \Phi_{11j} &= AP_{2j} + P_{2j}^T A^T + S_{0j} - e^{-2\alpha_j h_1} R_{0j} + 2\alpha_j P_{1j} \\ \Phi_{12j} &= P_{1j} - P_{2j} + \epsilon_j P_{2j}^T A \\ \Phi_{22j} &= -\epsilon_j P_{2j} - \epsilon_j P_{2j}^T + \sum_{i=0}^2 (h_{i+1} - h_i)^2 R_{ij}. \end{aligned} \quad (25)$$

Lemma 2: Given some scalars $\alpha_j > 0$ and $\epsilon_j > 0$, $j = 1, 2$, if there exist some matrices $P_{1j} > 0$, $R_{ij} > 0$, $S_{ij} > 0$, P_{2j} , P_{3j} , Y_{ij} , and M_j such that the following LMIs (26) and (27) hold, then each mode of the observation error equation given in (10) with $\hat{\delta}_{\text{con}}^i(t) = \delta_{\text{con}}^i(t)$ is exponentially stabilized with a

$$\Phi_{\text{con1}} = \begin{bmatrix} \Phi_{111} & \Phi_{121} & [\tilde{R}_{01} - BM_1 - Y_{11}^T] & Y_{11}^T & [Y_{11}^T + BM_1] & Y_{11}^T & 0 \\ * & \Phi_{221} & [-\epsilon_1 BM_1 - Y_{21}^T] & Y_{21}^T & [Y_{21}^T + \epsilon_1 BM_1] & Y_{21}^T & 0 \\ * & * & \tilde{S}_{11} - \tilde{S}_{01} - \tilde{R}_{01} & 0 & 0 & 0 & 0 \\ * & * & * & \hat{S}_{21} - \tilde{S}_{11} - \tilde{R}_{21} & 0 & 0 & \tilde{R}_{21} \\ * & * & * & * & -\tilde{R}_{11} & 0 & 0 \\ * & * & * & * & * & -\tilde{R}_{11} & 0 \\ * & * & * & * & * & * & -\tilde{S}_{21} - \tilde{R}_{21} \end{bmatrix} < 0 \quad (23)$$

$$\Phi_{\text{con2}} = \begin{bmatrix} \Phi_{112} & \Phi_{122} & [-BM_2 - Y_{12}^T] & Y_{12}^T & [Y_{12}^T + BM_2] & Y_{12}^T & \tilde{R}_{02} \\ * & \Phi_{222} & [-\epsilon_2 BM_2 - Y_{22}^T] & Y_{22}^T & [Y_{22}^T + \epsilon_2 BM_2] & Y_{22}^T & 0 \\ * & * & \tilde{S}_{22} - \tilde{S}_{12} - \tilde{R}_{12} & 0 & 0 & 0 & \tilde{R}_{12} \\ * & * & * & -\tilde{S}_{22} & 0 & 0 & 0 \\ * & * & * & * & -\tilde{R}_{22} & 0 & 0 \\ * & * & * & * & * & -\tilde{R}_{22} & 0 \\ * & * & * & * & * & * & \hat{S}_{12} - \tilde{S}_{02} - \tilde{R}_{02} - \tilde{R}_{12} \end{bmatrix} < 0 \quad (24)$$

decay rate α_j , shown in (26) and (27) at the bottom of the page, with

$$\begin{aligned}\tilde{R}_{ij} &= e^{-2\alpha_j h_{i+1}} R_{ij} \\ \tilde{S}_{ij} &= e^{-2\alpha_j h_{i+1}} S_{ij} \\ \hat{S}_{ij} &= e^{-2\alpha_j h_i} S_{ij} \\ \Phi_{11j} &= A^T P_{2j} + P_{2j}^T A + S_{0j} - e^{-2\alpha_j h_1} R_{0j} + 2\alpha_j P_{1j} \\ \Phi_{12j} &= P_{1j} - P_{2j}^T + \epsilon_j A^T P_{2j} \\ \Phi_{22j} &= -\epsilon P_{2j} - \epsilon P_{2j}^T + \sum_{i=0}^2 (h_{i+1} - h_i)^2 R_{ij}.\end{aligned}\quad (28)$$

Proof: Both proofs remain the same as for Theorem 1. Only some matrix manipulation and changes of variables are added. For more details, similar proofs are in [15] and [32].

■

IV. GLOBAL STABILIZATION OF THE NCS WITH MINIMUM DWELL TIME

The previous part was devoted to design the state feedback controller and observer to ensure exponential stability while the system does not switch. The present part gives material to design the switching rule of the controller guaranteeing stability along with some performances. The main idea is to exploit the ability of the controller to enlarge the delay by using buffer strategy and avoid some switching occurrences. This feature permits to respect some dwell time requirement for the global stability with switches.

A. NCS Model

This subsection gives the global model of the NCS. This model takes into account the interactions between the delay detection problem, the switching buffer size and the observer. In this part, there are no separation principle and the overall model has to be taken into account. It is assumed that the controller gains K_i and L_i are designed by using Lemmas 1 and 2 on the non switching model for decay rates α_i .

Let recall our NCS

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t - \delta_{\text{con}}(t)) \\ y(t) = Cx(t). \end{cases}\quad (29)$$

With its controller

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t - \hat{\delta}_{\text{con}}^i(t)) \\ \quad + L_i(y(t - \delta_{\text{obs}}^i(t)) - \hat{y}(t - \delta_{\text{obs}}^i(t))) \\ \hat{y}(t) = C\hat{x}(t), \\ u(t) = K_i\hat{x}(t) + ky_c(t). \end{cases}\quad (30)$$

Remind that the controller switches as soon as a bigger delay is detected but one can choose when it switch if the delay becomes smaller by increasing artificially the delay. The delay is measured with some lag (takes at maximum h_{max} if measured on the measurement channel). Because of this lag and because of the buffer size problem, four modes arise including two additional modes when the estimated delay does not equal the real one: SS_1 mode, $t \in [t_0, t_1]$: The delays are in S_1 and are correctly detected. In this case, the small buffer is used and the controller is in the mode $i = 1$. In that case, $\hat{\delta}_{\text{con}}(t) = \delta_{\text{con}}$. The controller gains, K_1 and L_1 , are designed for exponential stability with a decay rate α_1 . The equations of SS_1 with $\zeta(t) = \text{col}\{x(t), e(t)\}$ are given by

$$\begin{aligned}\dot{\zeta}(t) &= \bar{A}_0\zeta(t) + \bar{A}_{11}\zeta(t - \delta_{\text{con}}^1(t)) \\ &\quad + \bar{A}_{31}\zeta(t - \delta_{\text{obs}}^1(t))\delta_{\text{con}}^1(t), \delta_{\text{obs}}^1(t) \in [h_1, h_2]\end{aligned}\quad (31)$$

$$\text{with } \bar{A}_0 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \bar{A}_{11} = \begin{bmatrix} BK_1 & -BK_1 \\ 0 & 0 \end{bmatrix}, \bar{A}_{31} = \begin{bmatrix} 0 & 0 \\ 0 & -L_1C \end{bmatrix}.$$

SU_1 mode, $t \in [t_1, t_2]$: The system was in SS_1 then the delay grows and belongs now in S_2 . The value is not detected yet by the communication structure implying that the controller stays in mode 1. This mode may be unstable since the controller gains K_1 and L_1 are not designed for. Its decay rate is denoted by α_3 and may be negative. The second effect is that the control buffer is smaller than the communication delay, the observed

$$\Phi_{obs1} = \begin{bmatrix} \Phi_{111} & \Phi_{121} & [\tilde{R}_{01} - W_1C - Y_{11}^T] & Y_{11}^T & [Y_{11}^T + W_1C] & Y_{11}^T & 0 \\ * & \Phi_{221} & [-\epsilon_1 W_1C - Y_{21}^T] & Y_{21}^T & [Y_{21}^T + \epsilon_1 W_1C] & Y_{21}^T & 0 \\ * & * & \tilde{S}_{11} - \tilde{S}_{01} - \tilde{R}_{01} & 0 & 0 & 0 & 0 \\ * & * & * & \hat{S}_{21} - \tilde{S}_{11} - \tilde{R}_{21} & 0 & 0 & \tilde{R}_{21} \\ * & * & * & * & -\tilde{R}_{11} & 0 & 0 \\ * & * & * & * & * & -\tilde{R}_{11} & 0 \\ * & * & * & * & * & * & -\tilde{S}_{21} - \tilde{R}_{21} \end{bmatrix} < 0 \quad (26)$$

$$\Phi_{obs2} = \begin{bmatrix} \Phi_{112} & \Phi_{122} & [-W_2C - Y_{12}^T] & Y_{12}^T & [Y_{12}^T + W_2C] & Y_{12}^T & \tilde{R}_{02} \\ * & \Phi_{222} & [-\epsilon_2 W_2C - Y_{22}^T] & Y_{22}^T & [Y_{22}^T + \epsilon_2 W_2C] & Y_{22}^T & 0 \\ * & * & \tilde{S}_{22} - \tilde{S}_{12} - \tilde{R}_{12} & 0 & 0 & 0 & \tilde{R}_{12} \\ * & * & * & -\tilde{S}_{22} & 0 & 0 & 0 \\ * & * & * & * & -\tilde{R}_{22} & 0 & 0 \\ * & * & * & * & * & -\tilde{R}_{22} & 0 \\ * & * & * & * & * & * & \hat{S}_{12} - \tilde{S}_{02} - \tilde{R}_{02} - \tilde{R}_{12} \end{bmatrix} < 0 \quad (27)$$

control input and the plant control input are no more synchronized during this period because $\hat{\delta}_{\text{con}}(t) \neq \delta_{\text{con}}(t)$. This mode has a maximum dwell time since the delay detection takes less than the maximum communication delay. The equation of SU_1 are

$$\begin{aligned} \dot{\zeta}(t) &= \bar{A}_0 \zeta(t) + \bar{A}_{13} \zeta(t - \hat{\delta}_{\text{con}}^1(t)) \\ &\quad + \bar{A}_{23} \zeta(t - \delta_{\text{con}}^2(t)) + \bar{A}_{33} \zeta(t - \delta_{\text{obs}}^2(t)). \\ \hat{\delta}_{\text{con}}^1 &\in [h_1, h_2], \delta_{\text{con}}^2(t), \delta_{\text{obs}}^2(t) \in [h_2, h_3] \end{aligned} \quad (32)$$

$$\text{for } \bar{A}_0 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \bar{A}_{13} = \begin{bmatrix} 0 & 0 \\ -BK_1 & BK_1 \end{bmatrix}, \bar{A}_{23} = \begin{bmatrix} BK_1 & -BK_1 \\ BK_1 & -BK_1 \end{bmatrix}, \bar{A}_{33} = \begin{bmatrix} 0 & 0 \\ 0 & -L_1 C \end{bmatrix}.$$

SS_2 mode, $t \in [t_3, t_4]$: The system was in mode SU_1 and finally detect the correct value of the delay. The controller switches to the mode 2. In that case, the buffer works again properly, i.e., $\hat{\delta}_{\text{con}}(t) = \delta_{\text{con}}(t)$. The controller gains, K_2 and L_2 , are designed for exponential stability with a decay rate α_2 . The equations are

$$\begin{aligned} \dot{\zeta}(t) &= \bar{A}_0 \zeta(t) + \bar{A}_{12} \zeta(t - \delta_{\text{con}}^2(t)) \\ &\quad + \bar{A}_{32} \zeta(t - \delta_{\text{obs}}^2(t)) \delta_{\text{con}}^2(t), \delta_{\text{obs}}^2(t) \in [h_2, h_3] \end{aligned} \quad (33)$$

$$\text{with } \bar{A}_0 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \bar{A}_{12} = \begin{bmatrix} BK_2 & -BK_2 \\ 0 & 0 \end{bmatrix}, \bar{A}_{32} = \begin{bmatrix} 0 & 0 \\ 0 & -L_2 C \end{bmatrix}.$$

SU_2 mode, $t \in [t_3, t_s]$: The delay becomes smaller and belongs to S_1 but the controller does not switch to mode $i = 1$. It waits to meet the dwell time condition. The control gain K_2 is still applied for this mode since the delay is artificially kept in S_2 . The observer gain L_2 keeps the system stable since if the condition of theorem 1 holds for a given delay, it does also for a smaller one. The global decay rate is noted as α_4

$$\begin{aligned} \dot{\zeta}(t) &= \bar{A}_0 \zeta(t) + \bar{A}_{14} \zeta(t - \delta_{\text{con}}^2(t)) \\ &\quad + \bar{A}_{34} \zeta(t - \delta_{\text{obs}}^1(t)) \delta_{\text{con}}^2(t) \in [h_2, h_3], \delta_{\text{obs}}^1(t) \in [h_1, h_2] \end{aligned} \quad (34)$$

$$\text{for } \bar{A}_0 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \bar{A}_{14} = \begin{bmatrix} BK_2 & -BK_2 \\ 0 & 0 \end{bmatrix}, \bar{A}_{34} = \begin{bmatrix} 0 & 0 \\ 0 & -L_2 C \end{bmatrix}.$$

The four modes are summarized in the following figure where the arrows represent the possible transitions, the only controller transition is $SU_2 \rightarrow SS_1$.

To summarize with T being the sampling period

$$\begin{aligned} SS_1 : t \in [t_0, t_1] \quad i = 1, \quad \text{Buffer} &= h_2 - T \\ \hat{\delta}_{\text{con}}(t) &= \delta_{\text{con}}(t) \quad \delta_{\text{con}}(t) \in S_1, \quad \delta_{\text{obs}}(t) \in S_1 \\ SU_1 : t \in [t_1, t_2] \quad i = 1, \quad \text{Buffer} &= h_2 - T \\ \hat{\delta}_{\text{con}}(t) &\neq \delta_{\text{con}}(t) \quad \delta_{\text{con}}(t) \in S_2, \quad \delta_{\text{obs}}(t) \in S_2 \\ SS_2 : t \in [t_2, t_3] \quad i = 2, \quad \text{Buffer} &= h_3 - T \\ \hat{\delta}_{\text{con}}(t) &= \delta_{\text{con}}(t) \quad \delta_{\text{con}}(t) \in S_2, \quad \delta_{\text{obs}}(t) \in S_2 \\ SU_2 : t \in [t_3, t_4] \quad i = 2, \quad \text{Buffer} &= h_3 - T \\ \hat{\delta}_{\text{con}}(t) &= \delta_{\text{con}}(t) \quad \delta_{\text{con}}(t) \in S_2, \quad \delta_{\text{obs}}(t) \in S_1. \end{aligned}$$

B. Global Stability Conditions

In order to avoid unstable switching sequences, a minimum dwell time between the modes is computed here by using the exponential stability property. The global stability of the system will be studied by using the method of [17]. The LKF V_1, V_2, V_3 , and V_4 are, respectively, associated to the modes SS_1, SS_2, SU_1 , and SU_2 . A common structure is chosen

$$\begin{aligned} V_j(t, x_t, \dot{x}_t) &= x^T(t) P_j x(t) + \sum_{i=0}^2 \int_{t-h_{i+1}}^{t-h_i} e^{2\alpha_j(s-t)} x^T(s) S_{ij} x(s) ds \\ &\quad + \sum_{k=1}^3 \sum_{i=0}^2 (h_{i+1} - h_i) \int_{-h_{i+1}}^{-h_i} \int_{t+\theta}^t e^{2\alpha_j(s-t)} \dot{x}^T(s) R_{ij}^k \dot{x}(s) ds d\theta. \end{aligned} \quad (35)$$

LKF are used to compute the decay rate α_j of each modes and a scale parameter μ_{ij} between the modes

$$\begin{aligned} V_j(t, x_t, \dot{x}_t) &\leq \mu_{ij} V_i(t, x_t, \dot{x}_t) \quad \forall i, j = 1, \dots, 4 \\ V_j(t, x_t, \dot{x}_t) &\leq e^{-2\alpha_j(t-t_0)} V_j(t_0, x_{t_0}, \dot{x}_{t_0}) \quad \forall j = 1, \dots, 4. \end{aligned} \quad (36)$$

The following lemma provides conditions ensuring $V_j(t, x_t, \dot{x}_t) \leq \mu_{ij} V_i(t, x_t, \dot{x}_t)$ for the considered sequence.

Lemma 3: The property $V_j(t, x_t, \dot{x}_t) \leq \mu_{ij} V_i(t, x_t, \dot{x}_t)$, $(i, j) \in \{(1, 4), (4, 2), (2, 3), (3, 1)\}$ is guaranteed if the following conditions are satisfied:

$$\begin{aligned} \forall n \in \{0, 1, 2\}, \forall h \in \{h_n, h_{n+1}\}, \forall k \in \{1, 2, 3\} \\ P_i &\leq \mu_{ij} P_j \\ \forall (i, j) \in \{(1, 4), (4, 2), (2, 3), (3, 1)\} \\ e^{-2\alpha_i h} S_{ni} &\leq \mu_{ij} e^{-2\alpha_j h} S_{nj} \\ e^{-2\alpha_i h} R_{ni}^k &\leq \mu_{ij} e^{-2\alpha_j h} R_{nj}^k. \end{aligned} \quad (37)$$

The computation of the parameters α_i and μ_{ij} is done using the following lemma which is a specific version of Theorem 1.

Lemma 4: Given some scalars α_j and μ_{ij} , $(i, j) \in \{(1, 4), (4, 2), (2, 3), (3, 1)\}$, if there exist scalars $\mu_{ij}, i, j = 1, \dots, 4$, matrices $P_j > 0, R_{ij}^k > 0$ and $S_{ij} > 0$ ($i = 0, 1, 2$ and $k = 1, 2, 3$), P_{2j}, P_{3j}, Y_{1kj} , and Y_{2kj} ($j = 1, 2, 3, 4$ and $k = 1, 2, 3$) with proper dimensions, such that (37)–(39) conditions hold then the system (IV-A) has positive functionals (35) fulfilling (36)

$$\Phi|_{SS_1} = \begin{bmatrix} \Psi_{11} & \Psi_{21} \\ * & \Psi_{31} \end{bmatrix} < 0$$

$$\Phi|_{SS_2} = \begin{bmatrix} \Psi_{12} & \Psi_{22} \\ * & \Psi_{32} \end{bmatrix} < 0 \quad (38)$$

$$\Phi|_{SU_1} = \begin{bmatrix} \Psi_{13} & \Psi_{23} \\ * & \Psi_{33} \end{bmatrix} < 0$$

$$\Phi|_{SU_2} = \begin{bmatrix} \Psi_{14} & \Psi_{24} \\ * & \Psi_{34} \end{bmatrix} < 0 \quad (39)$$

where (40)–(45) are shown at the bottom of the page, where (46)–(51) are shown at the bottom of the next page, and, where

$$\begin{aligned}
 \tilde{R}_{ij} &= e^{-2\alpha_j(h_{i+1})} R_{ij} \\
 \tilde{S}_{ij} &= e^{-2\alpha_j h_{i+1}} S_{ij} \\
 \hat{S}_{ij} &= e^{-2\alpha_j h_i} S_{ij} \\
 \Phi_{11j} &= A^T P_{2j} + P_{2j}^T A + S_{0j} - e^{-2\alpha_j h_1} R_{0j} + 2\alpha_j P_j \\
 \Phi_{12j} &= P_j - P_{2j}^T + A^T P_{3j}, \quad \Phi_{22j} = -P_{3j} - P_{3j}^T \\
 &+ \sum_{i=0}^2 (h_{i+1} - h_i)^2 R_{ij}. \tag{52}
 \end{aligned}$$

Since the NCS global model switches in a predetermined order, the global performance/stability is achieved if all the functionals V_j decrease in every cycle. This gives the following condition to ensure a decay rate α_g over a complete cycle: $V_1(t_s, x_{t_s}, \dot{x}_{t_s}) \leq e^{-2\alpha_g(t_s-t_0)} V_1(t_0, x_{t_0}, \dot{x}_{t_0})$. This condition is illustrated in Fig. 3. The following theorem gives the stability conditions for a four-sequenced mode switching system.

Theorem 2: Consider a four-sequenced mode switching model described in (IV-A). Given some scalars α_j and μ_{ij} , $(i, j) \in \{(1, 4), (4, 2), (2, 3), (3, 1)\}$, if there exist scalars $\mu_{ij}, i, j = 1, \dots, 4$, matrices $P_j > 0$, $R_{ij}^k > 0$, and $S_{ij} > 0$ ($i = 0, 1, 2$ and $k = 1, 2, 3$), P_{2j} , P_{3j} , Y_{1kj} , and Y_{2kj}

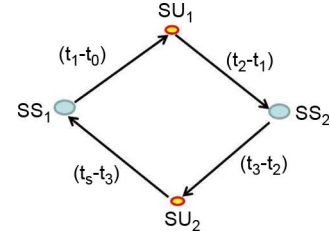


Fig. 2. Switch among four modes.

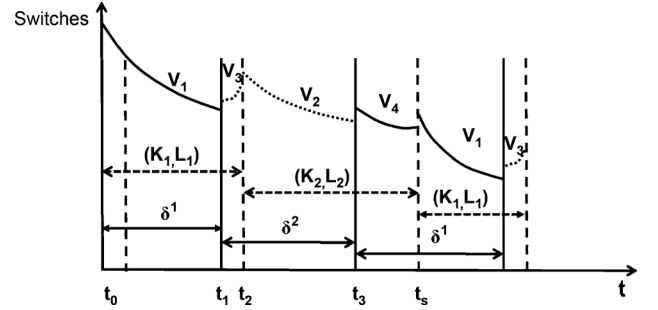


Fig. 3. Minimum dwell time: Small delay from t_0 to t_1 , then big delay from t_1 to t_3 , switches at t_2 and t_5 .

($j = 1, 2, 3, 4$ and $k = 1, 2, 3$) with proper dimensions, such that (37)–(39) and condition (53) hold for some $\alpha_g > 0$ then the model (IV-A) is exponentially stable with the decay rate α_g

$$\Psi_{11} = \begin{bmatrix} \Phi_{111} & \Phi_{121} & P_{21}^T A_{11} + P_{21}^T A_{31} + \sum_{k=1}^3 (\tilde{R}_{01}^k - Y_{1k1}^T) & \sum_{k=1}^3 Y_{1k1}^T & 0 \\ * & \Phi_{221} & P_{31}^T A_{11} + P_{31}^T A_{31} - \sum_{k=1}^3 Y_{2k1}^T & \sum_{k=1}^3 Y_{2k1}^T & 0 \\ * & * & \hat{S}_{11} - \tilde{S}_{01} - \sum_{k=1}^3 \tilde{R}_{01}^k & 0 & 0 \\ * & * & * & \hat{S}_{21} - \tilde{S}_{11} - \sum_{k=1}^3 \tilde{R}_{21}^k & \sum_{k=1}^3 \tilde{R}_{21}^k \\ * & * & * & * & -\tilde{S}_{21} - \sum_{k=1}^3 \tilde{R}_{21}^k \end{bmatrix} \tag{40}$$

$$\Psi_{21} = \begin{bmatrix} Y_{111}^T - P_{21}^T A_{11} & Y_{111}^T & Y_{121}^T & Y_{121}^T & Y_{121}^T & Y_{131}^T - P_{21}^T A_{31} & Y_{131}^T \\ Y_{211}^T - P_{31}^T A_{11} & Y_{211}^T & Y_{221}^T & Y_{221}^T & Y_{221}^T & Y_{231}^T - P_{31}^T A_{31} & Y_{231}^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

$$\Psi_{31} = \text{diag} \left(\left[-\tilde{R}_{11}^1, -\tilde{R}_{11}^1, -\tilde{R}_{11}^2, -\tilde{R}_{11}^2, -\tilde{R}_{11}^3, -\tilde{R}_{11}^3 \right] \right) \tag{42}$$

$$\Psi_{12} = \begin{bmatrix} \Phi_{112} & \Phi_{122} & \sum_{k=1}^3 \tilde{R}_{02}^k & P_{22}^T A_{12} + P_{21}^T A_{31} - \sum_{k=1}^3 Y_{1k2}^T & \sum_{k=1}^3 Y_{1k2}^T \\ * & \Phi_{222} & 0 & P_{32}^T A_{12} + P_{32}^T A_{32} - \sum_{k=1}^3 (Y_{2k2}^T) & \sum_{k=1}^3 Y_{2k2}^T \\ * & * & \hat{S}_{12} - \tilde{S}_{02} - \sum_{k=1}^3 (\tilde{R}_{02}^k + \tilde{R}_{12}^k) & \sum_{k=1}^3 \tilde{R}_{12}^k & 0 \\ * & * & * & \hat{S}_{22} - \tilde{S}_{12} - \sum_{k=1}^3 \tilde{R}_{12}^k & 0 \\ * & * & * & * & -\tilde{S}_{22} \end{bmatrix} \tag{43}$$

$$\Psi_{22} = \begin{bmatrix} Y_{112}^T - P_{22}^T A_{12} & Y_{112}^T & Y_{122}^T & Y_{122}^T & Y_{122}^T & Y_{132}^T - P_{22}^T A_{32} & Y_{132}^T \\ Y_{212}^T - P_{32}^T A_{12} & Y_{212}^T & Y_{222}^T & Y_{222}^T & Y_{222}^T & Y_{232}^T - P_{32}^T A_{32} & Y_{232}^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{44}$$

$$\Psi_{32} = \text{diag} \left(\left[-\tilde{R}_{22}^1, -\tilde{R}_{22}^1, -\tilde{R}_{22}^2, -\tilde{R}_{22}^2, -\tilde{R}_{22}^3, -\tilde{R}_{22}^3 \right] \right) \tag{45}$$

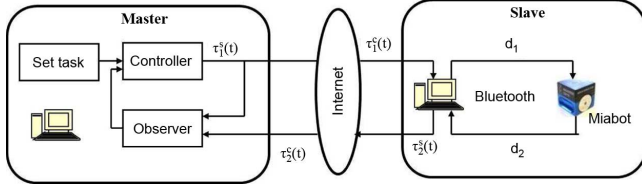


Fig. 4. Structure of the global system.

This optimization is possible since the conditions are harder to satisfy when α_j grows and μ_{ij} decreases.

The global approach for the switching controller design is summarized in the following algorithm.

Algorithm 1—Controller Design Steps:

- 1) Use lemmas 1 and 2 to compute the gains with given α performances for modes 1 and 2.
- 2) Use corollary 4 to obtain a compromise between the minimum μ_{ij} and the maximum α_i .
- 3) Implement the controller (7) by using the following switching law:

$$i = \begin{cases} 1, & \text{if } \delta_{\text{obs}}(t) \in S_1 \text{ and } t > t_s \\ 2, & \text{otherwise} \end{cases} \quad (56)$$

where t_s is extracted by applying the logarithm function on the condition of theorem 2.

Remark 6: These results did not take into account the packet dropout case for sake of clarity. Since the presented controller does not detect packet dropouts, it is treated using [30] to compute the worst case exponential decay rate for all modes. Remark also that these results can be extended to more zones and to the case where the control delay range is different from the observer delay range.

V. EXPERIMENTAL RESULTS

The experiments are done on two computers separated from about 40 kilometers away (see Fig. 4). The Master program runs on the remote computer with an advanced computing capability, the Slave one on the local one which also communicates with a light-inertia robot Miabot of the company Merlin by Bluetooth.

A. Structure of the Master

In order to implement the model for the remote control system, a four-threads program is designed to fulfill the functions of Controller and Observer in Fig. 4, while the explication of all the parameters refers to [15]. These four threads are concurrently working as shown in Fig. 5. (a) ConsThread is a periodic thread which gets the set points of the trajectory. (b) SenderThread is also a periodic thread which calculates the control data and its appliance time and send it out to the Slave. (c) ReceiverThread is a event-driven thread to receive controls from the Slave. (d) ObserverThread task is periodic and simulate the observers differential equations. It estimates the *present* position and speed of the remote motor.

B. Structure of the Slave

The Slave does not need power computation abilities, because it just needs to communicate with the Master and the Miabot. As we can see from Fig. 6, this program is divided into two

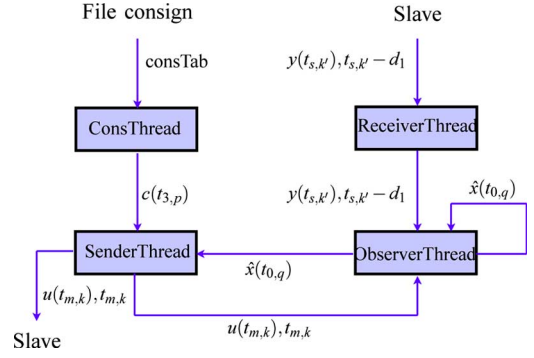


Fig. 5. Structure of the Master.

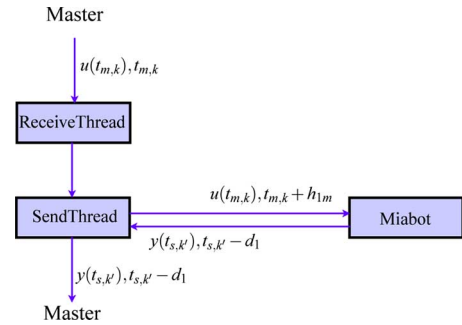


Fig. 6. Structure of the Slave.

threads: ReceiveThread and SendThread. (a) ReceiveThread is an event-driven thread which is activated by the control data arrived from the Master. (b) SendThread is used to apply the control to the Miabot as well as to get its real position, and then send the data back to the Master. As we need to apply the control data at the time given by the master, the buffer used is a list `list_Y` where the packets are sorted by order of appliance.

C. Experimental Study

The model of the Miabot is derived experimentally. In order to avoid saturation and nonlinear behaviors, the speed has been limited to 2 m/s. The obtained model is given by (57). The state value is composed of its position and speed, the control is the value sent by Master

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -10 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.024 \end{bmatrix} u(t - \delta_{\text{con}}(t)) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{cases} \quad (57)$$

Some delay measurement has been done between the master and the slave. During a day, the RTT (Round-trip-time=twice the communication delay) measured using the ICMP (Internet control message [25]) belonged to [4.1 ms 577 ms] with an average value of 52.37 ms. During the experimental time, the delay was oscillating between an average value of 40 and 100 ms. Taking into account these information, considering also the Bluetooth transmission delays (considered constant) and the sampling delays, we take $S_1 = [0.01, 0.08]$ and $S_2 = [0.08, 0.5]$ for the delay subset. Any packet data delayed by more than 0.5 s is considered lost.

According to Lemma (1) and (2), the maximum exponential convergence ensuring the modes stability are: $\alpha_1^c = 3.8$,

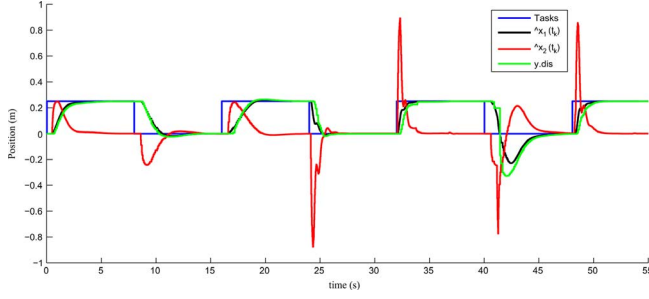


Fig. 7. Results of remote experiment.

$\alpha_1^o = 4.49$, $\alpha_2^c = \alpha_2^o = 0.72$. The corresponding control gains are too high to keep the speed lower than 2 m/s. To avoid actuator saturation and nonlinear behavior when the robot speed is too high, smaller gains are computed considering the following values: $\alpha_1^c = 2$, $\alpha_1^o = 2.5$, $\alpha_2^c = 0.6$, and $\alpha_2^o = 0.72$.

The resulting gains K_i and L_i ($i = 1, 2$) are given by

$$\begin{bmatrix} L_1 & L_2 \end{bmatrix} = \begin{bmatrix} -7.06 & -1.44 \\ 0.04 & -0.01 \end{bmatrix}, \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -1485 & -461 \\ -99 & 1 \end{bmatrix}. \quad (58)$$

Remark 7: Note that speed related component of the control gain K_2 has a positive value. This just mean that to keep the system stability when the delay is high, it is needed slightly degrade the performances of the system stable open-loop pole.

Once the gain are computed, the dwell time parameters α_i and μ_{ij} are computed following these steps.

- 1) The values of α_3 and α_4 are maximized for fixed $\mu_{ij} = 100$, α_1 and α_2 , satisfying corollary four conditions.
- 2) μ_{ij} are minimized using the criterion $\mu_{14} + \mu_{42} + \mu_{23} + \mu_{31}$ for the obtained α_i and LKF matrices, satisfying corollary four conditions.

These steps lead to the following solutions: $\alpha_1 = 2$, $\alpha_2 = 0.6$, $\alpha_3 = -4$, $\alpha_4 = 0.6$. The global performance α_g chosen is chosen to only keep stability: $\alpha_g = 0$.

D. Results of Remote Experiment

The result is shown in Fig. 7, in which the blue curve represents the set values; the green and red represent respectively the robot's estimated position and speed; the black corresponds to the real position of the Miabot. Fig. 8 illustrate the corresponding switched control signals from Master to Slave. The red curve is the real control while the green and the black ones are the controls calculated respectively for the two subsystems. We can see the switch points according to the values of time-delay. Fig. 9 depicts the variable global time-delays on the control communication channel and the corresponding switching signal.

On Fig. 7, one can notice three kinds of step responses. The first one corresponds to the case when the time-delay is greater than 80 ms, only the second subsystem is active. In this case, a decay rate α_2 is guaranteed. During the second step, only the first mode is active because delays are small. The performances are better: the response time is smaller since a decay rate of α_1 is achieved. In the last kind of response, where a some switches occur during the transient response. In that case, only the global stability is guaranteed. At the moment $t \in [42 \text{ ms}, 45 \text{ ms}]$, the

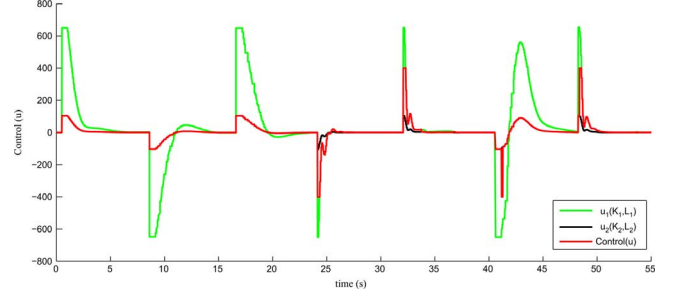


Fig. 8. Corresponding switched control.

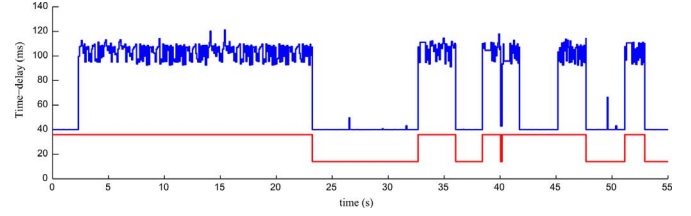


Fig. 9. Corresponding variable time-delays and signals of switching.

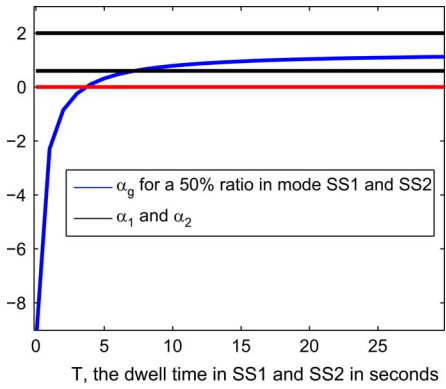


Fig. 10. Performances analysis.

time-delay becomes small, but the switching strategy (56) does not permit to switch back to mode 1 in order to guarantee the global stability.

Remark 8: Notice that despite the fact that the observer is based on delays measurements, it this keep a good estimation on the present state of the slave.

The last figure (see Fig. 10) provides a performance analysis in term of global decay rate α_g over one cycle ($SS_1 \rightarrow SU_1 \rightarrow SS_2 \rightarrow SU_2 \rightarrow SS_1$). It is assumed that the dwell time in the modes SU_1 and SU_2 are fixed to the maximum delay, i.e., 0.5s. The figure gives the lower bound α_g according to the dwell time T in the modes SS_1 and SS_2 . For $T \leq 3.7$ the controller dwell time condition is not satisfied which leads to a possible unstable behavior. For $T \in [3.7, 7.2]$ the performances are worse than $\alpha_2 = 0.6$ and for $T > 7.2$, the performance tends to be the mean value of the decay rates $(\alpha_1 + \alpha_2)/2 = 1.3$.

VI. CONCLUSION

A QoS-dependent controller has been presented to enhance the performances of a NCS system based on switching gains. This controller uses the time-stamps of packets to measure the

communication delay and to synchronize the observer with the plant. The resulting controller switches according to the delay and a dwell time strategy ensures the global stability.

For the sake of clarity, only two intervals for the delay range are considered to quantify the QoS but the conditions can be easily extended to multiple intervals.

An experimental platform has been presented and illustrates the results of the network-based control theory, showing the effectiveness of the dwell time strategy.

Possible extensions of this work are the use of more complex switching law (for example with hysteresis), reducing the conservatism of the conditions by considering output feedback controller (the complexity is mainly due to the observer) or releasing the constraints on the delays sets (taking different set for the control channel and the measurement channel).

REFERENCES

[1] Z. Chen, L. Liu, and X. Yin, "Networked control system with network time-delay compensation," in *Proc. Ind. Appl. Conf., 40th IAS Annu. Meet.*, 2005, pp. 2435–2440.

[2] J. Chiasson and J. J. Loiseau, *Applications of Time Delay Systems*. New York: Springer, 2007, vol. 352.

[3] H. J. Estrada-García, L. A. Marquez-Martinez, and C. H. Moog, "Master-slave synchronization for two inverted pendulums with communication time-delay," presented at the 7th IFAC Workshop Time Delay Syst., Nantes, France, 2007.

[4] A. Fattouh and O. Sename, " H^∞ -based impedance control of teleoperation systems with time delay," presented at the 4th Workshop Time Delay Syst., Rocquencourt, France, 2003.

[5] E. Fridman, "New Lyapunov-Krasovskii functionals for stability of linear retarded and neutral type systems," *Syst. Control Lett.*, vol. 43, pp. 309–319, 2001.

[6] E. Fridman, A. Seuret, and J.-P. Richard, "Robust sampled-data stabilization of linear systems: An input delay approach," *Automatica*, vol. 40, pp. 1441–1446, 2004.

[7] H. Gao, T. Chen, and J. Lam, "A new delay system approach to network-based control," *Automatica*, vol. 44, no. 1, pp. 39–52, 2008.

[8] J.-P. Georges, T. Divoux, and E. Rondeau, "Confronting the performances of a switched ethernet network with industrial constraints by using the network calculus," *Int. J. Commun. Syst. (IJCS)*, vol. 18, no. 9, pp. 877–903, 2005.

[9] K. Gu, V. Kharitonov, and J. Chen, *Stability of time-delay systems*. Boston, MA: Birkhauser, 2003.

[10] Y. He, M. Wu, J. H. She, and G. P. Liu, "Parameter-dependent lyapunov functional for stability of time-delay systems with polytopic-type uncertainties," *IEEE Trans. Autom. Control*, vol. 49, no. 5, pp. 828–832, May 2004.

[11] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," in *Proc. 38th Conf. Decision Control*, 1999, pp. 2655–2660.

[12] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proc. IEEE*, vol. 95, no. 1, pp. 138–162, Jan. 2007.

[13] S. Hirche, C.-C. Chen, and M. Buss, "Performance oriented control over networks-switching controllers and switched time delay-," presented at the 45th IEEE Conf. Decision Control, San Diego, CA, 2006.

[14] J. Q. Huang and F. L. Lewis, "Neural-network predictive control for nonlinear dynamic systems with time delays," *IEEE Trans. Neural Netw.*, vol. 14, no. 2, pp. 377–389, Feb. 2003.

[15] W.-J. Jiang, A. Kruszewski, J.-P. Richard, and A. Toguyeni, "A gain scheduling strategy for the control and estimation of a remote robot via internet," presented at the 27th Chinese Control Conf., Kunming, Yunnan, China, 2008.

[16] A. Lelevé, P. Fraisse, and P. Dauchez, "Telerobotics over IP networks: Towards a low-level real-time architecture," presented at the Int. Conf. Intell. Robots Syst., Maui, Hawaii, 2001.

[17] D. Liberzon, *Switching in Systems and Control*, T. Basar, Ed. Boston, MA: Birkhäuser, 2003.

[18] D. L. Mills, "Improved algorithms for synchronizing computer network clocks," *IEEE/ACM Trans. Network.*, vol. 3, no. 3, pp. 245–254, Jun. 1995.

[19] S.-I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*. New York: Springer, 2001, vol. 269.

[20] G. Niemeyer and J.-J. Slotine, "Towards force-reflecting teleoperation over the internet," *IEEE Int. Conf. Robot. Autom.*, vol. 3, p. 1909, 1998.

[21] J. Nilsson, "Real-Time Control Systems With Delays," Ph.D. dissertation, Dept. Autom. Control, Lund Inst. Technol., Lund, Sweden, 1998.

[22] J. Nilsson, B. Bernhardsson, and B. Wittenmark, "Stochastic analysis and control of real-time control systems with random time delay," *Automatica*, vol. 34, no. 1, pp. 57–64, 1998.

[23] N. J. Ploplys, P. A. Kawka, and A. G. Alleyne, "Closed-loop control over wireless networks," *IEEE Control Syst. Mag.*, vol. 24, no. 3, p. 58, Jun. 2004.

[24] J. Postel, "User datagram protocol," USC/Inform. Sci. Inst., CA, RFC 768, 1980.

[25] J. Postel, "Internet control message protocol," USC/Inform. Sci. Inst., RFC 792, 1981.

[26] J.-P. Richard, "Time delay systems: An overview of some recent advances and open problems," *Automatica*, vol. 39, pp. 1667–1694, 2003.

[27] J.-P. Richard and T. Divoux, "Systèmes commandés en réseau," *Hermès-Lavoisier, IC2, Systèmes Automatisés*, 2007.

[28] A. Seuret, E. Fridman, and J.-P. Richard, "Sampled-data exponential stabilization of neutral systems with input and state delays," presented at the IEEE 13th Med. Conf. Control Autom., Limassol, Cyprus, 2005.

[29] A. Seuret, F. Michaut, J.-P. Richard, and T. Divoux, "Networked control using GPS synchronization," presented at the Amer. Control Conf., Minneapolis, MN, 2006.

[30] A. Seuret and J.-P. Richard, "Control of a remote system over network including delays and packet dropout," presented at the IFAC World Congr., Seoul, Korea, 2008.

[31] X.-M. Sun, J. Zhao, and D. Hill, "Stability and l_2 -gain analysis for switched delay systems," *Automatica*, vol. 42, pp. 1769–1774, 2006.

[32] Y. G. Sun and L. Wang, "Stability of switched systems with time-varying delays: Delay-dependent common lyapunov functional approach," presented at the Amer. Control Conf., Minneapolis, MN, 2006.

[33] Y. Tipsuwan and M.-Y. Chow, "Control methodologies in network control systems," *Control Eng. Pract.*, vol. 11, pp. 1099–1011, 2003.

[34] E. Witrant, C. Canudas-De-Wit, and D. Georges, "Remote stabilization via communication networks with a distributed control law," *IEEE Trans. Autom. Control*, vol. 52, no. 8, p. 1480, 2007.

[35] P. Yan and H. Özbay, "Stability analysis of switched time delay systems," *SIAM J. Control Opt.*, vol. 47, no. 2, pp. 936–949, 2008.

[36] P. Yan, H. Özbay, and M. Sansal, "A switching control approach to stabilization of parameter varying time delay systems," in *48th IEEE Conf. Decision Control*, 2009, pp. 7222–7226.

[37] M. Yu, L. Wang, and T. Chu, "An LMI approach to network control systems with data packet dropout and transmission delays," presented at the Math. Theory Netw. Syst., Leuven, Belgium, 2004.

[38] S. Zampieri, "Trends in networked control systems," presented at the 17th World Congr., Int. Fed. Autom. Control, Seoul, Korea, 2008.

[39] W. A. Zhang and L. Yu, "Modelling and control of networked control systems with both network-induced delay and packet-dropout," *Automatica*, vol. 44, pp. 3206–3210, 2008.



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