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Finite-dimensional control of the heat equation: Dirichlet actuation and point measurement☆

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ABSTRACT

Recently finite-dimensional observer-based controllers were introduced for the 1D heat equation, where at least one of the observation or control operators was bounded. In this paper, for the first time, we manage with such controllers for the 1D heat equation with both operators being unbounded. We consider Dirichlet actuation and point measurement and use a modal decomposition approach via dynamic extension. We suggest a direct Lyapunov approach to the full-order closed-loop system, where the finite-dimensional state is coupled with the infinite-dimensional tail of the state Fourier expansion, and provide Linear Matrix Inequalities (LMIs) for finding the controller dimension and resulting exponential decay rate. A numerical example demonstrates the efficiency of the proposed method. In the discussion section, we show that the suggested controller design is well suited for the 1D heat equation with various boundary conditions.

1. Introduction

Finite-dimensional observer-based control for PDEs is attractive for applications and theoretically challenging. Such controllers for parabolic systems were designed by the modal decomposition approach in [1,3,4,11]. The existing results are mostly restricted to bounded control and observation operators, whereas efficient bounds on the observer and controller dimensions are missing. Thus, the bound suggested in [11] appeared to be highly conservative and difficult to compute

In our recent paper [16], the first constructive LMI-based method for finite-dimensional observer-based controller for the 1D heat equation was suggested, where the controller dimension and the resulting exponential decay rate were found from simple LMI conditions. Robustness of the finite-dimensional controller with respect to input and output delays was studied in [19]. However, the results of [16,19] were confined to cases where at least one of the observation or control operators is bounded. Sampled-data and delayed boundary control of 1D heat equation under boundary measurement was studied in [21] by using an infinite-dimensional PDE observer. However, finite-dimensional observer-based control of the heat equation in the challenging case where both operators are unbounded remained open. Note that finite-dimensional observer-based control of the 1D linear Kuramoto-Sivashinsky equation (KSE) with both observation and control operators unbounded was studied in [18].

In the present paper, for the first time, we manage with finite-dimensional observer-based controllers for the 1D heat equation with both operators unbounded. We consider Dirichlet actuation and point measurement and employ a modal decomposition approach via dynamic extension. We suggest a direct Lyapunov approach to the full-order closed-loop system, where the finite-dimensional state is coupled with the infinite-dimensional tail of the state Fourier expansion, and provide LMIs for finding the controller dimension and resulting exponential decay rate. In order to manage with point measurement, we consider H^1-stability and apply the Young inequality in a novel form (with fractional powers of the eigenvalues of a Sturm-Liouville operator). Note that for KSE, studied in [18] and [20], the use of fractional powers of the eigenvalues was not required. We also provide discussions on the extension of the method to 1D linear heat equation under various boundary conditions and on sampled-data implementation of the controller. A numerical example illustrates the efficiency of the proposed method.

Notations and mathematical preliminaries: Let $L^2(0,1)$ be the Hilbert space of Lebesgue measurable and square integrable functions $f:[0,1]\to\mathbb{R}$ with inner product $\langle f,g\rangle := \int_0^1 f(x)g(x)dx$ and induced norm $\|f\|^2 := \langle f,f\rangle$. $H^k(0,1)$ is the Sobolev space of functions $f:[0,1]\to\mathbb{R}$ having k square integrable weak derivatives,

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with the norm $\|f\|^2_{H^k} := \sum_{j=0}^k \left\|f^{(j)}\right\|^2$. The Euclidean norm on $\mathbb{R}^n$ will be denoted by $|\cdot|$. We denote $f \in H^1_0(0,1)$ if $f \in H^1(0,1)$ and $f(0) = f(1) = 0$. For $P \in \mathbb{R}^{n\times n}$, $P > 0$ means that P is symmetric and positive definite. The matrix norm of A is denoted by $|A|$. The sub-diagonal elements of a symmetric matrix are denoted by $*$. For $U \in \mathbb{R}^{n\times n}$, $U > 0$ and $x \in \mathbb{R}^n$ we denote $|x|^2_U = x^T U x$.

Consider the Sturm-Liouville eigenvalue problem

$$\phi'' + \lambda\phi = 0, \quad x \in [0,1] \tag{1}$$

with the following boundary conditions:

$$\phi(0) = \phi(1) = 0. \tag{2}$$

This problem induces a sequence of eigenvalues with corresponding eigenfunctions. The eigenfunctions form a complete orthonormal system in $L^2(0,1)$. The eigenvalues and corresponding eigenfunctions are given by

$$\phi_n(x) = \sqrt{2}\sin\left(\sqrt{\lambda_n}x\right), \ \ \lambda_n = n^2\pi^2, \ \ n \geq 1. \tag{3}$$

The following lemma will be used:

Lemma 1. *Let $h \in L^2(0,1)$ satisfy $h \overset{L^2}{=} \sum_{n=1}^\infty h_n\phi_n$. Then $h \in H^1_0(0,1)$ iff $\sum_{n=1}^\infty \lambda_n h_n^2 < \infty$. Moreover,*

$$\left\|h'\right\|^2 = \sum_{n=1}^\infty \lambda_n h_n^2. \tag{4}$$

The result of Lemma 1 follows by arguments of Lemma 2.1 in [16] for the particular choices $p(x) \equiv 1$ and $q(x) \equiv 0$ therein.

2. Boundary control of a heat equation

In this section we consider observer-based stabilization of a linear 1D heat equation under Dirichlet actuation and point measurement. Consider

$$z_t(x,t) = z_{xx}(x,t) + az(x,t), \ \ t \geq 0 \tag{5}$$

where $x \in [0,1]$, $z(x,t) \in \mathbb{R}$ and $a \in \mathbb{R}$ is the reaction coefficient. We consider Dirichlet actuation given by

$$z(0,t) = u(t), \quad z(1,t) = 0, \tag{6}$$

where $u(t)$ is a control input to be designed, and in-domain point measurement given by

$$y(t) = z(x_*,t), \ \ x_* \in (0,1). \tag{7}$$

Following [26], we introduce the change of variables

$$w(x,t) = z(x,t) - r(x)u(t), \quad r(x) := 1 - x \tag{8}$$

to obtain the following equivalent ODE-PDE system

$$\begin{aligned} &w_t(x,t) = w_{xx}(x,t) + aw(x,t) + ar(x)u(t) - r(x)v(t), \\ &\dot{u}(t) = v(t), \quad t \geq 0 \end{aligned} \tag{9}$$

with boundary conditions

$$w(0,t) = 0, \quad w(1,t) = 0. \tag{10}$$

and measurement

$$y(t) = w(x_*,t) + r(x_*)u(t). \tag{11}$$

Henceforth we will treat $u(t)$ as an additional state variable and $v(t)$ as the control input. Given $v(t)$, $u(t)$ can be computed by integrating $\dot{u}(t) = v(t)$, where we choose $u(0) = 0$. Note that this choice implies $z(\cdot,0) = w(\cdot,0)$.

We use completeness of the eigenfunction $\{\phi_n\}_{n=1}^\infty$, given in (3), in $L^2(0,1)$ to present the solution of (9) as

$$w(x,t) = \textstyle\sum_{n=1}^\infty w_n(t)\phi_n(x), \ \ w_n(t) = \langle w(\cdot,t), \phi_n\rangle. \tag{12}$$

We show below (see the paragraph after (23)) that the closed-loop system (9), (18) with control input (23) admits a unique classical solution. Therefore, the modes of the closed-loop system $\{w_n(t)\}_{n=1}^\infty$ are differentiable. By differentiating under the integral sign, integrating by parts and using (1) and (2) we obtain

$$\begin{aligned} &\dot{w}_n(t) = (-\lambda_n + a)w_n(t) + ab_n u(t) - b_n v(t), \ \ t \geq 0 \\ &b_n = \langle r, \phi_n\rangle = \sqrt{\tfrac{2}{\lambda_n}}, \ \ w_n(0) = \langle w(\cdot,0), \phi_n\rangle, \ \ n \geq 1. \end{aligned} \tag{13}$$

In particular note that

$$b_n \neq 0, \quad n \geq 1 \tag{14}$$

and

$$\sum_{n=N+1}^\infty b_n^2 \leq \frac{2}{\pi^2}\int_N^\infty \frac{dx}{x^2} = \frac{2}{\pi^2 N}, \ \ N \geq 1. \tag{15}$$

From (12) and (13), it can be seen that the modal decomposition approach converts the ODE-PDE system (9) into the sequence of ODEs, corresponding to $u(t)$ and the modes $\{w_n(t)\}_{n=1}^\infty$. Note that only finitely many modes in (13) are unstable, due to the growth of $\{\lambda_n\}_{n=1}^\infty$. Our aim is to stabilize the unstable modes, while ensuring that the stable modes are not destabilized in the process (a phenomenon known as spillover [10]).

Let $\delta > 0$ be a desired decay rate and let $N_0 \in \mathbb{N}$ satisfy

$$-\lambda_n + a < -\delta, \quad n > N_0. \tag{16}$$

Let $N \in \mathbb{N}$, $N_0 \leq N$. N_0 will define the dimension of the controller and N will define the dimension of the observer.

We construct a finite-dimensional observer of the form

$$\hat{w}(x,t) := \sum_{n=1}^N \hat{w}_n(t)\phi_n(x) \tag{17}$$

where $\hat{w}_n(t)$ satisfy the following ODEs for $t \geq 0$:

$$\begin{aligned} &\dot{\hat{w}}_n(t) = (-\lambda_n + a)\hat{w}_n(t) + ab_n u(t) - b_n v(t) \\ &\qquad\quad -l_n\left[\hat{w}(x_*,t) + r(x_*)u(t) - y(t)\right], \ \ n \geq 1, \\ &\hat{w}_n(0) = 0, \quad 1 \leq n \leq N. \end{aligned} \tag{18}$$

with $y(t)$ in (11) and scalar observer gains $\{l_n\}_{n=1}^N$.

Assumption 1. The point $x_* \in (0,1)$ satisfies

$$c_n = \phi_n(x_*) = \sqrt{2}\sin\left(\sqrt{\lambda_n}x_*\right) \neq 0, \ \ 1 \leq n \leq N_0. \tag{19}$$

Assumption 1 is satisfied for $N_0 = 1$ by any $x^* \in (0,1)$, whereas for $N_0 > 1$ the corresponding x^* must satisfy the following condition: $x^* \neq k/n < 1$, $k = 1, \ldots, N_0 - 1$, $n = 2, \ldots, N_0$. E.g, for $N_0 = 2$ the condition is $x^* \neq 0.5$. Assumption 1 implies that the measurement $y(t)$ contains information on the unstable modes $\{w_n(t)\}_{n=1}^{N_0}$. Thus, the innovation term $\hat{w}(x_*,t) + r(x_*)u(t) - y(t)$ appearing in the observer ODEs, will allow to obtain exponential convergence of the estimation error.

Let

$$\begin{aligned} &A_0 = \operatorname{diag}\{-\lambda_1 + a, \ldots, -\lambda_{N_0} + a\}, \\ &B_0 = [b_1, \ldots, b_{N_0}], \ \ L_0 = [l_1, \ldots, l_{N_0}]^T, \\ &C_0 = [c_1, \ldots, c_{N_0}], \ \ \tilde{B}_0 = [1, -b_1, \ldots, -b_{N_0}], \\ &\tilde{A}_0 = \begin{bmatrix} 0 & 0 \\ aB_0 & A_0 \end{bmatrix} \in \mathbb{R}^{(N_0+1)\times(N_0+1)}. \end{aligned} \tag{20}$$

Under Assumption 1 it can be verified that the pair (A_0, C_0) is observable, by the Hautus lemma. We choose $L_0 = \left[l_1, \ldots, l_{N_0}\right]^T \in \mathbb{R}^{N_0}$ which satisfies the Lyapunov inequality

$$P_o(A_0 - L_0C_0) + (A_0 - L_0C_0)^T P_o < -2\delta P_o, \tag{21}$$

with $0 < P_0 \in \mathbb{R}^{N_0 \times N_0}$. We choose $l_n = 0,\ n > N_0$. Since $b_n \neq 0,\ n \geq 1$ the pair $(\tilde{A}_0, \tilde{B}_0)$ is controllable. Let $K_0 \in \mathbb{R}^{1\times(N_0+1)}$ satisfy

$$P_c(\tilde{A}_0 + \tilde{B}_0 K_0) + (\tilde{A}_0 + \tilde{B}_0 K_0)^T P_c < -2\delta P_c, \tag{22}$$

with $0 < P_c \in \mathbb{R}^{(N_0+1)\times(N_0+1)}$. We propose a (N_0+1)-dimensional controller of the form

$$v(t) = K_0 \hat{w}^{N_0}(t),\ \ \hat{w}^{N_0}(t) = \left[u(t), \hat{w}_1(t), \ldots, \hat{w}_{N_0}(t)\right]^T \tag{23}$$

which is based on the N-dimensional observer (17).

For well-posedness of the closed-loop system (9) and (18) subject to the control input (23) we consider the operator

$$\begin{aligned} &\mathcal{A}_1 : \mathcal{D}(\mathcal{A}_1) \subseteq L^2(0,1) \to L^2(0,1), \quad \mathcal{A}_1 w = -w_{xx}, \\ &\mathcal{D}(\mathcal{A}_1) = \left\{ w \in H^2(0,1) | w(0) = w(1) = 0 \right\}. \end{aligned} \tag{24}$$

Since $\mathcal{A}_1$ is positive, it has a unique positive square root with domain $\mathcal{D}\left(\mathcal{A}_1^{\frac{1}{2}}\right) = H_0^1(0,1)$. The latter follows from (4) and Section 3.4 in [28]. Let $\mathcal{H} = L^2(0,1) \times \mathbb{R}^{N+1}$ be a Hilbert space with the norm $\|\cdot\|_{\mathcal{H}}^2 = \|\cdot\|^2 + |\cdot|^2$. Defining the state $\xi(t)$ as

$$\xi(t) = \mathrm{col}\left\{ w(\cdot,t), u(t), \hat{w}_1(t), \ldots, \hat{w}_N(t) \right\},$$

by arguments of [16] that employ Theorems 6.3.1 and 6.3.3 in [25] it can be shown that the closed-loop system (9) and (18) with control input (23) and $z(\cdot,0) = w(\cdot,0) \in \mathcal{D}\left(\mathcal{A}_1^{\frac{1}{2}}\right)$ has a unique classical solution

$$\xi \in C([0,\infty); \mathcal{H}) \cap C^1((0,\infty); \mathcal{H}) \tag{25}$$

such that

$$\xi(t) \in \mathcal{D}(\mathcal{A}_1) \times \mathbb{R}^{N+1}, \quad t > 0. \tag{26}$$

Let $e_n(t)$ be the estimation error defined by

$$e_n(t) = w_n(t) - \hat{w}_n(t),\ \ 1 \leq n \leq N. \tag{27}$$

By using (11), (12) and (17), the last term on the right-hand side of (18) can be written as

$$\hat{w}(x_*,t) + r(x_*)u(t) - y(t) = -\textstyle\sum_{n=1}^N c_n e_n(t) - \zeta(t), \tag{28}$$

where

$$\begin{aligned} &\zeta(t) = w(x_*,t) - \textstyle\sum_{n=1}^N w_n(t)\phi_n(x_*) \\ &\overset{(2),(10)}{=} \textstyle\int_0^{x_*} \left[w_x(x,t) - \sum_{n=1}^N w_n(t)\phi_n'(x) \right] dx. \end{aligned} \tag{29}$$

Then the error equations have the form

$$\begin{aligned} &\dot{e}_n(t) = (-\lambda_n + a)e_n(t) \\ &\quad -l_n\left(\textstyle\sum_{n=1}^N c_n e_n(t) + \zeta(t)\right), \quad t \geq 0. \end{aligned} \tag{30}$$

Note that $\zeta(t)$ satisfies the following estimate:

$$\begin{aligned} &\zeta^2(t) \overset{(29)}{\leq} \left\| w_x(\cdot,t) - \textstyle\sum_{n=1}^N w_n(t)\phi_n'(\cdot) \right\|^2 \\ &\quad \overset{(4)}{\leq} \textstyle\sum_{n=N+1}^\infty \lambda_n w_n^2(t). \end{aligned} \tag{31}$$

In order to compensate $\zeta(t)$ in the Lyapunov analysis (see (39) below) we will prove H^1-stability of the closed-loop system and apply the result of Lemma 1. Let

$$\begin{aligned} &e^{N_0}(t) = \mathrm{col}\left\{e_i(t)\right\}_{i=1}^{N_0},\ \ e^{N-N_0}(t) = \mathrm{col}\left\{e_i(t)\right\}_{i=N_0+1}^{N}, \\ &\hat{w}^{N-N_0}(t) = \mathrm{col}\left\{\hat{w}_i(t)\right\}_{i=N_0+1}^{N}, \\ &X(t) = \mathrm{col}\left\{ \hat{w}^{N_0}(t), e^{N_0}(t), \hat{w}^{N-N_0}(t), e^{N-N_0}(t) \right\}, \end{aligned} \tag{32}$$

and

$$\begin{aligned} &A_1 = \mathrm{diag}\left\{-\lambda_i + a\right\}_{i=N_0+1}^N,\ \ \tilde{L}_0 = \mathrm{col}\{0_{1\times1}, L_0\} \\ &B_1 = \left[b_{N_0+1}, \ldots, b_N\right]^T,\ \ C_1 = [c_{N_0+1}, \ldots, c_N], \\ &\mathrm{a} = \left[-a, 0_{1\times N_0}\right],\ \ \tilde{K}_0 = \left[K_0 + \mathrm{a}, \quad 0_{1\times(2N-N_0)}\right], \\ &\mathcal{L} = \mathrm{col}\left\{\tilde{L}_0, -L_0, 0_{2(N-N_0)\times1}\right\}, \\ &F = \begin{bmatrix} \tilde{A}_0 + \tilde{B}_0 K_0 & \tilde{L}_0 C_0 & 0 & \tilde{L}_0 C_1 \\ 0 & A_0 - L_0 C_0 & 0 & -L_0 C_1 \\ -B_1(K_0 + \mathrm{a}) & 0 & A_1 & 0 \\ 0 & 0 & 0 & A_1 \end{bmatrix}. \end{aligned} \tag{33}$$

From (13), (18), (23) and (33) we have the closed-loop system for $t \geq 0$:

$$\begin{aligned} &\dot{X}(t) = FX(t) + \mathcal{L}\zeta(t), \\ &\dot{w}_n(t) = (-\lambda_n + a)w_n(t) - b_n \tilde{K}_0 X(t),\ \ n > N. \end{aligned} \tag{34}$$

Remark 1. In the closed-loop system (34), the ODE system for $X(t)$ is coupled via $\zeta(t)$ with the infinite-dimensional part that depends on $X(t)$. This is different from the state-feedback case, where the finite-dimensional part is separated from the infinite-dimensional part, and thus the stability analysis can be done in two steps: 1) the analysis of the ODE system and 2) the convergence of the infinite-dimensional part with the exponentially decaying control signal [15]. To study the stability of the coupled system we will present a direct Lyapunov method initiated in [16], which works in the case of point actuation and measurement due to dynamic extension. Indeed, without dynamic extension, modal decomposition of (5) with boundary conditions (6) results in ODEs similar to (13), without $v(t)$, where $|b_n| \geq \lambda_n^{\frac{1}{2}}$. The growth of $\{b_n\}_{n=1}^\infty$ poses a problem in compensating cross terms which arise in the Lyapunov stability analysis (see (38) below). As can be seen in (15), dynamic extension leads to $\{b_n\}_{n=1}^\infty \in l^2(\mathbb{N})$ that results in finite coefficient $\frac{4\alpha_1}{\sqrt{N}\pi^{\frac{3}{2}}}$ multiplying $\left|\tilde{K}_0 X(t)\right|^2$ in the end of (38).

For H^1-stability analysis of the closed-loop system (34) we define the Lyapunov function

$$V(t) = |X(t)|_P^2 + \sum_{n=N+1}^\infty \lambda_n w_n^2(t), \tag{35}$$

where $P \in \mathbb{R}^{(2N+1)\times(2N+1)}$ satisfies $P > 0$. This function is chosen to compensate $\zeta(t)$ using the estimate (31). Differentiating $V(t)$ along the solution of (34) gives

$$\begin{aligned} &\dot{V} + 2\delta V = X^T(t)\left[PF + F^T P + 2\delta P\right]X(t) \\ &-2X^T(t)P\mathcal{L}\zeta(t) + 2\textstyle\sum_{n=N+1}^\infty \left(-\lambda_n^2 + (a+\delta)\lambda_n\right)w_n^2(t) \\ &-2\textstyle\sum_{n=N+1}^\infty \lambda_n w_n(t) b_n \tilde{K}_0 X(t), \quad t \geq 0. \end{aligned} \tag{36}$$

Note that since $\lambda_n = n^2\pi^2$, similar to (13) we have

$$\textstyle\sum_{n=N+1}^\infty \lambda_n^{-\frac{3}{4}} \leq \pi^{-\frac{3}{2}} \int_N^\infty x^{-\frac{3}{2}} dx = \frac{2}{\sqrt{N}\pi^{\frac{3}{2}}}. \tag{37}$$

Since $b_n = \sqrt{\frac{2}{\lambda_n}}$, the Young inequality implies

$$\begin{aligned} &-2\textstyle\sum_{n=N+1}^\infty \lambda_n w_n(t) b_n \tilde{K}_0 X(t) \leq \frac{1}{\alpha_1}\sum_{n=N+1}^\infty \lambda_n^{\frac{7}{4}} w_n^2(t) \\ &+2\alpha_1\left(\textstyle\sum_{n=N+1}^\infty \lambda_n^{-\frac{3}{4}}\right)\left|\tilde{K}_0 X(t)\right|^2 \\ &\overset{(37)}{\leq} \frac{1}{\alpha_1}\textstyle\sum_{n=N+1}^\infty \lambda_n^{\frac{7}{4}} w_n^2(t) + \frac{4\alpha_1}{\sqrt{N}\pi^{\frac{3}{2}}}\left|\tilde{K}_0 X(t)\right|^2 \end{aligned} \tag{38}$$

where $\alpha > 0$. From monotonicity of λ_n we have

$$\begin{aligned}
&2\sum_{n=N+1}^{\infty}\left(-\lambda_n^2+(a+\delta)\lambda_n\right)w_n^2(t)\\
&+2\sum_{n=N+1}^{\infty}\lambda_n w_n(t)(-b_n)\tilde{K}_0X(t)\\
&\overset{(38)}{\leq} -2\left(\lambda_{N+1}-a-\delta-\tfrac{1}{2\alpha_1}\lambda_{N+1}^{\frac{3}{4}}\right)\sum_{n=N+1}^{\infty}\lambda_n w_n^2(t)\\
&+\tfrac{4\alpha_1}{\sqrt{N}\pi^{\frac{3}{2}}}\left|\tilde{K}_0X(t)\right|^2 \overset{(31)}{\leq} \tfrac{4\alpha_1}{\sqrt{N}\pi^{\frac{3}{2}}}\left|\tilde{K}_0X(t)\right|^2\\
&-2\left(\lambda_{N+1}-a-\delta-\tfrac{1}{2\alpha_1}\lambda_{N+1}^{\frac{3}{4}}\right)\zeta^2(t),
\end{aligned} \tag{39}$$

provided $\lambda_{N+1}-a-\delta-\frac{1}{2\alpha_1}\lambda_{N+1}^{\frac{3}{4}} \geq 0$. Let $\eta(t)=\mathrm{col}\{X(t),\zeta(t)\}$. From (36), (38) and (39) we obtain

$$\dot{V}+2\delta V \leq \eta^T(t)\Psi^{(1)}\eta(t) \leq 0, \quad t \geq 0 \tag{40}$$

if

$$\begin{aligned}
&\Psi^{(1)}=\begin{bmatrix}\Phi^{(1)} & P\mathcal{L}\\ & -2(\lambda_{N+1}-a-\delta)+\frac{1}{\alpha_1}\lambda_{N+1}^{\frac{3}{4}}\end{bmatrix}<0,\\
&\Phi^{(1)}=PF+F^TP+2\delta P+\tfrac{4\alpha_1}{\sqrt{N}\pi^{\frac{3}{2}}}\tilde{K}_0^T\tilde{K}_0.
\end{aligned} \tag{41}$$

By Schur complement (41) holds iff

$$\begin{bmatrix}\Phi^{(1)} & P\mathcal{L} & 0\\ & -2(\lambda_{N+1}-a-\delta) & 1\\ & * & -\alpha_1\lambda_{N+1}^{-\frac{3}{4}}\end{bmatrix}<0. \tag{42}$$

Note that the LMI (42) has *N-dependent* coefficients and its dimension depends on N. Summarizing, we arrive at:

Theorem 1. *Consider (9) with boundary conditions (10), in-domain point measurement (11), control law (23) and $w(\cdot,0)\in\mathcal{D}(\mathcal{A}_1^{\frac{1}{2}})$. Let $\delta>0$ be a desired decay rate, $N_0\in\mathbb{N}$ satisfy (16) and $N\in\mathbb{N}$ satisfy $N_0\leq N$. Let L_0 and K_0 be obtained using (21) and (22), respectively. Let there exist a positive definite matrix $P\in\mathbb{R}^{(2N+1)\times(2N+1)}$ and scalar $\alpha_1>0$ which satisfy (42). Then the solution $w(x,t)$ and $u(t)$ to (9) under the control law (23), (18) and the corresponding observer $\hat{w}(x,t)$ defined by (17) satisfy*

$$\begin{aligned}
&\|w(\cdot,t)\|_{H^1}^2+|u(t)|^2 \leq Me^{-2\delta t}\|w(\cdot,0)\|_{H^1}^2,\\
&\left\|w(\cdot,t)-\hat{w}(\cdot,t)\right\|_{H^1}^2 \leq Me^{-2\delta t}\|w(\cdot,0)\|_{H^1}^2,
\end{aligned} \tag{43}$$

for some constant $M>0$. Moreover, (42) is always feasible for large enough N.

Proof. Feasibility of the LMI (42) implies, by the comparison principle,

$$V(t) \leq e^{-2\delta t}V(0), \ t \geq 0. \tag{44}$$

Since $u(0)=0$, for some $M_0>0$ we have

$$V(0) \overset{(4)}{\leq} M_0\|w_x(\cdot,0)\|^2 \leq M_0\|w(\cdot,0)\|_{H^1}^2. \tag{45}$$

By Wirtinger's inequality ([7], Section 3.10), for $t\geq 0$,

$$\|w_x(\cdot,t)\|^2 \leq \|w(\cdot,t)\|_{H^1}^2 \leq \frac{\pi^2+4}{\pi^2}\|w_x(\cdot,t)\|^2. \tag{46}$$

Since $w(\cdot,t)\in\mathcal{D}(\mathcal{A}_1)$ for all $t>0$ we have $\|w_x(\cdot,t)\|^2 \overset{(4)}{=} \sum_{n=1}^{\infty}\lambda_n w_n^2(t)$. Parseval's equality, (46) and monotonicity of $\{\lambda_n\}_{n=1}^{\infty}$ imply

$$\begin{aligned}
&V(t) \geq \sigma_{min}(P)|u(t)|^2\\
&+\min\left(\tfrac{\sigma_{min}(P)}{2\lambda_N},1\right)\|w_x(\cdot,t)\|^2, \quad t\geq 0.
\end{aligned} \tag{47}$$

Then (43) follows from (44), (45), (46), (47) and the presentation

$$w(\cdot,t)-\hat{w}(\cdot,t)=\sum_{n=1}^{N}e_n(t)\phi_n(\cdot)+\sum_{n=N+1}^{\infty}w_n(t)\phi_n(\cdot).$$

For feasibility of (42) with large enough N, note that (15) and (19) imply $|c_n|\leq\sqrt{2}$, $n\geq 1$ and $\{b_n\}_{n=1}^{\infty}\in l^2(\mathbb{N})$. Then, by arguments of Theorem 3.2 in [16], there exist some $\Lambda,\kappa>0$, independent of N, such that

$$\left|e^{(F+\delta I)t}\right| \leq \Lambda\cdot\sqrt{N}\left(1+t+t^2\right)e^{-\kappa t}. \tag{48}$$

Therefore, $P\in\mathbb{R}^{(2N+1)\times(2N+1)}$ which solves the Lyapunov equation

$$P(F+\delta I)+(F+\delta I)^T=-N^{-\frac{3}{4}}I \tag{49}$$

satisfies

$$|P| \leq \Lambda_1\cdot N^{\frac{1}{4}} \tag{50}$$

where $\Lambda_1>0$ is independent of N. We substitute (49), $\lambda_{N+1}=\pi^2(N+1)^2$ and $\alpha=N^{-\frac{3}{8}}$ into (41). By Schur complement, we find that (41) holds if and only if

$$\begin{aligned}
&-I+4\pi^{-\frac{3}{4}}N^{-\frac{1}{8}}\tilde{K}_0^T\tilde{K}_0\\
&+\tfrac{1}{2}\left(\lambda_{N+1}-a-\delta-N^{\frac{3}{8}}\pi^{\frac{3}{2}}(N+1)^{\frac{3}{2}}\right)^{-1}P\mathcal{L}\mathcal{L}^TP<0.
\end{aligned} \tag{51}$$

Since $\lambda_{N+1}-a-\delta\approx(N+1)^2$ and $\left|\tilde{K}_0\right|$, $|\mathcal{L}|$ are independent of N, by taking into account (50) we find that (51) holds for large enough N. □

Corollary 1. *Under the conditions of Theorem 1, the following estimates hold for $z(x,t)$ given in (8):*

$$\begin{aligned}
&\|z(\cdot,t)\|_{H^1}^2 \leq Me^{-2\delta t}\|z(\cdot,0)\|_{H^1}^2,\\
&\left\|z(\cdot,t)-\hat{w}(\cdot,t)\right\|_{H^1}^2 \leq Me^{-2\delta t}\|z(\cdot,0)\|_{H^1}^2,
\end{aligned} \tag{52}$$

where $M>0$ is some constant.

Proof. From (8) we have

$$\begin{aligned}
&\|z(\cdot,t)\|_{H^1} \leq \|w(\cdot,t)\|_{H^1}+|u(t)|\|r(\cdot)\|_{H^1},\\
&\left\|z(\cdot,t)-\hat{w}(\cdot,t)\right\|_{H^1} \leq \left\|w(\cdot,t)-\hat{w}(\cdot,t)\right\|_{H^1}\\
&+|u(t)|\|r(\cdot)\|_{H^1}.
\end{aligned} \tag{53}$$

From $u(0)=0$, (43) and (53), we obtain (52). □

Remark 2. Differently from Katz and Fridman [19], where Dirichlet actuation with non-local measurements were considered, we apply the Young inequality in (38) with fractional powers of λ_n which allows to compensate ζ by using (31) in the Lyapunov analysis. Note that for finite-dimensional observer-based control of the 1D linear Kuramoto-Sivashinsky equation (KSE), studied in [18], the use of fractional powers of the eigenvalues was not required. This is due to the faster growth rate of the eigenvalues corresponding to the fourth order spatial differential operator appearing in the KSE.

3. A numerical example

We demonstrate our approach to Dirichlet control of a 1D linear heat equation, where we choose $a=10$ leading to an unstable open-loop system. We choose point measurement at $x_*=\frac{1}{\pi}$ and $\delta=0.1$, which results in $N_0=1$. The observer and controller gains L_0 and K_0 are found from (21) and (22), respectively. The obtained gains are given by

$$L_0=0.7398, \quad K_0=\begin{bmatrix}-4.9910 & -5.3338\end{bmatrix}. \tag{54}$$

The LMI of Theorem 1 was verified using Matlab with $\delta=0.1$. It is feasible for $N=3$ implying that the closed-loop system is exponentially H^1-stable with a decay rate $\delta=0.1$.

For simulation of the closed-loop system (34) with $N=3$ we consider the initial conditions

$$u(0)=0, \quad w(\cdot,0)=10x(1-x)\cos(2x), \ x\in[0,1].$$

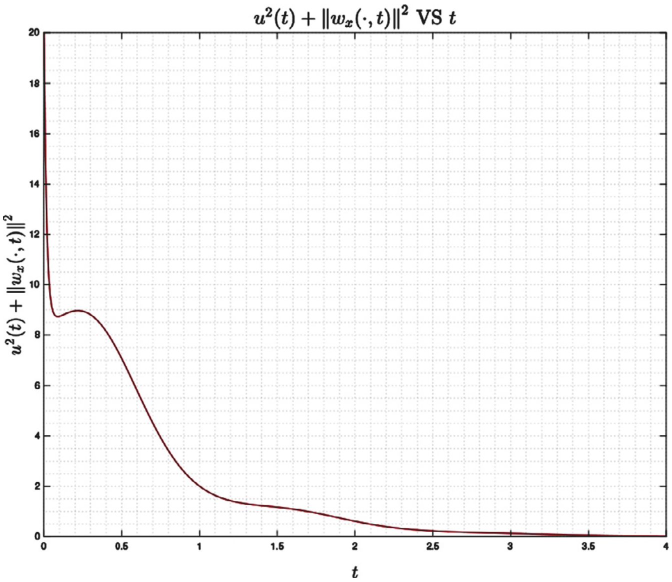

Fig. 1. $u^2(t)+\|w_x(\cdot,t)\|^2$ VS t.

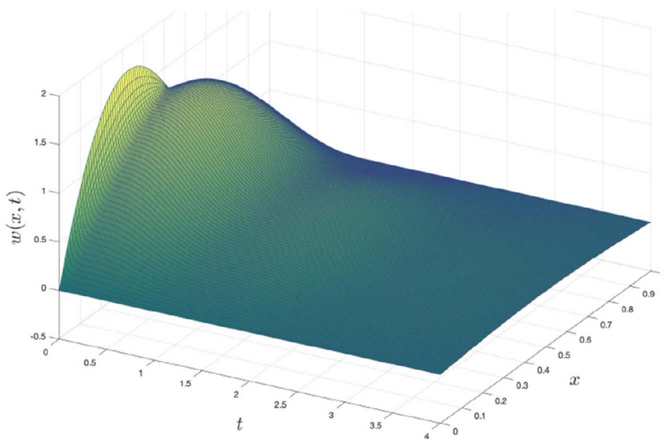

Fig. 2. $w(x,t)$ VS (x,t).

Note that $w(\cdot,0)\in\mathcal{D}\left(\mathcal{A}_1^{\frac{1}{2}}\right)=H_0^1(0,1)$. Using the modal decomposition (12) and (4) we approximate the solution norm using 50 modes as follows:

$$\|w(\cdot,t)\|^2 \overset{(12)}{\approx} \sum_{n=1}^{50} w_n^2(t), \quad \|w_x(\cdot,t)\|^2 \overset{(4)}{\approx} \sum_{n=1}^{50} \lambda_n w_n^2(t).$$

Then, the closed-loop system (34) (with the tail ODEs truncated after 50 modes) is simulated using MATLAB, subject to the observer-based control law (23). The value of $\zeta(t)$, defined in (29), is approximated by

$$\zeta(t)\approx\sum_{n=4}^{50} w_n(t)\phi_n(x_*).$$

We choose $t\in[0,4]$ as the simulation time interval. The evolution of $u^2(t)+\|w_x(\cdot,t)\|^2$ (which is equivalent to $u^2(t)+\|w(\cdot,t)\|_{H^1}^2$ by Sobolev's inequality) is given in Fig. 1. A surface plot of the solution $w(x,t)$ is given in Fig. 2. The numerical simulation validates our theoretical results. Moreover, simulation of the corresponding closed-loop system with $N=2$ shows stability. With $N=1$, the simulations did not show stability of the closed-loop system, meaning that our LMI-based condition is slightly conservative in this example.

4. Discussions of the proposed controller

4.1. Compatibility of the observer-based design to various boundary conditions

In this section we demonstrate that our observer-based controller is well suited for the 1D heat equation with various boundary conditions. As an example, consider (5) with Neumann actuation

$$z_x(0,t)=u(t), \quad z_x(1,t)=0, \tag{55}$$

and boundary measurement given by

$$y(t)=z(0,t). \tag{56}$$

By considering the Sturm-Liouville problem (1) with the boundary conditions

$$\phi'(0)=\phi'(1)=0, \tag{57}$$

it can be verified that the corresponding eigenvalues and eigenfunctions are given by

$$\begin{aligned} &\phi_1(x)=1,\ \lambda_1=0,\\ &\phi_n(x)=\sqrt{2}\cos\left(\sqrt{\lambda_n}x\right),\ \lambda_n=(n-1)^2\pi^2,\ n\geq 2. \end{aligned} \tag{58}$$

Given $h\in L^2(0,1)$ satisfying $h\overset{L^2}{=}\sum_{n=1}^{\infty}h_n\phi_n$, it can be easily verified by arguments of Katz and Fridman [16] that $h\in H^1(0,1)$ iff $\sum_{n=1}^{\infty}\lambda_n h_n^2<\infty$. Moreover, (4) holds.

For (55) we employ the change of variables

$$w(x,t)=z(x,t)-r(x)u(t), \quad r(x):=x-\frac{x^2}{2} \tag{59}$$

that leads to the following ODE-PDE system for $t\geq 0$:

$$\begin{aligned} &w_t(x,t)=w_{xx}(x,t)+aw(x,t)+r_1(x)u(t)-r(x)v(t),\\ &\dot{u}(t)=v(t), \quad r_1(x)=ar(x)-1 \end{aligned} \tag{60}$$

with boundary conditions

$$w_x(0,t)=0, \quad w_x(1,t)=0. \tag{61}$$

and measurement

$$y(t)=w(0,t). \tag{62}$$

We present the solution to (60) as (12). Differentiating under the integral sign, integrating by parts and using (1) and (57) we obtain the following ODEs for $t\geq 0$:

$$\dot{w}_n(t)=(-\lambda_n+a)w_n(t)+\bar{b}_n u(t)-b_n v(t),\ t\geq 0,$$

where

$$\begin{aligned} &b_1=\tfrac{1}{3},\ b_n=-\tfrac{\sqrt{2}}{\lambda_n},\ n\geq 2\\ &\bar{b}_1=\tfrac{a}{3}-1,\ \bar{b}_n=ab_n,\ n\geq 2,\\ &w_n(0)=\langle w(\cdot,0),\phi_n\rangle,\ n\geq 1. \end{aligned} \tag{63}$$

In particular, note that (14) holds.

Let $\delta>0$ be a desired decay rate and let $N_0\in\mathbb{N}$ satisfy (16). Let $N\in\mathbb{N},\ N_0\leq N$. We construct a finite-dimensional observer of the form (17) where $\left\{\hat{w}_n(t)\right\}_{n=1}^{N}$ satisfy the following ODEs for $t\geq 0$:

$$\begin{aligned} \dot{\hat{w}}_n(t)=&(-\lambda_n+a)\hat{w}_n(t)+\bar{b}_n u(t)-b_n v(t)\\ &-l_n\left[\hat{w}(0,t)-y(t)\right],\ n\geq 1,\\ \hat{w}_n(0)=&0, \quad 1\leq n\leq N. \end{aligned} \tag{64}$$

with $y(t)$ in (62) and scalar observer gains $\{l_n\}_{n=1}^{N}$.

Note that in this case

$$c_n=\phi_n(0)\neq 0,\ 1\leq n\leq N_0, \tag{65}$$

i.e. Assumption 1 always holds. Recall the notations (20) and let

$$\bar{B}_0=\left[\bar{b}_1,\ldots,\bar{b}_{N_0}\right]^T,\ \tilde{A}_0=\begin{bmatrix}0 & 0\\ a\bar{B}_0 & A_0\end{bmatrix}. \tag{66}$$

The pair (A_0,C_0) is observable, by the Hautus lemma. We choose $L_0=\left[l_1,\ldots,l_{N_0}\right]^T\in\mathbb{R}^{N_0}$ which satisfies the Lyapunov inequality (21) with $0<P_o\in\mathbb{R}^{N_0\times N_0}$. We choose $l_n=0,\ n>N_0$. Since $b_n\neq 0,\ n\geq 1$ the pair $(\tilde{A}_0,\tilde{B}_0)$ is controllable. Let $K_0\in\mathbb{R}^{1\times(N_0+1)}$ satisfy (22) with $\tilde{A}_0$ replaced by $\bar{A}_0$ and $0<P_c\in\mathbb{R}^{(N_0+1)\times(N_0+1)}$. We propose a (N_0+1)-dimensional controller of the form (23), which is based on the N-dimensional observer (17).

Recall the estimation error (27). By using (12), (17) and (62), the innovation term $\hat{w}(0,t)-y(t)$ in (64) can be written as the right-hand side of (28), where

$$\zeta(t)=w(0,t)-\textstyle\sum_{n=1}^N w_n(t)\phi_n(0). \tag{67}$$

Then the error equations have the form (30). By Sobolev's inequality (see Lemma 4.1 in [13]) and (4) we have for any $\Gamma>0$:

$$\begin{aligned}\zeta^2(t)&\overset{(67)}{\leq}\max_{x\in[0,1]}\left(w(x,t)-\textstyle\sum_{n=1}^N w_n(t)\phi_n(x)\right)^2\\ &\overset{\text{Sobolev}}{\leq}(1+\Gamma)\left\|w(\cdot,t)-\textstyle\sum_{n=1}^N w_n(t)\phi_n(\cdot)\right\|^2\\ &+\tfrac{1}{\Gamma}\left\|w_x(\cdot,t)-\textstyle\sum_{n=1}^N w_n(t)\phi_n'(\cdot)\right\|^2\\ &\overset{(4)}{\leq}\textstyle\sum_{n=N+1}^\infty \mu_n w_n^2(t),\ \ \mu_n=1+\Gamma+\frac{\lambda_n}{\Gamma},\ \ n\geq 1.\end{aligned} \tag{68}$$

Recall the notations (32), (33). The closed-loop system has the form (34) with F replaced by

$$\tilde{F}=\begin{bmatrix}\bar{A}_0+\tilde{B}_0K_0 & \tilde{L}_0C_0 & 0 & \tilde{L}_0C_1\\ 0 & A_0-L_0C_0 & 0 & -L_0C_1\\ -B_1(K_0+\mathrm{a}) & 0 & A_1 & 0\\ 0 & 0 & 0 & A_1\end{bmatrix}. \tag{69}$$

For H^1-stability analysis of the closed-loop system (34), with F replaced by $\tilde{F}$, we define the Lyapunov function (35), where $P\in\mathbb{R}^{(2N+1)\times(2N+1)}$ satisfies $P>0$. This Lyapunov function is chosen to compensate $\zeta(t)$ using the estimate (68). From here, the stability analysis follows arguments of (36)-(40) with two adjustments. First, taking into account $b_n,\ n\geq 2$, given in (63), the Young inequality (38) is replaced by:

$$\begin{aligned}&-2\textstyle\sum_{n=N+1}^\infty \lambda_n w_n(t)b_n\tilde{K}_0X(t)\leq\frac{1}{\alpha_1}\sum_{n=N+1}^\infty\lambda_n w_n^2(t)\\ &+2\alpha_1\left(\textstyle\sum_{n=N+1}^\infty\lambda_n^{-1}\right)\left|\tilde{K}_0X(t)\right|^2\\ &\leq\tfrac{1}{\alpha_1}\textstyle\sum_{n=N+1}^\infty\lambda_n w_n^2(t)+\frac{4\alpha_1}{N\pi^2}\left|\tilde{K}_0X(t)\right|^2\end{aligned} \tag{70}$$

where similar to (15)

$$\sum_{n=N+1}^\infty\lambda_n^{-1}=\frac{1}{\pi^2}\sum_{n=N}^\infty\frac{1}{n^2}\leq\frac{1}{\pi^2}\left[\frac{1}{N}+\int_N^\infty\frac{dx}{x^2}\right]\leq\frac{2}{\pi^2N}.$$

Second, with notation

$$\theta_n=\frac{\lambda_n^2-\left(a+\delta+\frac{1}{2\alpha_1}\right)\lambda_n}{\mu_n},\quad n\geq 2$$

(39) is replaced by:

$$\begin{aligned}&2\textstyle\sum_{n=N+1}^\infty\left[-\lambda_n^2+(a+\delta+\frac{1}{2\alpha_1})\lambda_n\right]w_n^2(t)\\ &=-2\textstyle\sum_{n=N+1}^\infty\theta_n\mu_n w_n^2(t)\\ &\leq-2\theta_{N+1}\textstyle\sum_{n=N+1}^\infty\mu_n w_n^2(t)\overset{(68)}{\leq}-2\theta_{N+1}\zeta^2(t)\end{aligned} \tag{71}$$

provided $\lambda_n-a-\delta-\frac{1}{2\alpha_1}\geq 0,\ n\geq N+1$.

Therefore, (40) holds with Ψ^1 changed by the following Ψ^2:

$$\begin{aligned}&\Psi^2=\begin{bmatrix}\Phi^{(2)} & P\mathcal{L}\\ & -2\theta_{N+1}\end{bmatrix}<0,\\ &\Phi^{(2)}=P\tilde{F}+\tilde{F}^TP+2\delta P+\tfrac{4\alpha_1}{N\pi^2}\tilde{K}_0^T\tilde{K}_0.\end{aligned} \tag{72}$$

By applying Schur complement to (72) we obtain

$$\begin{bmatrix}\Phi^{(2)} & P\mathcal{L} & 0\\ & -2\frac{\lambda_{N+1}^2-(a+\delta)\lambda_{N+1}}{\mu_{N+1}} & 1\\ & * & -\frac{\alpha_1\mu_{N+1}}{\lambda_{N+1}}\end{bmatrix}<0. \tag{73}$$

which guarantees H^1-stability of the closed-loop system with decay rate δ.

LMI (73) is always feasible for large enough N. Indeed, by (63) and (65) we have $|c_n|\leq\sqrt{2},\ n\geq 1$ and $\{b_n\}_{n=1}^\infty\in l^2(\mathbb{N})$. Thus, by arguments of Theorem 3.2 in [16], there exist some $\Lambda,\kappa>0$, independent of N, such that (48) holds with F replaced by $\tilde{F}$. Then, $P\in\mathbb{R}^{(2N+1)\times(2N+1)}$ which solves the Lyapunov equation

$$P(\tilde{F}+\delta I)+(\tilde{F}+\delta I)^TP=-N^{-1}I \tag{74}$$

satisfies $|P|\leq\Lambda_1$ where $\Lambda_1>0$ is independent of N. Substituting (74), $\lambda_{N+1}=\pi^2N^2$ and $\alpha_1=N^{-\frac{1}{2}}$ into (72), we find that (72) is feasible for large enough N.

For other boundary conditions of the type Neumann-Dirichlet or Dirichlet-Neumann, our observer-based control design goes through with small modifications. Indeed, for both cases we will have $\lambda_n\approx n^2$. By choosing a suitable $r(x)$ in the change of variables $w(x,t)=z(x,t)-r(x)u(t)$ we will obtain $|c_n|=O(1)$ and $|b_n|\lesssim\frac{1}{n}$, implying $\{b_n\}_{n=1}^\infty\in l^2(\mathbb{N})$. Then, our observer-based control design, leading to LMIs, will guarantee H^1-stability of the closed-loop system. Furthermore, the resulting LMIs will always be feasible for large enough N.

4.2. Sampled-data implementation of the controller

For practical application of the controllers, their sampled-data implementation is important. Sampled-data control of PDEs is becoming an active research area. General results on sampled-data control of PDEs were presented in [22,23]. Sampled-data controllers for PDEs implemented by zero-order hold devices were suggested in [2,8,9,13–15,19,21,27]. Event-triggered sampled-data control of parabolic and hyperbolic PDEs has been studied in [5,6,12].

In this section we briefly discuss a possible sampled-data implementation of our finite-dimensional controller. Due to dynamic extension the resulting controller becomes proportional integral. Differently from the proportional sampled-data controllers that can be implemented via zero-order hold devices, for sampled-data implementation of our proportional integral controller we suggest to employ a generalized hold device (see e.g. [24] for ODEs and [23] for PDEs). Some preliminary results on such implementation can be found in [17]. By using a time-delay approach to sampled-data control [7], it is shown in [17] that the resulting observer-based controller preserves H^1-stability of the corresponding closed-loop system for fast enough sampling. Moreover, upper bounds on the lengths of the sampling intervals and the resulting decay rate of the closed-loop system can be found from LMIs.

Detailed study of the sampled-data implementation is not in the scope of the present paper.

5. Conclusions

This paper presented the first constructive LMI-based method for finite-dimensional boundary controller design under the point measurement for 1D heat equation. The method was based on modal decomposition approach via dynamic extension. Sampled-data and network-based implementations of the proposed controller are currently under study. Extension of results to other parabolic PDEs may be a topic for future research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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