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Brief paper

Global stabilization of a 1D semilinear heat equation via modal decomposition and direct Lyapunov approach ☆

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ABSTRACT

In this paper we consider state-feedback global stabilization of a semilinear 1D heat equation with a nonlinearity exhibiting a linear growth bound. We study both non-local and boundary control via a modal decomposition approach. For both cases, we suggest a direct Lyapunov method applied to the full-order closed-loop system. The nonlinear terms are compensated by using Parseval's inequality, leading to efficient and constructive linear matrix inequality (LMI) conditions for obtaining the controller dimension and gain. For non-local control we provide sufficient conditions that guarantee global stabilization for any linear growth bound via either linear or nonlinear controller, provided the number of actuators is large enough. We prove that the nonlinear controller achieves at least the same performance as the linear one. For the case of boundary control, we employ a multi-dimensional dynamic extension, whereas in the numerical example we manage with a larger linear growth bound. The introduced direct Lyapunov approach gives tools for a variety of robust control problems for semilinear parabolic PDEs.

1. Introduction

Semilinear parabolic PDEs with a nonlinearity exhibiting a linear growth bound arise in many physical models (Christofides, 2001). Global state-feedback stabilization of such PDEs was studied by using modal decomposition in Christofides (2001), Hagen (2006), Hagen and Mezic (2003), Karafyllis (2021) and Karafyllis and Krstic (2019) and spatial decomposition (Fridman & Bar Am, 2013; Fridman & Blighovsky, 2012). The advantage of modal decomposition over spatial decomposition is in its efficiency for boundary control or for non-local control, where the shape functions need not cover the whole spatial domain.

The challenge of modal decomposition for nonlinear PDEs lies in coupling of the solution modes, which is introduced by the nonlinearity. Such coupling may cause a spillover behavior (Hagen & Mezic, 2003). In Hagen (2006) and Hagen and Mezic (2003) an LQR-based controller was designed for a 1D semilinear heat equation. For boundary control, a novel multi-dimensional dynamic extension was introduced recently in Karafyllis (2021). This extension allows a multi-dimensional integral controller dynamics with more design freedom than the existing scalar dynamic extensions (Coron & Trélat, 2004; Curtain & Zwart, 1995; Hagen, 2006). The control design in Karafyllis (2021) is based on simple control Lyapunov functions for both linear and nonlinear controllers. Sufficient conditions for stabilization are derived by using a small gain theorem, as in Karafyllis and Krstic (2019). The proposed method was demonstrated to work if the linear growth bound is not too large. Note that local stabilization of nonlinear PDEs was studied e.g. in Al Jamal and Morris (2018), Bekiaris-Liberis and Vazquez (2019), Coron and Trélat (2004) and Vazquez and Krstic (2008).

Recently, a finite-dimensional observer-based controller for the 1D linear heat equation via a modal decomposition was studied in Katz and Fridman (2020), where spillover was introduced into the closed-loop dynamics by the finite-dimensional observer, which leads to coupling between the finite-dimensional part of the system with the infinite dimensional tail. A direct Lyapunov approach was suggested to derive efficient LMIs for finding the controller and observer dimensions. An advantage of such a direct Lyapunov approach lies in the ability of its easy extension to delayed or sampled-data control (Katz & Fridman, 2021a), input-to-state and L_2-gain stabilization (Katz & Fridman, 2021b) and H_1-stabilization (Katz & Fridman, 2021c). However, a direct Lyapunov approach for global stabilization of semilinear parabolic PDEs via modal decomposition seems to be missing in the literature.

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In this paper we consider state-feedback global stabilization of a 1D heat equation with a nonlinearity exhibiting a linear growth bound. We study both non-local and boundary control via modal decomposition. For both cases, we introduce a direct Lyapunov method to the full-order closed-loop system. The nonlinear terms are compensated by using Parseval's inequality, leading to efficient and constructive LMIs for finding the controller dimension and gain. For non-local actuation, we consider first an arbitrary finite number of actuators and a linear controller. In this case we cannot guarantee the feasibility of the LMIs for any linear growth bound. We further consider the case where the controller dimension is equal to the number of the control inputs. Here, for either linear or nonlinear controller, we present sufficient conditions that guarantee a global stabilization for any linear growth bound, provided the number of actuators is large enough (similarly to the spatial decomposition approach). We prove that the nonlinear controller achieves at least the same performance as the linear one. A numerical example shows that a nonlinear controller slightly improves the results under the linear one, but on account of the controller complexity and the knowledge of nonlinearity.

For boundary control, we employ a dynamic extension as initiated in Karafyllis (2021) with an m-dimensional dynamic extension, leading to a $m+N$ dimensional linear PI controller. Here, as in Karafyllis (2021), we are unable to guarantee feasibility of the resulting LMIs for any linear growth bound (see Remark 3.2). Nevertheless, a numerical example from Karafyllis (2021) illustrates the efficiency of our method, where we manage with a much larger linear growth bound by using a linear controller with a larger N.

Notations and preliminaries: $L^2(0,1)$ is the Hilbert space of Lebesgue measurable and square integrable functions $f:[0,1]\to\mathbb{R}$ with the inner product $\langle f,g\rangle := \int_0^1 f(x)g(x)dx$ and induced norm $\|f\|^2 := \langle f,f\rangle$. $H^1(0,1)$ is the space of functions $f:[0,1]\to\mathbb{R}$ with one square integrable weak derivative, with the norm $\|f\|_{H^1}^2 := \sum_{j=0}^1 \left\|f^{(j)}\right\|^2$. The Euclidean norm on $\mathbb{R}^n$ is denoted by $|\cdot|$. For $P\in\mathbb{R}^{n\times n}$, $P>0$ means that P is symmetric and positive definite. The sub-diagonal elements of a symmetric matrix will be denoted by $*$. For $U\in\mathbb{R}^{n\times n}$ and $x\in\mathbb{R}^n$ let $|x|_U^2 = x^TUx$. $\mathbb{N}$ is the set of natural numbers.

Recall that the Sturm–Liouville eigenvalue problem

$$\phi''+\lambda\phi=0,\quad x\in[0,1],\quad \phi(0)=\phi(1)=0, \tag{1.1}$$

induces a sequence of eigenvalues with corresponding eigenfunctions

$$\lambda_n=n^2\pi^2,\quad \phi_n(x)=\sqrt{2}\sin\left(\sqrt{\lambda_n}x\right),\ n\in\mathbb{N}. \tag{1.2}$$

The eigenfunctions form a complete orthonormal system in $L^2(0,1)$. Given $N\in\mathbb{N}$ and $h\in L^2(0,1)$ satisfying $h \overset{L^2}{=} \sum_{n=1}^\infty h_n\phi_n$ we denote

$$\|h\|_N^2=\sum_{n=N+1}^\infty h_n^2. \tag{1.3}$$

2. Stabilization of a semilinear heat equation — non-local actuation

In this section we consider non-local actuation with either an arbitrary number $m\in\mathbb{N}$ of actuators or $m=N$ actuators, whereas N is the controller dimension.

2.1. Non-local stabilization with $m\in\mathbb{N}$ actuators

Consider stabilization of the following nonlinear 1D heat equation:

$$\begin{aligned}&z_t(x,t)=z_{xx}(x,t)+f(z(x,t))z(x,t)+b(x)u(t),\\&z(0,t)=0,\quad z(1,t)=0\end{aligned} \tag{2.1}$$

where $t\geq 0$, $x\in(0,1)$, $z(x,t)\in\mathbb{R}$,

$$\begin{aligned}&b(x)=[b_1(x),\dots,b_m(x)]\in\mathbb{R}^{1\times m},\quad b_i\in L^2(0,1),\\&u(t)=\mathrm{col}\{u_1(t),\dots,u_m(t)\}.\end{aligned} \tag{2.2}$$

Here, $u(t)$ is the control input to be designed. Throughout the paper we assume that $f:\mathbb{R}\to\mathbb{R}$ is a locally Lipschitz function satisfying

$$\|f\|_{L^\infty}\leq\sigma \tag{2.3}$$

for some $\sigma>0$. Through out the paper, except for Section 2.C, we will assume that f is unknown.

Remark 2.1. As a special case of (2.1), one can consider the following semilinear heat equation

$$\begin{aligned}&z_t(x,t)=z_{xx}(x,t)+g(z(x,t))+b(x)u(t),\\&z(0,t)=0,\quad z(1,t)=0,\end{aligned} \tag{2.4}$$

where $g:\mathbb{R}\to\mathbb{R}$ is a differentiable function with $g(0)=0$ and locally Lipschitz derivative satisfying $\left\|g'\right\|_{L^\infty}\leq\sigma$ for some $\sigma>0$. By the fundamental theorem of calculus, we have

$$\begin{aligned}&g(z(x,t))=\int_0^1\tfrac{d}{ds}\left[g(sz(x,t))\right]ds=f(z(x,t))z(x,t),\\&f(z(x,t))\overset{\Delta}{=}\int_0^1 g'(sz(x,t))ds\end{aligned}$$

with a locally Lipschitz f subject to (2.3). One can also easily consider (2.1) with an additional linear reaction term, as done in Section 3 (for comparison purposes with (Karafyllis, 2021), where a similar case was studied) and with variable diffusion and reaction coefficients as in Karafyllis (2021) and Katz and Fridman (2020). The functions $\{b_i\}_{i=1}^m$ can be any $L^2(0,1)$ functions, including point-like, functions with local support, spatially distributed, etc.

We present the solution to (2.1) as

$$z(x,t)=\sum_{n=1}^\infty z_n(t)\phi_n(x),\ \ z_n(t)=\langle z(\cdot,t),\phi_n\rangle \tag{2.5}$$

with $\{\phi_n\}_{n\in\mathbb{N}}$ defined in (1.2). Differentiating under the integral sign, integrating by parts and using (1.1) we have

$$\begin{aligned}\dot z_n(t)&=\int_0^1 z_t(x,t)\phi_n(x)dx\overset{(2.1)}{=}\int_0^1 z_{xx}(x,t)\phi_n(x)dx\\&\quad+\int_0^1 f(z(x,t))z(x,t)\phi_n(x)dx+\int_0^1 b(x)u(t)\phi_n(x)dx\\&=-\lambda_n z_n(t)+B_nu(t)+f_n(t),\ \ z_n(0)=\langle z(\cdot,0),\phi_n\rangle,\end{aligned} \tag{2.6}$$

where

$$\begin{aligned}&B_n=\left[\langle b_1,\phi_n\rangle,\dots,\langle b_m,\phi_n\rangle\right],\\&f_n(t)=\langle f(z(\cdot,t))z(\cdot,t),\phi_n\rangle.\end{aligned} \tag{2.7}$$

Let $\delta>0$ be a desired decay rate. Since $\lim_{n\to\infty}\lambda_n=\infty$, there exists some $N\in\mathbb{N}$ such that

$$-\lambda_n+\sigma+\delta<0,\quad n>N. \tag{2.8}$$

We introduce the notation

$$A_0=\mathrm{diag}\{-\lambda_n\}_{n=1}^N,\ \ B_0=\mathrm{col}\{B_1,\dots,B_N\}\in\mathbb{R}^{N\times m}. \tag{2.9}$$

Assumption 1. Assume that (A_0,B_0) is controllable.

We propose a controller of the form

$$u(t)=-K_0z^N(t),\ z^N(t)=\mathrm{col}\,\{z_i(t),\}_{i=1}^N, \tag{2.10}$$

where the controller gain $K_0 \in \mathbb{R}^{m\times N}$ will be obtained further from LMIs (see (2.24)–(2.26)).

For well-posedness of the closed-loop system (2.1), (2.10), we define the operator

$$\mathcal{A}:\mathcal{D}(\mathcal{A})\subseteq L^2(0,1)\to L^2(0,1),\ \mathcal{A}h=-h'',$$
$$\mathcal{D}(\mathcal{A})=H^2(0,1)\cap H_0^1(0,1).$$

The operator $-\mathcal{A}$ is a sectorial operator, generating an analytic semigroup (Pazy, 1983). Furthermore, $\mathcal{A}$ is positive and has a unique square root $\mathcal{A}^{\frac{1}{2}}$ with $\mathcal{D}\left(\mathcal{A}^{\frac{1}{2}}\right) = H_0^1(0,1)$ (Tucsnak & Weiss, 2009). Consider the Hilbert space $\mathcal{H}=H_0^1(0,1)\times\mathbb{R}^N$ with norm $\|\cdot\|_{\mathcal{H}}^2=\|\cdot\|_{H^1}^2+|\cdot|^2$. Let $z(\cdot,0)\in\mathcal{D}\left(\mathcal{A}^{\frac{1}{2}}\right)$. Defining

$$\xi(t)=\mathrm{col}\left\{z(\cdot,t),z^N(t)\right\}$$

the closed loop system can be presented as

$$\dot{\xi}(t)+\begin{bmatrix}\mathcal{A} & 0\\ 0 & -(A_0-B_0K_0)\end{bmatrix}\xi(t)=\begin{bmatrix}F_1(\xi)\\ F_2(\xi)\end{bmatrix},$$
$$F_1(\xi)=f(\xi_1)\xi_1-b(x)K_0\xi_2,$$
$$F_2(\xi)=\mathrm{col}\,\{\langle f(\xi_1)\xi_1,\phi_1\rangle,\dots,\langle f(\xi_1)\xi_1,\phi_N\rangle\}.$$

Recall that f is a locally Lipschitz function. Let $(\nu_1,\nu_2)^T$ and $(\eta_1,\eta_2)^T$ in a ball $\mathcal{B}\subseteq\mathcal{H}$. By the Sobolev inequality

$$\begin{aligned}&\|f(\nu_1)\nu_1-f(\eta_1)\eta_1\|\le\|f(\nu_1)(\nu_1-\eta_1)\|\\&+\|(f(\nu_1)-f(\eta_1))\eta_1\|\overset{(2.3),\ \text{Sobolev}}{\le}\sigma\|\nu_1-\eta_1\|\\&+c\cdot\|\eta_1\|\,\|\nu_1-\eta_1\|_{H^1}\le c_1\cdot\|\nu-\eta\|_{\mathcal{H}}\end{aligned} \tag{2.11}$$

for some constants $c,c_1>0$ (note that (2.11) is a local estimate under the assumption $\nu,\eta\in\mathcal{B}$). Furthermore, we have

$$\|f(\nu_1)\nu_1\|\le\sigma\|\nu\|_{\mathcal{H}}. \tag{2.12}$$

From (2.11), (2.12) and Theorems 6.3.1 and 6.3.3 in Pazy (1983), the closed-loop system has a unique classical solution

$$\xi\in C([0,\infty),L^2(0,1))\cap C^1((0,\infty),L^2(0,1))$$

such that $\xi(t)\in\mathcal{D}(\mathcal{A})\times\mathbb{R}^N$ for $t>0$.

Let

$$f^N(t)=\mathrm{col}\,\{f_1(t),\dots,f_N(t)\}. \tag{2.13}$$

Then, using (2.6), (2.9), (2.10) and (2.13), the closed-loop system for $t\ge0$ can be presented as

$$\begin{aligned}&\dot{z}^N(t)=(A_0-B_0K_0)z^N(t)+f^N(t),\\&\dot{z}_n(t)=-\lambda_nz_n(t)-B_nK_0z^N(t)+f_n(t),\quad n>N.\end{aligned} \tag{2.14}$$

For L^2-stability analysis of the closed-loop system (2.14), we consider the Lyapunov function

$$V(t)=\left|z^N(t)\right|_P^2+\sum_{n=N+1}^{\infty}z_n^2(t), \tag{2.15}$$

where $0<P\in\mathbb{R}^{N\times N}$. Differentiation of $V(t)$ along the solution of (2.14) gives

$$\begin{aligned}&\dot{V}(t)+2\delta V(t)=\left(z^N(t)\right)^T[P(A_0-B_0K_0)\\&+(A_0-B_0K_0)^TP+2\delta P]\,z^N(t)+2\left(z^N(t)\right)^TPf^N(t)\\&+2\sum_{n=N+1}^{\infty}(-\lambda_n+\delta)z_n^2(t)+2\sum_{n=N+1}^{\infty}z_n(t)f_n(t)\\&-2\sum_{n=N+1}^{\infty}z_n(t)B_nK_0z^N(t).\end{aligned} \tag{2.16}$$

By using the Young inequality, we have

$$\begin{aligned}&2\sum_{n=N+1}^{\infty}z_n(t)f_n(t)=2\sum_{n=N+1}^{\infty}\left(\sqrt{\sigma}z_n(t)\right)\left(\frac{1}{\sqrt{\sigma}}f_n(t)\right)\\&\le\sigma\sum_{n=N+1}^{\infty}z_n^2(t)-\frac{1}{\sigma}\left|f^N(t)\right|^2+\frac{1}{\sigma}\sum_{n=1}^{\infty}f_n^2(t).\end{aligned} \tag{2.17}$$

Then, from Parseval's equality, we obtain

$$\begin{aligned}&\frac{1}{\sigma}\sum_{n=1}^{\infty}f_n^2(t)\overset{(2.7)}{=}\frac{1}{\sigma}\int_0^1f^2(z(x,t))z^2(x,t)dx\\&\overset{(2.3)}{\le}\sigma\int_0^1z^2(x,t)dx=\sigma\left|z^N(t)\right|^2+\sigma\sum_{n=N+1}^{\infty}z_n^2(t).\end{aligned} \tag{2.18}$$

Combining (2.17) and (2.18), we have

$$\begin{aligned}&2\sum_{n=N+1}^{\infty}z_n(t)f_n(t)\le2\sigma\sum_{n=N+1}^{\infty}z_n^2(t)\\&+\sigma\left|z^N(t)\right|^2-\frac{1}{\sigma}\left|f^N(t)\right|^2.\end{aligned} \tag{2.19}$$

Similarly, by the Young inequality

$$\begin{aligned}&-2\sum_{n=N+1}^{\infty}z_n(t)B_nK_0z^N(t)\\&\le\alpha\sum_{n=N+1}^{\infty}z_n^2(t)+\frac{1}{\alpha}\sum_{n=N+1}^{\infty}\left|B_nK_0z^N(t)\right|^2,\end{aligned} \tag{2.20}$$

where $\alpha>0$. Considering the last term on the right-hand side, we have

$$\begin{aligned}&\frac{1}{\alpha}\sum_{n=N+1}^{\infty}\left|B_nK_0z^N(t)\right|^2\\&=\frac{1}{\alpha}\left(K_0z^N(t)\right)^T\left[\sum_{n=N+1}^{\infty}B_n^TB_n\right]K_0z^N(t)\\&=\frac{1}{\alpha}\|b\|_N^2\left|K_0z^N(t)\right|^2,\ \|b\|_N^2=\left[\sum_{i=1}^m\|b_i\|_N^2\right],\end{aligned} \tag{2.21}$$

where we use (1.3) and

$$\sum_{n=N+1}^{\infty}\sum_{i=1}^mb_{n,i}^2\overset{(2.7)}{=}\sum_{i=1}^m\|b_i\|_N^2.$$

Let $\eta(t)=\mathrm{col}\left\{z^N(t),f^N(t)\right\}$. From (2.16)–(2.21), we have for $t\ge0$

$$\dot{V}(t)+2\delta V(t)\le\eta^T(t)\Phi^{(1)}\eta(t)+2\sum_{n=N+1}^{\infty}\Upsilon_n^{(1)}z_n^2(t)\le0, \tag{2.22}$$

provided $\Upsilon_n^{(1)}=-\lambda_n+\delta+\sigma+\frac{\alpha}{2}<0,\ n>N$ and

$$\begin{aligned}&\Phi^{(1)}=\begin{bmatrix}\phi^{(1)}+\sigma I+\frac{1}{\alpha}\|b\|_N^2K_0^TK_0 & P\\ * & -\sigma^{-1}I\end{bmatrix}<0,\\&\phi^{(1)}=P(A_0-B_0K_0)+(A_0-B_0K_0)^TP+2\delta P.\end{aligned} \tag{2.23}$$

From monotonicity of $\lambda_n,\ n\in\mathbb{N}$ we have $\Upsilon_n^{(1)}<0,\ n>N$ iff

$$\Upsilon_{N+1}^{(1)}=-\lambda_{N+1}+\delta+\sigma+\tfrac{\alpha}{2}<0. \tag{2.24}$$

To obtain equivalent LMIs for the design of the gain K_0, we multiply $\Phi^{(1)}$ from the left and right by $\mathrm{diag}\left\{P^{-1},I\right\}$. Then, introducing the notations

$$P^{-1}=Q,\ Y=P^{-1}K_0^T=QK_0^T \tag{2.25}$$

and applying Schur complement, it can be seen that (2.23) holds iff

$$\Phi^{(2)} = \begin{bmatrix} \phi^{(2)} & Q & Y \\ * & -\sigma^{-1}I & 0 \\ * & * & -\frac{\alpha}{\|b\|_N^2} \end{bmatrix} < 0, \tag{2.26}$$

$$\phi^{(2)} = A_0Q + QA_0^T - YB_0^T - B_0Y^T + 2\delta Q + \sigma I.$$

In particular, (2.24) and (2.26) are LMIs in Q, Y and α. If (2.24) and (2.26) are feasible, the controller gain is obtained by $K_0 = Y^TQ^{-1}$. Finally, we show that (2.23) and (2.24) are always feasible for small enough $\sigma > 0$. First, by (2.8) we can fix $\alpha > 0$ such that (2.24) holds. Then, by Assumption 1 we choose $K_0 \in \mathbb{R}^{m\times N}$ such that $A_0 - B_0K_0 + \delta I$ is Hurwitz. Let $P \in \mathbb{R}^{N\times N}$ satisfy

$$P(A_0 - B_0K_0 + \delta I) + (A_0 - B_0K_0 + \delta I)^TP = -\chi I \tag{2.27}$$

where $\chi > 0$ satisfies $-\chi I + \frac{1}{\alpha}\|b\|_N^2 K_0^TK_0 < 0$. Substituting (2.27) into (2.23) and applying Schur complement, we find that (2.23) holds iff

$$-\chi I + \frac{1}{\alpha}\|b\|_N^2 K_0^TK_0 + \sigma\left(I + P^2\right) < 0. \tag{2.28}$$

The latter clearly holds for small enough $\sigma > 0$. Summarizing, we arrive at:

Theorem 2.1. *Consider the heat equation (2.1) with a locally Lipschitz $f : \mathbb{R} \to \mathbb{R}$ satisfying (2.3) with some $\sigma > 0$, $b(x)$ given in (2.2) and the control law (2.10). Assume $z(\cdot, 0) \in H_0^1(0, 1)$. Let $\delta > 0$ be a desired decay rate. Let $N \in \mathbb{N}$ satisfy (2.8). Let there exist a scalar $\alpha > 0$ and matrices $0 < Q \in \mathbb{R}^{N\times N}$, $Y \in \mathbb{R}^{N\times m}$ such that the LMIs (2.24) and (2.26) hold. Let $K_0 = Y^TQ^{-1}$. Then the solution $z(x, t)$ to the closed-loop system (2.1), (2.10) satisfies*

$$\|z(\cdot, t)\| \le Me^{-\delta t}\|z(\cdot, 0)\| \tag{2.29}$$

for some $M > 0$, meaning that the closed-loop system is exponentially stable with a decay rate $\delta > 0$. The LMIs (2.24) and (2.26) are always feasible for small enough $\sigma > 0$.

Proof. Feasibility of the LMIs (2.24) and (2.26) implies $\dot{V}(t) + 2\delta V(t) \le 0$. By the comparison principle

$$V(t) \le e^{-2\delta t}V(0).$$

By (2.15) and Parseval's equality, we have

$$\sigma_{\min}(P)\|z(\cdot, t)\|^2 \le V(t) \le \sigma_{\max}(P)\|z(\cdot, t)\|^2 \tag{2.30}$$

Hence, (2.29) follows.

Remark 2.2. Differently from (2.15), one can also consider the Lyapunov function

$$V_1(t) = \left|z^N(t)\right|_P^2 + \sum_{n=N+1}^{\infty} \lambda_n z_n^2(t)$$

which is equivalent to the $H^1(0, 1)$ norm (see Lemma 1 in Katz and Fridman 2021c). Analysis similar to (2.16)–(2.29) will then result in LMI conditions for H^1-stability of the closed-loop system. For brevity, we present our results only for L^2-stability of the closed-loop system.

Remark 2.3. An immediate corollary of Theorem 2.1 is that if the LMIs (2.24) and (2.26) hold, then increasing the dimension of the controller (2.10) does not deteriorate the performance of the resulting closed-loop system. Indeed, let $K_0 = Y^TQ^{-1} \in \mathbb{R}^{m\times N}$ be obtained from the LMIs. Considering (2.10) with K_0 and N replaced by $\bar{K}_0 = \begin{bmatrix} K_0 & 0_{m\times 1}\end{bmatrix} \in \mathbb{R}^{m\times(N+1)}$ and $N + 1$, we obtain $u(t) = -K_0z^N(t)$. In particular, the resulting closed-loop system for $t \ge 0$ can be presented as (2.14). The same Lyapunov function (2.15) leads to the LMIs of Theorem 2.1. The latter are feasible by assumption.

2.2. Non-local stabilization with $m = N$ actuators — linear state-feedback

Here, we show that for (2.1) with $f : \mathbb{R} \to \mathbb{R}$ satisfying (2.3) with an arbitrarily large $\sigma > 0$, we can still obtain feasibility of (2.23) and (2.24), under additional assumptions and provided that N is large enough.

Recall A_0 and B_0, given in (2.9). We make the following assumptions:

Assumption 2. Assume that B_0 is invertible and satisfies

$$\left[B_0^{-1}\right]^T B_0^{-1} \le \beta_1 N^{\gamma_1} I \tag{2.31}$$

for some $\gamma_1, \beta_1 > 0$, independent of N.

Assumption 3. Assume that $\|b\|_N^2$, given in (2.21), satisfies

$$\|b\|_N^2 \le \beta_2 N^{\gamma_2} \tag{2.32}$$

for some $\beta_2, \gamma_2 > 0$, independent of N.

Below, we show that feasibility of (2.23) and (2.24) is guaranteed for large enough N, provided $\gamma_1 + \gamma_2 < 2$.

Remark 2.4. Note that Assumptions 2 and 3 are satisfied for the particular case $b_i = \phi_i$, $1 \le i \le N$, with ϕ_i given in (1.2). In this case $B_0 = I_N$ and $\|b\|_N^2 = 0$ for any $N \in \mathbb{N}$. In the case of *linear systems*, these $\{b_i\}_{i=1}^{\infty}$ were considered in the seminal paper (Curtain, 1982, Section D).

Remark 2.5. Subject to smoothness properties of the shape functions $\{b_i\}_{i=1}^{\infty}$, Assumption 3 can be verified using Fourier analysis. For example, if $\{b_i\}_{i=1}^{\infty}$ are functions of bounded variation with $\sup_{i\in\mathbb{N}} \mathrm{Var}(b_i) \le M$, where $M > 0$, then it is well known (Grafakos, 2008) that $|\langle b_i, \phi_n\rangle| \le \frac{M}{2\pi n}$ for any $i, n \in \mathbb{N}$. Therefore,

$$\|b\|_N^2 \le \frac{M^2}{4\pi^2}\sum_{i=1}^{N}\sum_{n=N+1}^{\infty}\frac{1}{n^2} \le \frac{M^2}{4\pi^2}\sum_{i=1}^{N}\frac{1}{N} \le \frac{M^2}{4\pi^2}.$$

Assumption 2 is related to the generalized Petrov–Galerkin finite-elements method (Reddy, 2010), where B_0 can be thought of as the stiffness matrix, corresponding to trial functions $\{b_i\}_{i=1}^{\infty}$ and basis functions $\{\phi_n\}_{n=1}^{\infty}$. Explicit characterization of functions $\{b_i\}_{i=1}^{\infty}$ which satisfy Assumption 2 is outside of the scope of this paper and is left for future research.

We show next that under Assumptions 2 and 3 with $\gamma_1+\gamma_2 < 2$ for any $\sigma > 0$ there always exists a value of N and appropriate $K_0 \in \mathbb{R}^{N\times N}$ such that (2.10) stabilizes (2.1)

Proposition 1. *Let Assumptions 2 and 3 hold with $\gamma_1 + \gamma_2 < 2$. Then for any $\sigma > 0$ and $\delta > 0$ there exists $N \in \mathbb{N}$ such that (2.10) with $K_0 = -kB_0^{-1}$ and $k \in R$, subject to*

$$-\lambda_1 - k + \delta + \sigma < 0, \tag{2.33}$$

exponentially stabilizes (2.1) with a decay rate $\delta > 0$.

Proof. Choosing $P = I$ in (2.23), applying Schur complement and taking into account Assumptions 2 and 3, we have that (2.23) holds iff

$$2\left(A_0 - [k - \delta - \sigma]I\right) + \frac{k^2\beta_1\beta_2N^{\gamma_1+\gamma_2}}{\alpha}I < 0. \tag{2.34}$$

Feasibility of (2.24) and (2.34) implies (2.29). Note that (2.34) holds provided

$$2\left(-\lambda_1 - k + \delta + \sigma\right) + \frac{k^2\beta_1\beta_2N^{\gamma_1+\gamma_2}}{\alpha} < 0. \tag{2.35}$$

Recall that we assume $\gamma_1+\gamma_2<2$. Choosing $\alpha=N^{\gamma_1+\gamma_2+\epsilon}$ such that $0<\epsilon<2-\gamma_1-\gamma_2$, (2.24) and (2.35) hold iff

$$
\begin{aligned}
&-\lambda_1-k+\delta+\sigma+\tfrac{k^2\beta_1\beta_2}{2N^{\epsilon}}<0,\\
&-\lambda_{N+1}+\delta+\sigma+\tfrac{N^{\gamma_1+\gamma_2+\epsilon}}{2}<0.
\end{aligned}
\tag{2.36}
$$

Since $\lambda_{N+1}=\pi^2(N+1)^2$, we see that (2.36) hold for large enough N. The minimal value of N which satisfies (2.36) can be estimated, given δ, σ, k, β_1, γ_1, β_2 and γ_2.

2.3. Non-local stabilization with $m = N$ actuators — nonlinear state-feedback

Here, we consider the nonlinear state-feedback

$$
u(t)=-B_0^{-1}\left(k_1z^N(t)+f^N(t)\right), \tag{2.37}
$$

where $f^N(t)$ is given in (2.13) and $k_1\in\mathbb{R}$ satisfies

$$
-\lambda_1-k_1+\delta<0. \tag{2.38}
$$

Note that the gain k_1 for the nonlinear state-feedback is smaller than k for the linear state-feedback (see (2.33)).

Remark 2.6. In order to use the controller (2.37), f in (2.1) must be known explicitly. Otherwise, one can only use the linear state-feedback $K_0=-kB_0^{-1}$ in Section B.

Using (2.37), the closed-loop system for $t\geq 0$ can be presented as

$$
\begin{aligned}
&\dot z^N(t)=(A_0-k_1I)z^N(t),\\
&\dot z_n(t)=-\lambda_nz_n(t)-k_1B_nB_0^{-1}z^N(t)\\
&\qquad -B_nB_0^{-1}f^N(t)+f_n(t),\quad n>N_0.
\end{aligned}
\tag{2.39}
$$

For L^2-stability analysis of the closed-loop system (2.39), we consider the Lyapunov function (2.15) with $P=\rho I$ for $0<\rho\in\mathbb{R}$. In comparison with the linear state-feedback case, the finite-dimensional part of the closed-loop system (2.39) is decoupled from the tail modes $\{z_n(t)\}_{n=N+1}^{\infty}$. The choice $P=\rho I$ in the Lyapunov functional will allow us to reduce the number of LMIs which guarantee exponential stability of the closed-loop system.

Differentiation of $V(t)$ along the solution of (2.39) and arguments similar to (2.16)–(2.21) give

$$
\begin{aligned}
&\dot V(t)+2\delta V(t)\leq 2\rho\left(z^N(t)\right)^T\Big[A_0-k_1I+\delta I\\
&+\tfrac{\sigma}{2\alpha_2\rho}I+\tfrac{k_1^2}{2\rho\alpha}\|b\|_N^2\left(B_0^{-1}\right)^TB_0^{-1}\Big]z^N(t)\\
&+\left(f^N(t)\right)^T\left[\tfrac{1}{\alpha_1}\|b\|_N^2\left(B_0^{-1}\right)^TB_0^{-1}-\tfrac{1}{\alpha_2\sigma}I\right]f^N(t)\\
&+2\sum_{n=N+1}^{\infty}\left(-\lambda_n+\delta+\tfrac{\alpha_2+\alpha_2^{-1}}{2}\sigma+\tfrac{\alpha}{2}+\tfrac{\alpha_1}{2}\right)z_n^2(t)
\end{aligned}
\tag{2.40}
$$

where we used

$$
\begin{aligned}
&-2\sum_{n=N+1}^{\infty}z_n(t)B_nB_0^{-1}f^N(t)\\
&\leq\alpha_1\sum_{n=N+1}^{\infty}z_n^2(t)+\tfrac{1}{\alpha_1}\|b\|_N^2\left|B_0^{-1}f^N(t)\right|^2
\end{aligned}
\tag{2.41}
$$

and, from arguments similar to (2.17)–(2.19),

$$
\begin{aligned}
&2\sum_{n=N+1}^{\infty}z_n(t)f_n(t)\leq\left(\alpha_2+\alpha_2^{-1}\right)\sigma\sum_{n=N+1}^{\infty}z_n^2(t)\\
&\qquad+\tfrac{\sigma}{\alpha_2}\left|z^N(t)\right|^2-\tfrac{1}{\alpha_2\sigma}\left|f^N(t)\right|^2.
\end{aligned}
\tag{2.42}
$$

Recall Assumptions 2 and 3. Taking into account (2.38) and using $\alpha=\frac{1}{\sqrt{\rho}}$, $\rho\to\infty$, $\dot V(t)+2\delta V(t)\leq 0$ holds provided

$$
\tfrac{\alpha_2\beta_1\beta_2N^{\gamma_1+\gamma_2}}{\alpha_1}-\tfrac{1}{\sigma}<0,\ \ -\lambda_{N+1}+\delta+\tfrac{\alpha_2+\alpha_2^{-1}}{2}\sigma+\tfrac{\alpha_1}{2}<0. \tag{2.43}
$$

Theorem 2.2. *Consider the heat equation (2.1) with a locally Lipschitz $f:\mathbb{R}\to\mathbb{R}$ satisfying (2.3) with some $\sigma>0$, $b(x)$ given in (2.2) with $m=N$ and the nonlinear control law (2.37) subject to (2.38) with a desired decay rate $\delta>0$. Assume $z(\cdot,0)\in H_0^1(0,1)$. Let Assumptions 2 and 3 hold with $\gamma_1+\gamma_2<4$. Given $N\in\mathbb{N}$, let there exist $0<\alpha_1,\alpha_2\in\mathbb{R}$ such that (2.43) hold. Then the solution $z(x,t)$ to (2.1) under the control law (2.37) satisfies (2.29) for some $M>0$. Moreover, for any $\sigma>0$, (2.43) are always feasible for an appropriate N.*

Proof. Taking into account $\gamma_1+\gamma_2<4$, let $\alpha_2=N^{-0.5(\gamma_1+\gamma_2+\epsilon)}$ and $\alpha_1=N^{0.5(\gamma_1+\gamma_2+\epsilon)}$, where $0<\epsilon<4-\gamma_1-\gamma_2$. Substituting into (2.43) gives $\frac{\beta_1\beta_2}{N^{\epsilon}}-\frac{1}{\sigma}<0$, $-\lambda_{N+1}+\delta+\frac{N^{\xi}+N^{-\xi}}{2}\sigma+\frac{N^{\xi}}{2}<0$, where $\xi=\frac{\gamma_1+\gamma_2+\epsilon}{2}<2$. Since $\lambda_{N+1}=\pi^2(N+1)^2$, the latter holds for large enough N, which can be explicitly estimated.

We want to compare (2.24), (2.35) (resulting from linear state-feedback) and (2.43) (resulting from nonlinear state-feedback) in terms of N required for feasibility of LMIs. We show next that nonlinear state-feedback leads to a value of N that is not larger than for the case of linear state-feedback, subject to the technical assumption $\sigma\geq 2\lambda_1$. This assumption is mild since in Sections B and C we are interested in feasibility for arbitrarily large values of σ. This result is consistent with (Karafyllis, 2021), where it was shown that a nonlinear controller performs better than a linear one. Theoretical analysis which characterizes scenarios in which the nonlinear controller performs better than the linear one is desirable. However, it is beyond the scope of this paper and remains a direction for future research.

Proposition 2. *Let $\sigma=(\mu+1)\lambda_1$, $\mu\geq 1$. Assume that $k\in\mathbb{R}$ satisfies (2.24)* and *(2.33), (2.35) hold for some N and $\alpha=\alpha_*$. Then (2.43) hold with N, $\alpha_2=1$ and $\alpha_1=\alpha_*$.*

Proof. It suffices to show that $\frac{\beta_1\beta_2N^{\gamma_1+\gamma_2}}{\alpha_*}<\frac{1}{\sigma}$. From (2.33) we have $\lambda_1+k-\delta-\sigma=\varepsilon\Rightarrow k=\mu\lambda_1+\delta+\epsilon$ for some $\varepsilon>0$. From (2.35) we obtain

$$
\begin{aligned}
&\tfrac{\beta_1\beta_2N^{\gamma_1+\gamma_2}}{\alpha_*}<\tfrac{2\epsilon}{k^2}=\tfrac{2\epsilon}{(\mu\lambda_1+\delta+\varepsilon)^2}\leq\tfrac{1}{\sigma}\\
&\iff 0\leq\varepsilon^2+2\epsilon\left(\delta-\lambda_1\right)+(\mu\lambda_1+\delta)^2.
\end{aligned}
\tag{2.44}
$$

The term on the right-hand side is a quadratic polynomial with discriminant $-4\left(\mu+1\right)\lambda_1\left(2\delta+[\mu-1]\lambda_1\right)\leq 0$.

3. Stabilization of a semilinear heat equation — boundary actuation

Inspired by Karafyllis (2021), where a trigonometric change of variables was suggested, we consider two cases of variables change for dynamic extension: trigonometric and polynomial.

3.1. Trigonometric change of variable

Here we consider the following nonlinear 1D heat equation

$$
\begin{aligned}
&z_t(x,t)=z_{xx}(x,t)+az(x,t)+f(z(x,t))z(x,t),\\
&z(0,t)=0,\quad z(1,t)=\textstyle\sum_{i=1}^m u_i(t)
\end{aligned}
\tag{3.1}
$$

where $a\in\mathbb{R}$ is the reaction coefficient and $m\in\mathbb{N}$. Inspired by Karafyllis (2021), let

$$
\psi_i(x)=(-1)^{i+1}\sin(\sqrt{\mu_i}x),\ \ \mu_i=\pi^2\left(i-\frac{1}{2}\right)^2,\ i\in\mathbb{N}. \tag{3.2}
$$

It can be easily verified that $\{\psi_i\}_{i=1}^{\infty}$ satisfy

$$\begin{aligned}&\psi_i''(x)+\mu_i\psi_i(x)=0,\\&\psi_i(0)=\psi_i'(1)=0,\ \ \psi_i(1)=1,\ \ \|\psi_i\|=\tfrac{1}{\sqrt{2}}.\end{aligned}\tag{3.3}$$

In particular, $\{\psi_i\}_{i=1}^{\infty}$ is an orthogonal family, being a sequence of eigenfunctions of a Sturm–Liouville problem. Furthermore, note that $\mu_i\neq\lambda_n$ for $n,i\in\mathbb{N}$. We consider the following dynamic extension

$$\begin{aligned}&w(x,t)=z(x,t)-\psi^T(x)u(t),\\&\psi(x)=\mathrm{col}\,\{\psi_i(x)\}_{i=1}^{m},\ \ u(t)=\mathrm{col}\,\{u_i(t)\}_{i=1}^{m}.\end{aligned}\tag{3.4}$$

Substituting (3.4) into (3.1) we obtain

$$\begin{aligned}&w_t(x,t)=w_{xx}(x,t)-\psi^T(x)\left(-\left(\Xi+aI\right)u(t)+\dot u(t)\right)\\&\qquad+f\left(w(x,t)+\psi^T(x)u(t)\right)\left[w(x,t)+\psi^T(x)u(t)\right],\\&w(0,t)=w(1,t)=0,\ \ \Xi=\mathrm{diag}\left\{-\mu_1,\ldots,-\mu_m\right\}.\end{aligned}\tag{3.5}$$

We will henceforth treat $u(t)$ as an additional state variable, subject to the dynamics

$$\dot u(t)=\left(\Xi+aI\right)u(t)+v(t),\ \ u(0)=0\tag{3.6}$$

where $v(t)\in\mathbb{R}^{m\times1}$ is the new control input. From (3.5) and (3.6) we obtain the following ODE-PDE system, which is equivalent to (3.1):

$$\begin{aligned}&\dot u(t)=\left(\Xi+aI\right)u(t)+v(t),\\&w_t(x,t)=w_{xx}(x,t)+aw(x,t)-\psi^T(x)v(t)\\&\qquad+f\left(w(x,t)+\psi^T(x)u(t)\right)\left[w(x,t)+\psi^T(x)u(t)\right],\\&u(0)=0,\ \ w(0,t)=w(1,t)=0.\end{aligned}\tag{3.7}$$

We present the solution to (3.7) as

$$w(x,t)=\sum_{n=1}^{\infty}w_n(t)\phi_n(x)\tag{3.8}$$

with $\{\phi_n\}_{n\in\mathbb{N}}$ defined in (1.2). Differentiating under the integral sign, integrating by parts and using (1.1) we have

$$\begin{aligned}&\dot w_n(t)=(-\lambda_n+a)w_n(t)-B_nv(t)+f_n^{(1)}(t)+f_n^{(2)}(t),\\&w_n(0)=\langle w(\cdot,0),\phi_n\rangle\end{aligned}\tag{3.9}$$

where

$$\begin{aligned}&B_n=\left[\langle\psi_1,\phi_n\rangle,\ldots,\langle\psi_m,\phi_n\rangle\right],\\&f_n^{(1)}(t)=\textstyle\int_0^1 f\left(w(x,t)+\psi^T(x)u(t)\right)w(x,t)\phi_n(x)dx,\\&f_n^{(2)}(t)=\textstyle\int_0^1 f\left(w(x,t)+\psi^T(x)u(t)\right)\psi^T(x)u(t)\phi_n(x)dx.\end{aligned}\tag{3.10}$$

By Parseval's equality and orthogonality of $\{\psi_i\}_{i=1}^{\infty}$

$$\begin{aligned}&\sum_{n=1}^{\infty}\left[f_n^{(2)}(t)\right]^2\\&=\textstyle\int_0^1\left|f\left(w(x,t)+\psi^T(x)u(t)\right)\psi^T(x)u(t)\right|^2dx\\&\le\sigma^2\left\|\psi^T(x)u(t)\right\|^2=\sigma^2\sum_{i=1}^{m}u_i^2(t)\left\|\psi_i\right\|^2\overset{(3.3)}{=}\tfrac{\sigma^2}{2}\left|u(t)\right|^2.\end{aligned}\tag{3.11}$$

Let $\delta>0$ be a desired decay rate and $N\in\mathbb{N}$ such that

$$-\lambda_n+a+\frac{3}{2}\sigma+\delta<0,\quad n>N.\tag{3.12}$$

Recall A_0 and B_0, given in (2.9), and let

$$\begin{aligned}&X(t)=\mathrm{col}\left\{u(t),w_1(t),\ldots,w_N(t)\right\},\\&F^{N,(j)}(t)=\mathrm{col}\left\{0_{m\times1},f_1^{(j)}(t),\ldots,f_N^{(j)}(t)\right\},\ j\in\{1,2\},\\&\bar A=\mathrm{diag}\left\{\Xi,A_0\right\}+aI,\ \ \bar B=\mathrm{col}\left\{I_{m\times m},-B_0\right\}.\end{aligned}\tag{3.13}$$

By Lemma 2.1 in Karafyllis (2021), the pair $(\bar A,\bar B)$ is controllable. Let the controller gain $\bar K\in\mathbb{R}^{m\times(m+N)}$ be obtained from LMIs (see (3.26) and (3.28)). We propose a controller of the form

$$v(t)=-\bar KX(t)\tag{3.14}$$

leading to the following closed-loop system for $t\ge0$:

$$\begin{aligned}&\dot X(t)=(\bar A-\bar B\bar K)X(t)+F^{N,(1)}(t)+F^{N,(2)}(t),\\&\dot w_n(t)=(-\lambda_n+a)w_n(t)+B_n\bar KX(t)\\&\qquad+f_n^{(1)}(t)+f_n^{(2)}(t),\quad n>N.\end{aligned}\tag{3.15}$$

Well-posedness of the closed-loop system (3.15) has been shown in Karafyllis (2021) (see Theorem 2.2 and Theorem 3.1 therein). In particular, given $z(\cdot,0)\in\mathcal{D}(\mathcal{A})$, (3.1) has a unique solution $z\in C([0,\infty)\times[0,1])\cap C^1((0,\infty)\times[0,1])$ such that $z(\cdot,t)\in C^2(0,1)$ for all $t>0$ and satisfies the boundary conditions in (3.1).

For L^2-stability analysis of the closed-loop system (3.15), we consider the Lyapunov function

$$V(t)=|X(t)|_P^2+\sum_{n=N+1}^{\infty}w_n^2(t)\tag{3.16}$$

where $0<P\in\mathbb{R}^{(N+m)\times(N+m)}$. Differentiation of $V(t)$ along the solution to (3.15) gives

$$\begin{aligned}&\dot V(t)+2\delta V(t)=X^T(t)\left[P(\bar A-\bar B\bar K)+(\bar A-\bar B\bar K)^TP\right.\\&\left.+2\delta P\right]X(t)+2X^T(t)PF^{N,(1)}(t)\\&+2X^T(t)PF^{N,(2)}(t)+2\sum_{n=N+1}^{\infty}\left(-\lambda_n+a+\delta\right)w_n^2(t)\\&+2\sum_{n=N+1}^{\infty}w_n(t)f_n^{(1)}(t)+2\sum_{n=N+1}^{\infty}w_n(t)f_n^{(2)}(t)\\&+2\sum_{n=N+1}^{\infty}w_n(t)B_n\bar KX(t).\end{aligned}\tag{3.17}$$

By using the Young inequality, we have

$$\begin{aligned}&2\sum_{n=N+1}^{\infty}w_n(t)f_n^{(1)}(t)\le\sigma\sum_{n=N+1}^{\infty}w_n^2(t)\\&\qquad-\tfrac{1}{\sigma}\left|F^{N,(1)}(t)\right|^2+\tfrac{1}{\sigma}\sum_{n=1}^{\infty}\left(f_n^{(1)}\right)^2(t).\end{aligned}\tag{3.18}$$

Then, from Parseval's equality, we obtain

$$\begin{aligned}&\sum_{n=1}^{\infty}\left(f_n^{(1)}\right)^2(t)=\textstyle\int_0^1 f^2(w(x,t)+\psi^T(x)u(t))w^2(x,t)dx\\&\overset{(2.3)}{\le}\sigma^2\textstyle\int_0^1 w^2(x,t)dx=\sigma^2\left|X(t)\right|_{\Lambda_1}^2+\sigma^2\displaystyle\sum_{n=N+1}^{\infty}w_n^2(t),\\&\Lambda_1=\mathrm{diag}\left\{0_{m\times m},I_{N\times N}\right\}.\end{aligned}\tag{3.19}$$

From (3.18) (3.19) we obtain

$$\begin{aligned}&2\sum_{n=N+1}^{\infty}w_n(t)f_n^{(1)}(t)\le2\sigma\sum_{n=N+1}^{\infty}w_n^2(t)\\&-\tfrac{1}{\sigma}\left|F^{N,(1)}(t)\right|^2+\sigma\left|X(t)\right|_{\Lambda_1}^2\end{aligned}\tag{3.20}$$

Similarly, we have

$$2\sum_{n=N+1}^{\infty} w_n(t)f_n^{(2)}(t) \le \sigma \sum_{n=N+1}^{\infty} w_n^2(t) - \tfrac{1}{\sigma}\left|F^{N,(2)}(t)\right|^2 + \tfrac{1}{\sigma}\sum_{n=1}^{\infty}\left[f_n^{(2)}(t)\right]^2 \overset{(3.11)}{\le} \sigma \sum_{n=N+1}^{\infty} w_n^2(t) - \tfrac{1}{\sigma}\left|F^{N,(2)}(t)\right|^2 + \tfrac{\sigma}{2}X^T(t)\Lambda_2 X(t), \quad \Lambda_2 = \mathrm{diag}\left\{I_{m\times m}, 0_{N\times N}\right\}. \tag{3.21}$$

Finally, by arguments similar to (2.20) and (2.21), we have

$$\sum_{n=N+1}^{\infty} w_n(t)B_n\bar{K}X(t) \le \alpha \sum_{n=N+1}^{\infty} w_n^2(t) + \tfrac{1}{\alpha}\left\|\psi\right\|_N^2\left|\bar{K}X(t)\right|^2 \tag{3.22}$$

where $\alpha > 0$ and $\|\psi\|_N^2 = \sum_{i=1}^{m}\|\psi_i\|_N^2$.

Remark 3.1. The term $\|\psi\|_N^2$ can be upper bounded. Indeed, integrating by parts twice, it can be easily verified that

$$\langle\psi_i, \phi_n\rangle = -\tfrac{1}{\mu_i}\left\langle\psi_i'', \phi_n\right\rangle = \tfrac{1}{\mu_i}\phi_n'(1) + \tfrac{\lambda_n}{\mu_i}\left\langle\psi_i, \phi_n\right\rangle.$$

Since $\mu_i \neq \lambda_n$ for $n, i \in \mathbb{N}$, we find that

$$\langle\psi_i, \phi_n\rangle = \tfrac{(-1)^n\sqrt{2\lambda_n}}{\mu_i - \lambda_n}. \tag{3.23}$$

In particular, we have

$$\|\psi_i\|_N^2 = 2\sum_{n=N+1}^{\infty}\tfrac{\lambda_n}{(\lambda_n-\mu_i)^2}$$

which can be upper bounded by using the integral test for series convergence.

Let $\eta(t) = \mathrm{col}\left\{X(t), F^{N,(1)}(t), F^{N,(2)}(t)\right\}$. From (3.17)–(3.22) we obtain

$$\dot{V}(t) + 2\delta V(t) \le \eta^T(t)\Phi^{(3)}\eta(t) + 2\sum_{n=N+1}^{\infty}\Upsilon_n^{(2)}w_n^2(t) \le 0, \tag{3.24}$$

provided $\Upsilon_n^{(2)} = -\lambda_n + a + \delta + \frac{3}{2}\sigma + \frac{\alpha}{2} < 0, \ n > N$ and

$$\Phi^{(3)} = \begin{bmatrix} \phi^{(3)} + \frac{1}{\alpha}\|\psi\|_N^2\bar{K}^T\bar{K} & P & P \\ * & -\sigma^{-1}I & 0 \\ * & * & -\sigma^{-1}I \end{bmatrix} < 0,$$
$$\phi^{(3)} = P(\bar{A} - \bar{B}\bar{K}) + (\bar{A} - \bar{B}\bar{K})^T P + 2\delta P + \sigma\Lambda_3,$$
$$\Lambda_3 = \Lambda_1 + \tfrac{1}{2}\Lambda_2 \tag{3.25}$$

hold. From monotonicity of $\lambda_n, \ n \in \mathbb{N}$ we have $\Upsilon_n^{(2)} < 0, \ n > N$ iff

$$\Upsilon_{N+1}^{(2)} = -\lambda_{N+1} + a + \delta + \tfrac{3}{2}\sigma + \tfrac{\alpha}{2} < 0. \tag{3.26}$$

To obtain equivalent LMIs for the design of the gain $\bar{K}$, we multiply $\Phi^{(3)}$ from the left and right by $\mathrm{diag}\left\{P^{-1}, I, I\right\}$. Using the notations

$$P^{-1} = Q, \ \bar{Y} = P^{-1}\bar{K}^T = Q\bar{K}^T, \tag{3.27}$$

noting that Λ_3, given in (3.25), is positive definite and applying Schur complement, we find that (3.25) holds iff

$$\Phi^{(4)} = \begin{bmatrix} \phi^{(4)} & Q & \bar{Y} \\ * & -\sigma^{-1}\Lambda_3^{-1} & 0 \\ * & * & -\alpha\|\psi\|_N^{-2} \end{bmatrix} < 0,$$
$$\phi^{(4)} = \bar{A}Q + Q\bar{A}^T - \bar{B}\bar{Y}^T - \bar{Y}\bar{B}^T + 2\delta Q + 2\sigma I. \tag{3.28}$$

In particular, (3.26) and (3.28) are LMIs in Q, $\bar{Y}$ and α. If (3.26) and (3.28) are feasible, the controller gain is obtained by $\bar{K} = \bar{Y}^T Q^{-1}$. Finally, we claim that (3.25) and (3.26) are feasible, for small enough $\sigma > 0$. By (3.12), choose $\alpha > 0$ such that (3.26) holds. From controllability of $(\bar{A}, \bar{B})$ we can choose $\bar{K} \in \mathbb{R}^{m\times(m+N)}$ such that $\bar{A} - \bar{B}\bar{K} + \delta I$ is Hurwitz. Let $P \in \mathbb{R}^{(N+m)\times(N+m)}$ be such that (2.27) holds with A_0, B_0, K_0 replaced by $\bar{A}, \bar{B}, \bar{K}$, respectively, where $\chi > 0$ satisfies $-\chi I + \frac{1}{\alpha}\|\psi\|_N^2\bar{K}^T\bar{K} < 0$. Substituting into (3.25) and applying Schur complement, (3.25) holds iff

$$-\chi I + \frac{1}{\alpha}\|\psi\|_N^2\bar{K}^T\bar{K} + \sigma\left[\Lambda_3 + 2P\right] < 0. \tag{3.29}$$

The latter holds for small enough $\sigma > 0$. Summarizing:

Theorem 3.1. *Consider (3.7) with a locally Lipschitz f satisfying (2.3) with some $\sigma > 0$ and the control law (3.14). Assume $z(\cdot, 0) \in \mathcal{D}(\mathcal{A})$. Let $\delta > 0$ be a desired decay rate. Let $N \in \mathbb{N}$ satisfy (3.12). Let a scalar $\alpha > 0$, matrix $0 < Q \in \mathbb{R}^{(N+m)\times(N+m)}$ and vector $\bar{Y} \in \mathbb{R}^{(N+m)\times 1}$ be such that the LMIs* (3.26) and (3.28) *hold. Then $u(t), w(x,t)$ to (3.7) under the control law (3.14) with $\bar{K} = \bar{Y}^T Q^{-1}$ satisfy*

$$u^2(t) + \|w(\cdot,t)\|^2 \le Me^{-2\delta t}\|w(\cdot,0)\|^2 \tag{3.30}$$

with some $M > 0$. The LMIs (3.26) and (3.28) *are always feasible provided $\sigma > 0$ is small enough.*

Remark 3.2. For boundary control we do not consider stabilization with $m = N$ and arbitrarily large $\sigma > 0$ for two reasons. First, in case of a linear controller, the chosen functions $\{\psi_i\}_{i=1}^{\infty}$ lead to B_0 (see (2.9), (3.10) and (3.13)) with entries (3.23). This matrix is a Hilbert-type matrix whose singular values decay exponentially fast. Therefore, (2.31) is not expected to hold for large $N \in \mathbb{N}$. Finding a change of variables (see (3.4)) that does not lead to a badly conditioned B_0, which satisfies (2.31), remains an open problem. Second, the use of a nonlinear boundary controller similar to (2.37) (with $u(t)$ replaced by $v(t)$) will introduce nonlinear terms into (3.6), which may destabilize it. Note that even if a nonlinear controller gives some improvement compared to a linear one in the case of non-local actuation, it needs knowledge of $f(z)$ and is more difficult for robustness analysis in the presence of delays (Fridman, 2014).

3.2. Polynomial change of variables

One can replace the change of variables (3.4) with the following standard dynamic extension

$$w(x,t) = z(x,t) - \sum_{j=1}^{m} x^j u_j(t). \tag{3.31}$$

For simplicity we briefly describe the case $m = 1$. The ODE-PDE system, resulting from (3.31) is given by

$$\begin{aligned} &\dot{u}(t) = v(t) \\ &w_t(x,t) = w_{xx}(x,t) + aw(x,t) - xv(t) + axu(t) \\ &\qquad + f\left(w(x,t) + xu(t)\right)\left[w(x,t) + xu(t)\right] \\ &u(0) = 0, \ w(0,t) = w(1,t) = 0. \end{aligned} \tag{3.32}$$

Presenting the solution to (3.32) as (3.8), we obtain

$$\dot{w}_n(t) = (-\lambda_n + a)w_n(t) + b_n\left(au(t) - v(t)\right) + f_n^{(1)}(t) + f_n^{(2)}(t), \quad w_n(0) = \langle w(\cdot,0), \phi_n\rangle, \ b_n = \tfrac{(-1)^{n+1}\sqrt{2}}{\sqrt{\lambda_n}}. \tag{3.33}$$

Let $\delta > 0$ be a desired decay rate and $N \in \mathbb{N}$ such that (3.12) holds. We introduce the notations

$$\tilde{A}_0 = \mathrm{diag}\left\{0_{1\times 1}, A_0\right\} + aI, \ B_0 = [b_1, \ldots, b_N]^T, \quad \tilde{B}_0 = \mathrm{col}\left\{1, -B_0\right\}, \ \bar{K}_a = \bar{K} + [a, 0_{1\times N}], \tag{3.34}$$

where A_0 is given in (2.9). We suggest a controller of the form (3.14), with $X(t)$ given in (3.13). Then, the closed-loop system for

$t \geq 0$ is given by

$$\begin{aligned}&\dot{X}(t)=(\tilde{A}_0-\tilde{B}\bar{K}_a)X(t)+F^{N,(1)}(t)+F^{N,(2)}(t),\\&\dot{w}_n(t)=(-\lambda_n+a)w_n(t)+b_n\bar{K}_aX(t)\\&\qquad+f_n^{(1)}(t)+f_n^{(2)}(t),\quad n>N.\end{aligned} \tag{3.35}$$

with $F^{N,(1)}(t)$ and $F^{N,(2)}(t)$ given in (3.13). For L^2-stability analysis of the closed-loop system (3.15), we consider the Lyapunov function (3.16). Then, by arguments similar to (3.17)–(3.22), it can be verified that

$$\dot{V}(t)+2\delta V(t)\leq \eta^T(t)\Phi^{(5)}\eta(t)+2\sum_{n=N+1}^{\infty}\Upsilon_n^{(2)}w_n^2(t)\leq 0, \tag{3.36}$$

with $\eta(t)$ and $\Upsilon_n^{(2)},\ n>N$ given before (3.24). The estimate (3.36) holds provided (3.26) and

$$\begin{aligned}&\Phi^{(5)}=\begin{bmatrix}\phi^{(5)}+\frac{2}{\alpha\pi^2N}\bar{K}_a^T\bar{K}_a & P & P\\ * & -\sigma^{-1}I & 0\\ * & * & -\sigma^{-1}I\end{bmatrix}<0,\\&\phi^{(5)}=P(\tilde{A}_0-\tilde{B}_0\bar{K}_a)+(\tilde{A}_0-\tilde{B}_0\bar{K}_a)^TP+2\delta P\\&\qquad+\sigma\ \mathrm{diag}\left\{\tfrac{1}{3},I_N\right\}.\end{aligned} \tag{3.37}$$

hold. Note that equivalent LMIs for the design of the gain $\bar{K}$ can be obtained by arguments similar to (3.27)–(3.28). Moreover, (3.26) and (3.37) are always feasible provided $\sigma>0$ is small enough.

The change of variables (3.31) can be used with $m\geq 2$. In this case, it can be easily verified that (3.11) is replaced by the following estimate

$$\begin{aligned}&\sum_{n=1}^{\infty}\left[f_n^{(2)}(t)\right]^2\leq\sigma^2\left\|\sum_{j=1}^{m}x^ju_j(t)\right\|^2\\&=\sigma^2u^T(t)Hu(T)\leq\sigma^2\left\|H\right\|\left|u(t)\right|^2,\ H=\left(\tfrac{1}{i+j+1}\right)_{i,j=1}^{m}.\end{aligned} \tag{3.38}$$

The matrix H is the well-known Hilbert matrix, which is positive-definite and highly ill-conditioned. Moreover, H approximates the "one-way infinite" Hilbert matrix (thought of as an operator on $\ell^2(\mathbb{N})$). It is known (Choi, 1983) that the norm of the latter is π. Therefore, we expect $\|H\|\approx\pi$. Hence, for large m, (3.38) likely leads to a more conservative upper bound than (3.11).

4. Numerical examples

We start with the case of non-local actuation (2.1) with either $m=1$ or $m=2$. Let $\delta=10^{-4}$. The LMIs of Theorem 2.1 were verified, using Matlab, for different values of $\sigma>0$ (which correspond to unstable open-loop systems). For $m=1$ we choose $b_1=\mathbb{1}_{[0.3,0.9]}$ (an indicator function). Note that differently from the spatial decomposition approach (see e.g. Fridman and Blighovsky 2012), the shape function b_1 does not cover the entire interval $[0,1]$. For $m=2$, we choose $b_i=\mathbb{1}_{[\frac{i-1}{2},\frac{i}{2}]},\ i\in\{1,2\}$.

For $m=N=1$, the LMIs were verified to be feasible for $\sigma=10$. The resulting gain K_0 of controller (2.10) is given by $K_0=8.7469$. The maximal value of σ for which the feasibility is preserved with $m=N=1$ is $\sigma=34$, with the gain $K_0=165.27$. For $m=N=2$, the LMIs were verified to be feasible for $\sigma=40$. The maximal value of for which feasibility is preserved with $m=N=2$ is $\sigma=72$. The resulting gains K_0 are given by

$$\sigma=40:\begin{bmatrix}120.3 & 109.05\\120.3 & -109.95\end{bmatrix},\ \sigma=72:\begin{bmatrix}1587.8 & 259.02\\1587.8 & -259.04\end{bmatrix}. \tag{4.1}$$

In the case of nonlinear control, the maximal values of σ which preserve the feasibility were $\sigma=35.2$ for $m=N=1$ and $\sigma=73.3$ for $m=N=2$. Thus, in this example, a nonlinear control allows for slightly larger values of σ. Note that for the case $m=N=2$, where the shape functions cover the interval $[0,1]$, the obtained results are similar to the ones obtained by the spatial decomposition approach.

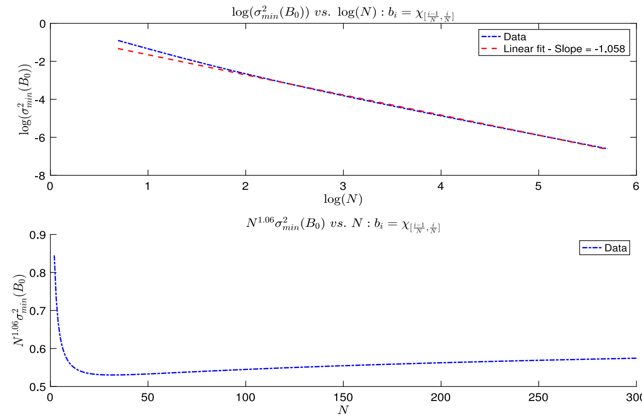

Fig. 1. Assumption 2 with $b_i^{(N)}=\mathbb{1}_{[\frac{i-1}{N},\frac{i}{N}]},\ 1\leq i\leq N$.

Table 1
Boundary actuation, $m=2$.

$\sigma_{\max}$	0.303	1.03	1.305	1.43	1.502	1.545	1.575
N	2	4	6	8	10	12	14

Next, we verify numerically the validity of Assumptions 2 and 3 with the shape functions $b_i^{(N)}=\mathbb{1}_{[\frac{i-1}{N},\frac{i}{N}]},\ 1\leq i\leq N$. For these functions, Assumption 3 holds trivially, since $\sum_{i=1}^{N}\left\|b_i^{(N)}\right\|_N^2=1-\sum_{i,n=1}^{N}\left|\left\langle b_i^{(N)},\phi_n\right\rangle\right|^2\leq 1$, meaning that $\beta_2=1$ and $\gamma_2=0$. Assumption 2 can be equivalently written as $\frac{1}{\beta_1}I\leq N^{\gamma_1}B_0^TB_0\iff\frac{1}{\beta_1}\leq N^{\gamma_1}\sigma_{\min}^2(B_0)$. We verify the right-hand side for $1\leq N\leq 300$. The results are given in Fig. 1.

The numerical results indicate that Assumption 2 holds with $\beta_1=2$ and $\gamma_1\approx 1.06$. In particular, $\gamma_1+\gamma_2<2$, meaning that the assumptions of Proposition 1 and Theorem 2.2 are satisfied.

Consider now the case of boundary actuation (3.1) with $a=5\pi^2$ and $m=2$ (Example 3.3 from Karafyllis, 2021). Let $\delta=10^{-4}$. The LMIs of Theorem 3.1 were verified for different values of N to obtain the maximal value of $\sigma>0$ which preserves the feasibility. The results are given in Table 1.

For $N=2$ we recover the upper bound in Karafyllis (2021). However, the latter was obtained using a nonlinear controller, whereas we use a simpler linear one (which does not require the nonlinearity to be known explicitly). Note that in Karafyllis (2021), the author assumes $m=N$ and proposes a controller which is based on inverting the matrix $(\langle\psi_i,\phi_n\rangle)_{i,n=1}^{N}$. As explained in Remark 3.2, the latter matrix is extremely ill-conditioned when N is large, which will likely lead to conservative bounds on σ. Our approach allows to obtain larger values of $\sigma_{\max}$ by increasing N while holding m fixed. This is because m and N are independent in our approach. As seen from Table 1 we achieve a value of σ which is five times larger than in Karafyllis (2021).

For $m=1$ we also compare the dynamic extensions (3.4) and (3.31) for $a=0$ and $a=3\pi^2$ to obtain the maximal value of σ which preserves feasibility of the LMIs. For $a=0$, (3.4) results in $\sigma_{\max}=6.03$, whereas (3.31) results in a slightly larger $\sigma_{\max}=6.33$. However, for a larger $a=3\pi^2$, (3.4) results in $\sigma_{\max}=0.42$, whereas (3.31) results in a smaller $\sigma_{\max}=0.31$. Thus, the dynamic extension (3.4) allows larger σ than (3.31) when a is not small.

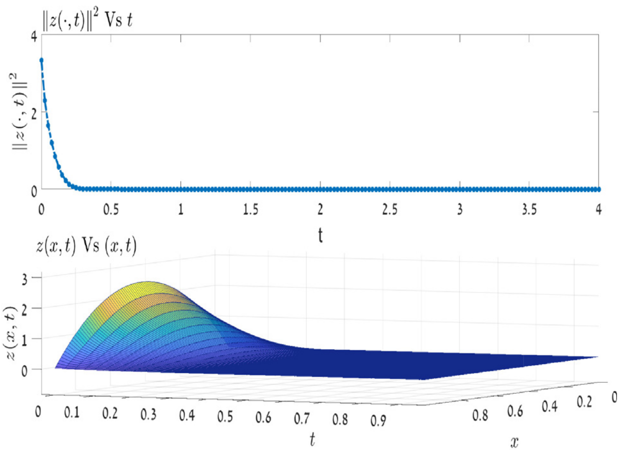

Fig. 2. $b_1^{(N)} = \mathbb{1}_{[0.3,0.9]},\ \ m = N = 1.$

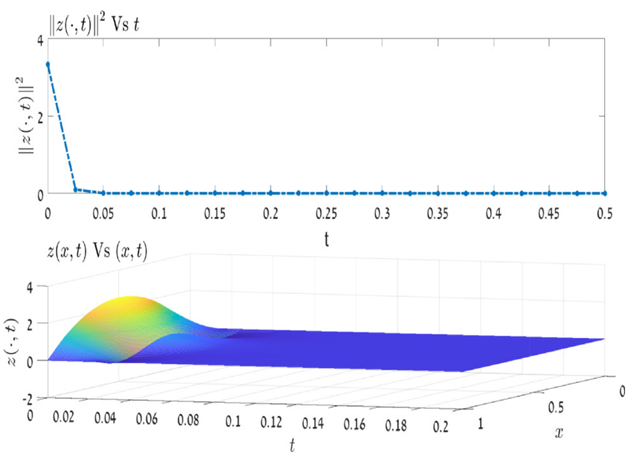

Fig. 3. $b_i = \mathbb{1}_{[\frac{i-1}{2},\frac{i}{2}]},\ \ i \in \{1,2\}\,,\ \ m = N = 2.$

For simulations of the closed-loop system, consider (2.1) with $f(z) = \sigma sin(z)$ and initial condition $z(x,0) = 10x(1-x)$. Let $\sigma = 10$ and $m = N = 1$. Let the shape function be $b_1 = \mathbb{1}_{[0.3,0.9]}$, the controller (2.10) gain be $K_0 = 8.7469$ as obtained from LMIs of Theorem 2.1. The PDE (2.1) and ODEs of $z^N(t)$ (see (2.14)) were simulated using the FTCS (Forward Time Centered Space) finite-difference scheme. The simulation results are given in Fig. 2 and confirm our theoretical analysis.

Stability of the closed-loop system with the same gain was preserved in simulations up to $\sigma = 16.7$, that may mirror a slight conservatism of our LMIs. Next, consider $\sigma = 40$ and $M = N = 2$. Let the shape functions be given by $b_i = \mathbb{1}_{[\frac{i-1}{2},\frac{i}{2}]},\ \ i \in \{1,2\}$. We use the controller (2.10) with gain (4.1) as obtained from LMIs of Theorem 2.1. Simulations of the PDE (2.1) and ODEs of $z^N(t)$ are given in Fig. 3. Stability with the same gain is preserved in simulations up to $\sigma = 110$.

Simulations of the closed-loop system under boundary actuation (as presented in Section 3) also confirm the theoretical results and are omitted due to space constraints.

5. Conclusion

A direct Lyapunov approach was suggested to global stabilization of 1D parabolic PDEs with a nonlinearity exhibiting a linear growth bound. The presented method can be extended in the future to various robust control problems for semilinear parabolic PDEs.

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